Plastic zone and effective distance under mixed mode fracture -
Volumetric approach-

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Abstract: Majority of the structures contain singularities, and that cause a high stress concentration. Near the root of defect, the singularity of stresses enables to activate strong plastic deformation. In the field of fracture mechanics, the investigation of plastic deformation at a notch tip under mixed mode loading is important to the development of understanding of material fracture and failure.

In this paper, a study was conducted using U-notched circular specimens subjected to compression load. The analysis consists in studying the region of high stress concentration under elastoplastic behaviors. Two local methods are used: the volumetric approach and Irwin models for different values of angle and notch radius under mixed-mode I+II.

At the notch root, there exists a plastic zone which affects the evolution of the crack at notch tip. The value of this plastic zone will be compared to effective distance generated by relative stress gradient. A new model are proposed to evaluate the plastic zone is proposed using the effective stress and notch stress intensity factor. The effective distance can be determined accepting that this distance is supposed to be larger than the plastic zone diameter.

Keywords: Effective distance, Irwin method, Mixed mode, Plastic zone, Relative stress gradient, Volumetric approach.

I. Introduction

Majority of the structures contain singularities, and that cause a high concentration of constraints. Near the root of defect, the singularity of stresses enables to activate strong plastic deformation. In the field of fracture mechanics, the investigation of plastic deformation at a crack tip under mixed mode loading is important to the development of understanding of material fracture and failure.

Plastic deformation at a crack tip in material produces a plastic zone around the crack which keeps radially decaying stresses away from the crack tip. Various theoretical analyses have been reported over the last few decades to gain a comprehensive understanding of the mixed mode fracture, as done by Golos and Wasiluk[1], Erdogan and Sih[2] proposed a criterion based on the maximum tangential stress, Sih[2, 3] presented a criterion based on the energy study, Theocaris et al.[4] used von Mises yield criterion for predicting the radius of plastic zone. But Irwin considered that the presence of a plastic zone at the bottom of crack, fact that the length of the crack behaves as if it was longer than its physical size and the stress distribution is equivalent to a crack elastic length (a+r) [5,6], so its effective length, \( a_{eff} \) is:

\[
a_{eff} = a + r_{eff} \quad \text{With} \quad R_p = 2 r_{eff}
\]

For simple estimation of the size of plastic zone along ‘θ’ equal to zero degree, considering a first approximation that plastic zone is circular with diameter \( R_p \), for a perfectly elastic plastic material, according to:

\[
R_p(\theta_0) = \lambda \left( \frac{K}{\sigma_{V}} \right)^2
\]

i.e., (1)

With: \( \sigma_{V} \) stress tangential, \( K \) stress intensity factor and \( \lambda \) varies between 0.30 – 0.39 (Irwin: 0.318 and Dugdale[7]: 0.342).

II. Materials and methodology

1. Material

The material studied is a high strength steel named 45CDS6 according to French standard. Mechanical properties are listed in Table 1.
TABLE 1 MECHANICAL PROPERTIES OF 45CDS6 STEEL

<table>
<thead>
<tr>
<th>E (MPa)</th>
<th>v</th>
<th>σy(MPa)</th>
<th>σu(MPa)</th>
<th>A%</th>
<th>Density (Kg/m³)</th>
<th>KIC(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>210065</td>
<td>0.28</td>
<td>1463</td>
<td>1662</td>
<td>2.8</td>
<td>7800</td>
<td>97</td>
</tr>
</tbody>
</table>

The microanalysis of the material gives the following chemical composition:

TABLE 2

<table>
<thead>
<tr>
<th>%C</th>
<th>%Mn</th>
<th>%Si</th>
<th>%Cr</th>
<th>%Mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>0.60</td>
<td>1.60</td>
<td>0.60</td>
<td>0.25</td>
</tr>
</tbody>
</table>

2. Specimens

The experiments have carried out by examining various U-notched circular ring specimens (see Fig.1.), with various geometries and boundary conditions (Fig.2.) [8].

With: external radius Re=20mm, internal radius Ri=10mm, thickness B=7mm, and notch length a=4mm.

Different notch radii are introduced using a wire-cutting electrical discharge machine and using wires of different diameter. The notch root radius was measured using a profile projector.

Five notch radius values are used: ρ = {0.15, 0.3, 0.5, 1, 2 mm}. With: 0° < α < 33° (mixed mode I+II). [9]

Fig.1. U-notched circular ring specimen

Fig.2. Loading mode of the specimen

The specimens are submitted to compression load in order to determine the critical loads when the fracture occurs. These loads are introducing to the simulation computation to finally evaluate the stress triaxiality evolution (Fig.3.).

I. FINITE ELEMENT ANALYSIS

The nonlinear finite element simulations are performed using ABAQUS 6.10[10]. The geometry of the U-notched circular was simplified by considering a plane part worthless thickness such as z=1.
To study the transitional stages of mode I to the mode II, it’s necessary to define new orientations of stress. It exist an angle corresponding to each mode of application of load named $\theta_0$ (fig.4)[15]. Method XFEM is very effective to study this kind of problem (applicable on Abaqus/CAE).

The following figure (fig.5) summarizes the values of the angles drawn on (Abaqus/CAE) and raised [10].

The numerical values obtained according to $\theta_0$ are compared to the experimental values measured by an optical microscope [4]. In mode II (for $\alpha=33^\circ$), for a radius notch $\rho=0$:

$\theta_0 \approx 70.35^\circ$

This value is compared to others results, such as: $\theta_0 \approx 70.39^\circ$ [8, 9] and $\approx 70.5^\circ$ [11], and $\approx 70.33^\circ$ [12].

The evolutions of stress with various radii are presented in the sections at the bottom, in mixed-mode (I+II) fracture crack initiation from notches is governed by the tangential stress. The stress evolutions are replotted versus the notch tip distance for each angle $\beta$, for ($\rho=1$) (Fig.6.). The maximum stress values decrease when the notch radius increases [15].

**II. EFFECTIVE DISTANCE AND VOLUMETRIC METHOD**

Stress distributions around the notch defect have been converted into so called notch stress intensity factor using the notch fracture mechanics and particularly the volumetric method.
The volumetric method is a local fracture criterion, which supposes that the fracture process requires a certain fracture volume. This volume is assumed as a cylinder with effective distance at its diameter. The elastic-plastic stress distribution along the ligament is plotted in the bi-logarithmic diagram as can be seen in figure 7.

Three distinct zones in the diagram can be distinguished:
- Zone I: the elastic-plastic stress opening stress increases and attains a peak value.
- Zone II: the elastic-plastic stress drops gradually in the elastic regime.
- Zone III: starts at a certain distance which is named the effective distance. It represents linear behaviour in the bi-logarithmic diagram.

![Fig.7. Schematic elastic-plastic stress distribution along notch ligament and stress intensity concept](image)

The notch stress intensity factor is defined as the function of effective distance and effective stress:

\[ K_p = \sigma_{eff} \sqrt{2 \pi X_{eff}} \]

i.e., (2)

By definition, the effective distance is the diameter of the process volume assuming it has a cylindrical shape. To determine this effective distance [4] studied the evolution of the function of the gradient relative of stress to the bottom of notch. This function represents a minimum corresponding to the effective distance \( X_{eff} \):

\[ \chi(r) = \frac{1}{\sigma_{xx}(r)} \frac{d \sigma_{xx}(r)}{dr} \]

i.e.,

\[ \chi(r) = \frac{1}{\sigma_{xx}(r)} \frac{d \sigma_{xx}(r)}{dr} \]

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Where: \( \chi(r) \) and \( \sigma_{xx}(r) \), are relative stress gradient and maximum principal stresses or crack opening stress, respectively.

Average volume of the stress distribution over the effective distance. However stresses are multiplied by a weight function in order to take into account the influence of stress gradient due to geometry and loading mode. The effective stress is defined as:

\[ \sigma_{eff} = \frac{1}{X_{eff}} \int_0^{X_{eff}} \sigma_{xx}(r)(1 - r \chi(r)) dr \]

i.e., (4)

### III. VON-MISES STRESS AND PLASTIC RADIUS

The criterion of plasticity Rp is defined by the point of intersection of the stress Von Mises and the yield stress of material.
This model of behavior of criterion of plasticity considers that the threshold from which the plastic flow develops, the constraint equivalent to the zone of development of rupture is in intersection with the yield stress (with $\sigma_y=1463$MPa).

The notch generates a concentration to the front of defect, which involves a fall of the resistance of the U-notched circular ring specimens. There is then risk of rupture. The values of $R_p$ then present an agreement with the distance where the material joined the mode of plasticity.

The examination of (Fig.8.) shows that the extent of the plastic zone at the bottom of notch varies according to the values of bifurcation angle.

![Figure 8](https://example.com/figure8.png)

**Figure 8** shows the distribution of Von Mises stress for different angles of notch: $0^\circ$, $10^\circ$, $18^\circ$, $25^\circ$ and $33^\circ$. The change in parameters of notch, radius and angle, changes the morphology of plastic zone near the bottom of notch.

### III. Results and discussion

#### 1. $R_p$ and $R_p^{\infty}(\theta_o)$ analytical

The parameters introduced into the elastoplastic zone $R_p(\theta_o)$ defined by Irwin i.e., (1) are valid only in case of a crack ($p=0$ and $\psi=0$). After results analysis and for a perfectly elastoplastic material according to a rupture under notch effect, $R_p(\theta_o)$ can be proposed and written in the following form:

$$R_p^{\infty}(\theta_o) = \frac{1}{2\pi} \left( \frac{K_p}{\sigma_{eff}} \right)^2$$

i.e., (5)

For critical loads, the expression i.e., (5) will be written:

$$R_p^{\infty}(\theta_o) = \frac{1}{2a} \frac{K_p}{\sigma_{eff}}$$

i.e., (6)

With: $K_p$ critical stress intensity factor, $\sigma_{eff}$ critical effective stress.

By the analytical method i.e., (2) and i.e., (6), we calculate the values of $R_p^{\infty}(\theta_o)$, and its well be compared with the values of $R_p$ by Von Mises yield criterion. The table 3 summarizes the various values of $R_p$ and $R_p^{\infty}(\theta_o)$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\rho$</th>
<th>$\sigma_{eff}$</th>
<th>$R_p$</th>
<th>$R_p^{\infty}(\theta_o)$</th>
<th>$R_p$</th>
<th>$R_p^{\infty}(\theta_o)$</th>
</tr>
</thead>
</table>

![Table 3](https://example.com/table3.png)

**Table 3**

Comparison of plastic zone by Von Mises criterion and analytical method.

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Table 3 shows the evolution of the plastic zone by the two methods (analytical and by the constraint of Von Mises) for various specimens. It is shown clearly that the distribution varies with a weak variation, but the values are very close, and the results are in agreement.

\[ \frac{R_p}{R_p(0)} \approx 1 \quad \rightarrow \quad R_p \approx R_p^*(0) \]

2. \( R_p \) and effective distance

The values of effective distance \( X_{eff} \) determined according to the procedure described by the i.e., (3). To interpret the various criteria for \( R_p \) and their convergence towards the determination of the effective distance \( X_{eff} \) to the bottom of the notch, the table 4 summarizes all those values.

| Mode | \( \rho \) | \( X_{eff} \) | \( R_p(\text{mm}) \) | \( X_{eff}/R_p \) | Mode | \( \rho \) | \( X_{eff} \) | \( R_p(\text{mm}) \) | \( X_{eff}/R_p \) |
|------|------|------|------|------|------|------|------|------|------|------|
| 0.15 | 0.585 | 0.585 | 1.0  | | 0.15 | 0.635 | 0.564 | 1.1  | |
| 0.3  | 0.6004 | 0.6004 | 1.0  | | 0.3  | 0.548 | 0.513 | 1.1  | |
| \( \alpha = 0^\circ \) | 0.5  | 0.6603 | 0.57 | 1.2 | \( \alpha = 10^\circ \) | 0.5  | 0.665 | 0.532 | 1.3  | |
| 1    | 0.7202 | 0.5401 | 1.3  | | 1    | 0.669 | 0.729 | 0.9  | |
| 2    | 1.0199 | 0.84  | 1.2  | | 2    | 0.943 | 0.794 | 1.2  | |
| \( \alpha = 18^\circ \) | 0.5  | 0.6603 | 0.699 | 0.9 | \( \alpha = 25^\circ \) | 0.5  | 0.665 | 0.697 | 1.0  | |
| 1    | 0.7202 | 0.778 | 0.9  | | 1    | 0.669 | 0.95  | 0.7  | |
| 2    | 1.0199 | 1.04  | 1.0  | | 2    | 0.943 | 1.222 | 0.8  | |
| \( \alpha = 33^\circ \) | 0.5  | 0.694 | 0.595 | 1.2 | | 0.3  | 0.312 | 0.312 | 1.0  | |
| 1    | 0.734 | 0.825 | 0.9  | | 2    | 1.283 | 1.433 | 0.9  | |

The examination of the evolution of the various values shows that the size of the plastic zone increases with the radius of notch (table 4)[14]. The analyses above (Fig.9.), assume that the zone of fracture process is larger than the plastic zone (for \( \alpha = 10^\circ \) and \( \rho = 2 \text{mm} \)).

In both cases, the values of \( X_{eff} \) are considered to be larger than the plastic zone; the fracture process zone is covering the plastic zone.

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IV. Conclusion

In order to evaluate the elastoplastic zone, the modeling of a problem is carried out by using two methods to describe it; by using Von Mises’s stress analysis and by analytical method; good agreement between the two forms of the plastic zone were achieved. Furthermore, the result of analysis shows that the effective distance \( X_{\text{eff}} \) may be determined by taking into account the following principles:

- The effective distance \( X_{\text{eff}} \) can be determined accepting that this distance is supposed to be larger than the plastic zone diameter.
- The effective distance is the limit of the most highly stressed zone.

Moreover, the calculation of the size of the plastic zone depends mainly on the type of specimen, the radius and angle of notch too.

References

10. Abaqus/CAE 6.10 Logiciel d’éléments finis développé pour la visualisation et de modélisation pour les dits solveurs.

Fig.9. Comparison between effective distance \( X_{\text{eff}} \) and plastic zone \( R_p \)