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Behavior of R.C. Beams with Inclined Cantilever

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Abstract: *In this study, equilibrium configurations of a cantilever beam subjected to an end load with angle of inclination is investigated. It is shown analytically that if the beam is sufficiently flexible, there are multiple equilibrium solutions for a specific beam and loading condition. A method is also presented for the determination of these deflected configurations. The cantilever beam studied in this research is considered to be initially straight andprismatic in addition to being homogeneous, elastic, and isotropic. The procedure outlined in this paper is utilized to show that for each combination of load and beam parameters, there is certain number of equilibrium configurations for a cantilever beam. The ranges of these combinations, along with some examples of the deflected shapes of the beams, are provided for several load inclination angles.*

I. Introduction

A cantilever beam is a beam that is only supported on one of its ends. Cantilever construction is popular in many kinds of architectural design and in other kinds of engineering, where professionals use terms like end load, intermediate load, and end moment to identify how much a cantilever beam will hold. The term moment is related to torque and to a theoretical load on a beam.

In residential architecture, cantilever beam design is often used for creating balconies and other extensions above ground level. Famous architects like Frank Lloyd Wright [(3)] were known to take advantage of cantilever beam_construction to provide for parts of a building that protrude from a supported section. The use of cantilever beam setups and similar cantilever engineering is also often seen in bridges and similar projects. Carpenters might think of cantilever beam_design in terms of wooden beams, but in other kinds of projects a cantilever design is applied to a concrete_slab or a metal girder.

A specific use of cantilever beam construction is often part of a temporary construction project. While a bridge or other project is in transition, cantilever design provides for structural integrity while only one side of a beam is supported Later, that beam may be supported on both sides. Engineers can show diagrams of how cantilever design will help ensure stability mid-way through a building project as part of a safety study.

Architects and engineers also use cantilever beam structures for the overhangs that are often a part of various buildings. Airports, university campuses, office complexes, and other areas will often include exterior structures that use cantilever beam construction for different intentions in building_design. These overhanging elements can provide shelter from the elements or a decorative aspect to a building. In some cases, an "open style" cantilever design fits into a modern or artistic design for a space, where planners have blended practicality with aesthetic appeal.

Those who are interested in observing how cantilever design is used in modern engineering can easily find many visible instances of this engineering method at work in their local communities. A detailed study of this kind of design can better prepare a student for entrance into an engineering or architectural program. It can also increase a student's understanding of how professionals implement this kind of design to both residential and commercial projects.

The ultimate load considered as the ultimate capacity of the section to carrying bending or shears which is critical equ. The theoretical values of the cracking shear load (Qcrth) can be determined according to (Egyptian code 2002) [(1)].

$Q\ th = 0.75\sqrt{f_c/\gamma_c}$ kg/cm^2 (1)

$q_{ush} = 2.2\sqrt{f_c/\gamma_c}$ kg/cm^2 (2)

The equation is according ACI Code (1995&2002) [(2)] the theoretical values of the ultimate load (P_{uth}) can be determined according to the smallest value of the following cases (a), or (b) as follows.

(a) - Due to bending by (ACI Code 1995&2002)

$Puth = 2.7\ fy\ (1 - 0.9\ fy/105)$ (3)

(b) Due to shear by (ACI Code 1995&2002)

$P_{uth} = 1.6 * \sqrt{f'_c} * b * d$ (4)

Then the critical theoretical values of the ultimate load (Puth) can be determined due to bending.
The theoretical values of the cracking load (Pcrth) can be determined
According to (ACI Code 318-1995& 2002) & ACI 363R-92.
Where Pcth = 2.5Mcr, Mcr = (fctr .Ig)/yct, fctr =0.94√*f*/ c Mpa.

I. Experimental Work

Experimental tests have been carried out on rectangular reinforced concrete beams under static loading up to failure. The study takes into consideration the following parameters:
1-Percentage of longitudinal tension reinforcement (ρ)
2- Inclination (α) of cantilever in tested beam

The above mentioned parameters were chosen to declare and to determine their effects on the general behavior of strength reinforced concrete beams under static loading particularly on their shear, flexural and torsion states in the well-known stages of the loading up to failure.

Ten reinforced concrete beams were prepared. They have normal strength of concrete (f_c = 250) and with straight cantilever (α=0.0°) and inclined cantilevers (α= 30°, 45°, 60°,90°)the main steel percentage of tested beam were A_{s1} = 2Φ16mm (ρ = 1.24) and A_{s2} = 4Φ16mm(ρ = 1.24) for all beam.

II. Materials

2.1 Concrete

Concrete mixes design was made to produce concrete having 28 days' strengths of about 250kg/cm^2. The mix proportions by weight are represented in table (1).
For normal strength concrete (f_c = 250 kg/cm^2), the constituent materials were:

a- Ordinary Portland cement, its properties agree with ECP 203.
b- Local gravel; the used gravel was 20 mm nominal maximum size, 2.65 specific gravity, and 1.66 t/m^3 volume weight.
c- Local sand; the used sand is a medium type one which has a specific gravity and volume weight of 2.65 and 1.65 t/m^3 respectively. The sieve analysis of the used sand and gravel are given in tables(1)
d- Potable water was used.

Table (1), Concrete mix proportion.

concrete	Amount of constituent materials/m^3					
	Cement (kg)	Sand (kg)	Gravel. (kg)	Water (liter)	f_{cu}**(kg/cm^2)**	
					7days	**28days**
C	350	650	1350	140	200	286

2.2 Tested beams

Tested beams consisted of ten simply supported concrete beams with cantilever. These beams were arranged in five groups (A, B, C, Dand E) Beams of groups A, B, C, D and E having a variable length resulting the cantilever inclination (α). All beams having an overall depth of 30 cm and 12 cm width. The beam width was deliberately kept constant at 12cm to maintain the same fracture energy for all beam specimens. This is because the surface layer of a beam contains a lower content of large aggregates and a relatively higher content of mortar. Therefore, the crack propagation will occur farther in the surface than in the interior.

Group (A, B, C, D And E)

This group includes ten beams having the same concrete compressive strength (f_{c28}), horizontal web reinforcement ratio (ρ_h), and vertical web reinforcement ratio (ρ_v) group (A) includes two beams having straight cantilever(α=0.0°) and groups (B,C,D and E) having different inclined cantilever with angle (α) equal to (30°, 45°, 60°and 90°)respectively .All groups (A, B, C, D and E**)** having pairs of beam with different longitudinal steel ratio (ρ) A_{s1} = 2Φ16mm (ρ = 1.24) and A_{s2} = 4Φ16mm(ρ = 1.24).
Complete details for all the tested beams are presented in table (2) as well as in Figs. from (1) to (10

Table (2), Details of the tested beams .

Series No.	specimen			beams (cm)	cantilevers (cm)	anchorage (cm)	web reinforcement ratio			F_c
	No	α	As	L_b	L_c	l_b	ρ	ρ_v	ρ_h	Fc (kg / cm2)
A	A1	0.0°	2ϕ16	100	50	10	1.24	0.16	0.35	295
	A2		4ϕ16				2.48			
B	B1	30°	2ϕ16	100	54	10	1.24	0.16	0.35	290
	B2		4ϕ16				2.48			
C	C1	45°	2ϕ16	100	68	10	1.24	0.16	0.35	285
	C2		4ϕ16				2.48			
D	D1	60°	2ϕ16	100	100	10	1.24	0.16	0.35	275
	D2		4ϕ16				2.48			
E	E1	90°	2ϕ16	100	50	10	1.24	0.16	0.35	285
	E2		4ϕ16				2.48			

Where:

L_b Length of the beam. l_b Length of anchorage length beyond the support. ρ longitudinal main steel ratio.
L_c Length of the cantilever f_c Concrete compressive strength, average of 3 cubes α the angle of inclination
ρ_v, ρ_h Web reinforcement ratio (vertical and horizontal), respectively.

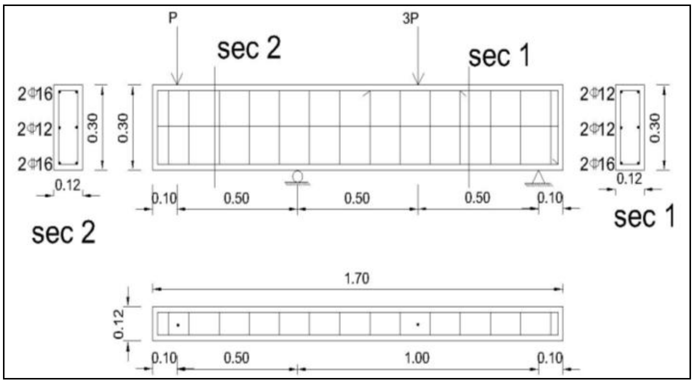


Fig. (1): Details of beams A1 (α=0), As = 2Φ16mm (ρ=1.24)

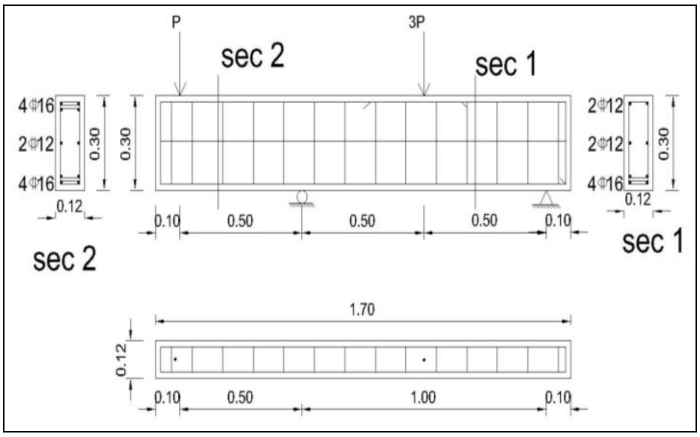


Fig. (2): Details of beams A2 (α=0), As = 4Φ16mm (ρ=2.48)

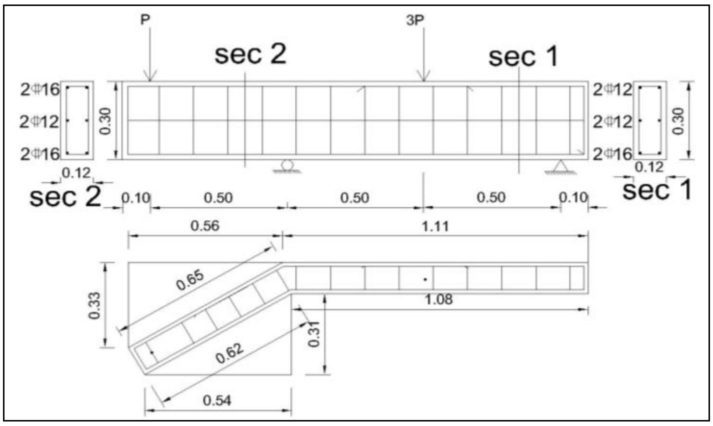


Fig.(3): Details of beams B1(α=30), As = 2Φ16mm(ρ=1.24)

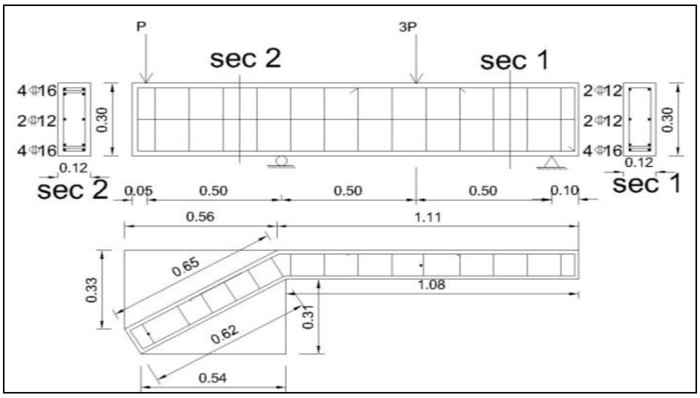


Fig.(4): Details of beams B2(α=30), As = 4Φ16mm(ρ=2.48)

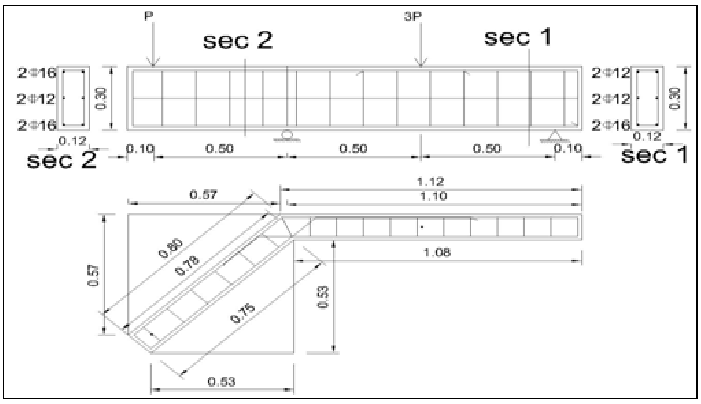


Fig. (5): Details of beams C1 (α=45), As = 2Φ16mm (ρ=1.24)

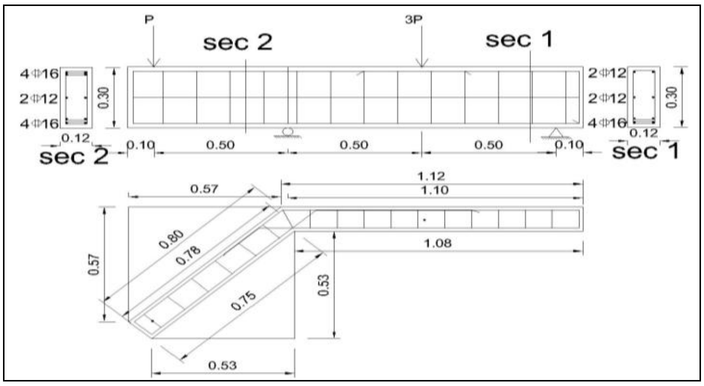


Fig. (6): Details of beams C2 (α=45), As = 4Φ16mm (ρ=2.48)

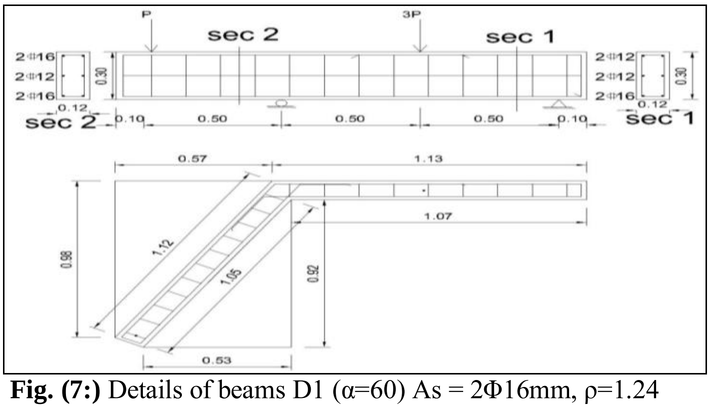


Fig. (7:) Details of beams D1 (α=60) As = 2Φ16mm, ρ=1.24

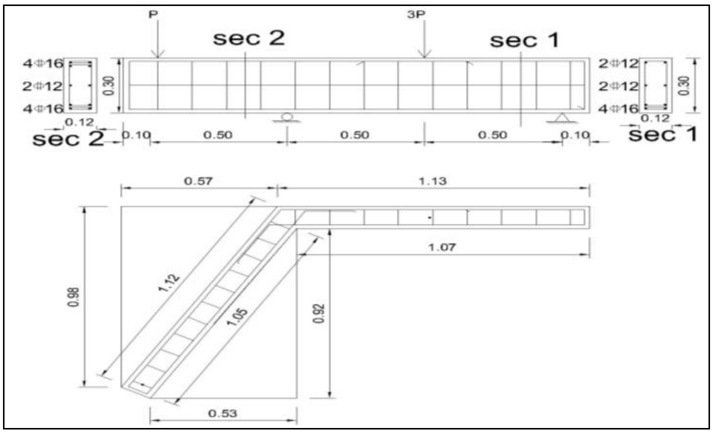


Fig. (8): Details of beams D2 (α=60) A_s = 4Φ16mm, ρ=2.48

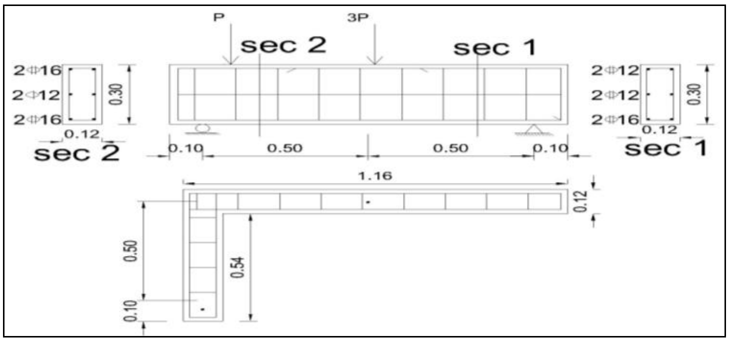


Fig. (9): Details of beams E1 (α=90) As = 2Φ16mm, ρ=1.24

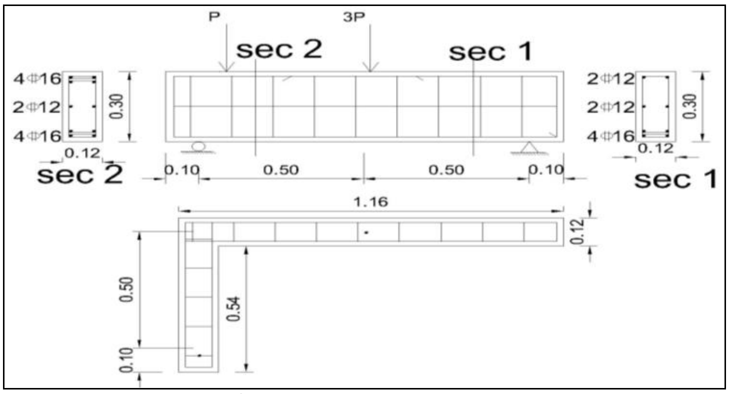


Fig. (10): Details of beams E2 (α=90) As = 4Φ16mm, ρ=2.48

2.3 Test procedure

All beams with cantilever were tested under two point static loading one of the loads at cantilever end and other load effect on the mid span of tested beams the loads was increased up to (0.5, 1.5) tons respectively then; the load were applied in increments each of (0.5, 1.5) ton for the tested beam with cantilever concrete the load was kept constant every two successive increments for five minutes. During this period, the mid span deflection was recorded, cracks propagation was traced, and reading of strain gauges were recorded. For each beam, the total duration of loading up to failure was different depending upon the inclination beam cantilever (α), percentage of longitudinal reinforcement ratios (ρ).

2.3 Measured deformation of beams:

Strains of concrete and steel were measured by means of electrical strain gauges at the shown positions in Fig (11).The gauge length was 52mm, and the 800mm resistance was 600 ohms and gauge factor (2 ± 0.75%) . Strain gauges were connected to strain indicator with its box resistance.

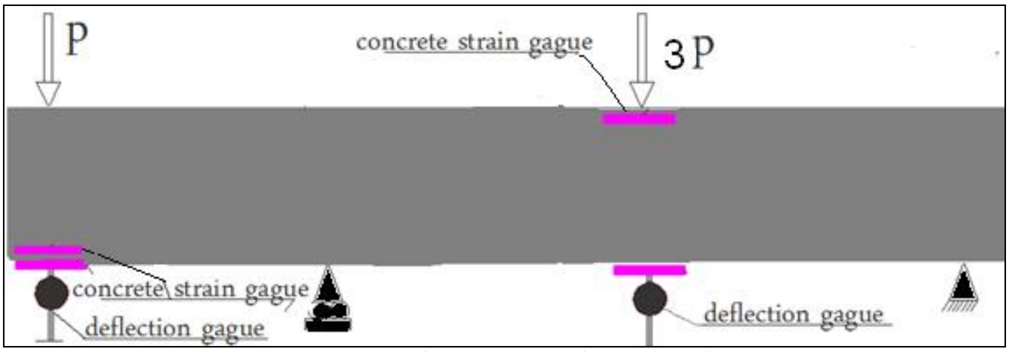


Fig. (11). Method of measuring deformation of beams.

III. Test Result

3.1 Crack pattern and mode of failure

The crack pattern and mode of failure are explained for the tested reinforced concrete beams with inclined cantilever.

All beams failed in shear, in spite of the different amount of reinforcement(ρ) that considered in the tests, in the early stages of loading, no flexural cracks were observed in the region of bending moment as the applied load increased. With a further increase of load, diagonal cracks formed in the shear span area and developed towards the loading points and supports.

It is worth mentioning that for all beams which failed in diagonal tension, the inclination of the major crack making an angle between 38° to 63° for all beams.

The failure modes of beams in groups A to E are presented in table (3); the most common failure for the tested beams is a diagonal tension failure. The shear failure in beams is always initiated by splitting action Diagonal tension was observed in beams of groups (A, B, C, D, and E) at critical shear zone between the cantilever span and mid span of beams .

The amount of longitudinal steel ratio (ρ) has no effect on final mode of failure. The presence of stirrups is very essential to resist shearing stresses. Consequently, it has a considerable effect on pattern of cracks and modes of failure. The stirrups importance once already appeared at instant of the first inclined crack formation.

Therefore, stirrups must be arranged in such a way that any probable diagonal tension crack should be encountered with at least more than one stirrup. I.e. to ensure that any potential diagonal tension crack encounters a stirrup and does not open excessively and consequently the risk of the beam at the level of tension steel or the sudden failure without warning is prevented.

The observed failure which accompanied by a vertical displacement as well as vertical sliding between the two adjacent portions of the beam just to the load points preceded by diagonal crack is denoted by diagonal tension failure.

The cracking and ultimate loads were recorded in table (3) and the deflection and strain for concrete were given in Table (4). Mode of failure for each beam was as follow:

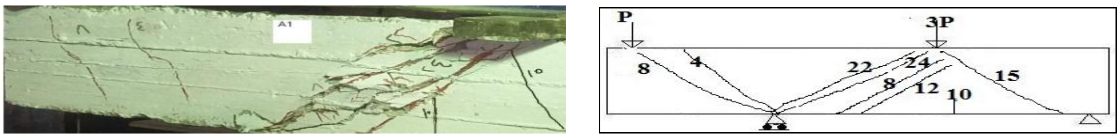


Fig (12) Crack pattern of beam (A1)

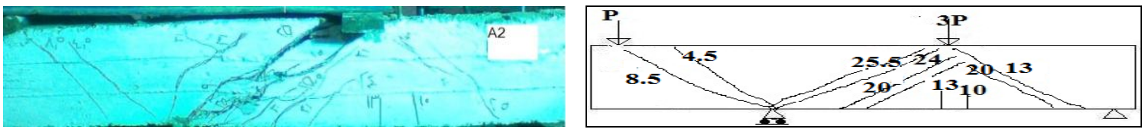


Fig (13) Crack pattern of beam (A2)

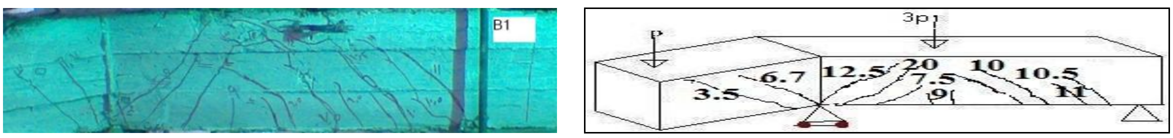


Fig (14) Crack pattern of beam (B1)

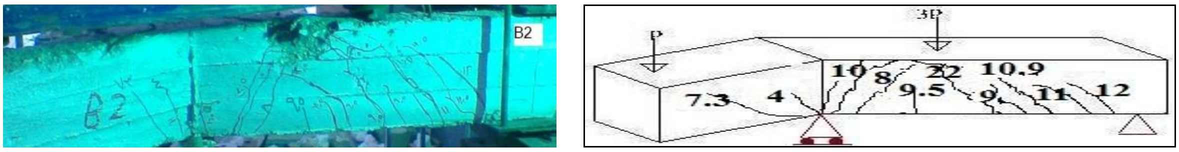


Fig (15) Crack pattern of beam (B2)

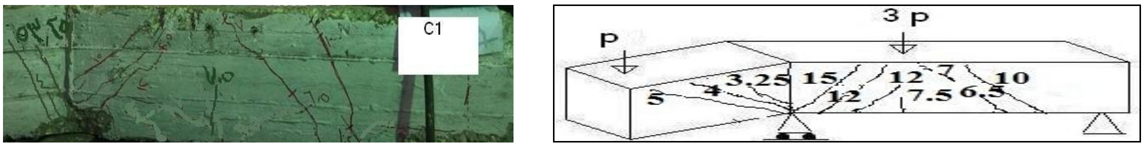


Fig (16) Crack pattern of beam (C1)

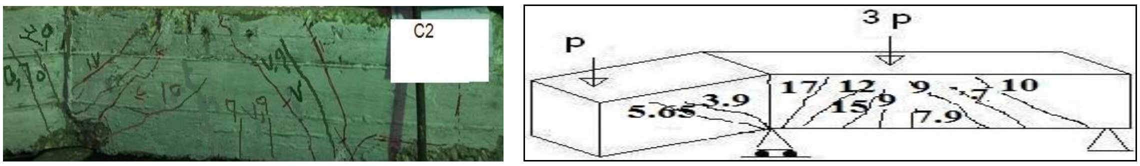


Fig (17) Crack pattern of beam (C2)

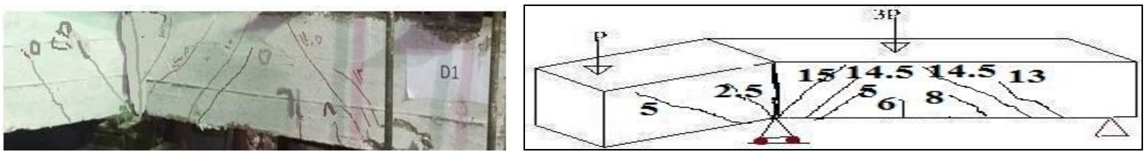


Fig (18) Crack pattern of beam (D1)

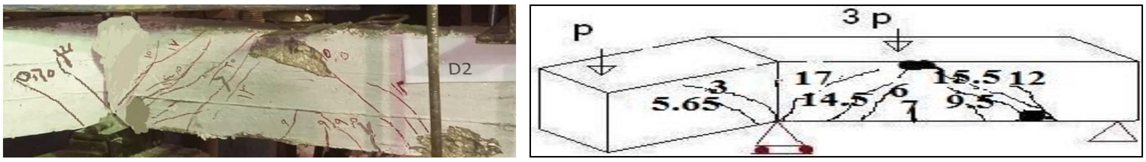


Fig (19) Crack pattern of beam (D2)

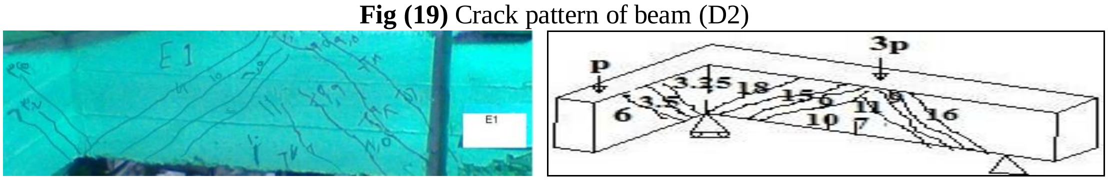


Fig (20) Crack pattern of beam (E1)

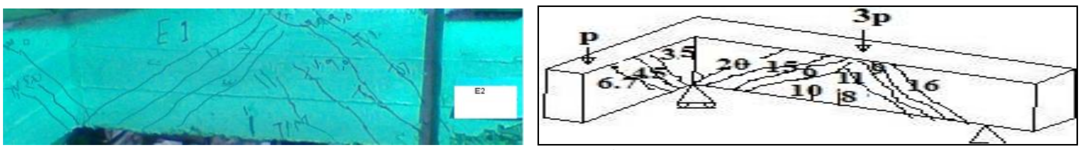


Fig (21) Crack pattern of beam (E2)

Table (3), Test results of reinforced concrete tested beams.

Table (4-1), Test results of tested beams with inclined cantilevers .

Series No.	specimen			cracking load (t)			failure load (t)		deflection (mm)				strain* 10 -3		slope (red)/1000		mode of failure	
	No	α	As	Qcrb	Pcrb	Qcrc	Pub	Puc	Δcrb	Δub	Δcrc	Δuc	€cbx 10 -3	€ccx 10 -3	θccr *10 -3	θuc *10 -3	beam	cantilever
A	A1	0.0°	2ϕ16	8	10	4	24	8	2.8	6.5	6.2	12.5	2.35	0.9	5	14	**D.T**	**D.T**
	A2	0.0°	4ϕ16	9	10	5	25.5	8.5	2.6	6	6	12	2.38	1.1	8	16	**D.T**	**D.T**
B	B1	30°	2ϕ16	7.5	9	3.5	20	6.7	2.9	7.5	7.3	14	2.1	0.75	9	20	**D.T**	**D.T**
	B2	30°	4ϕ16	8	9.5	4	22	7.3	2.8	7	7	14	2.15	0.9	14	24	**D.T**	**D.T**
C	C1	45°	2ϕ16	6.5	6.5	3.25	15	5	3	8	7.6	14	1.8	0.6	16	25	**D.T**	**D.T**
	C2	45°	4ϕ16	7.9	7.9	3.9	17	5.65	2.9	7.5	7.2	14	1.85	0.9	20	29	**D.T**	**D.T**
D	D1	60°	2ϕ16	5	6	2.5	15	5	3.3	9	7.9	16	1.75	0.6	15	29	**D.T**	**D.T**
	D2	60°	4ϕ16	6	7	3	17	5.65	3.1	8	7.8	14.5	1.8	0.85	19	33	**D.T& S.C**	**D.T**
E	E1	90°	2ϕ16	6.5	7	3.25	18	6	3.6	9.5	8.2	16.5	2	0.8	23	35	**D.T**	**D.T**
	E2	90°	4ϕ16	7	8	3.5	20	6.7	3.4	9.1	8	15	2.1	0.9	25	40	**D.T**	**D.T**

D.T Diagonal tension. D.T&S.C Diagonal tension and shear compression.

3.2 Deflection characteristics.

The measured values of maximum deflection are plotted versus the applied load from starting the loading up to failure as shown in Fig. (22) To Fig. (27).

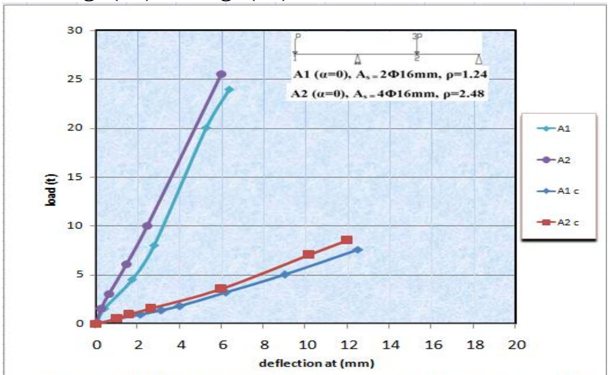


Figure (22) Load - Mid Span Deflection relationship for beam of Group (A)

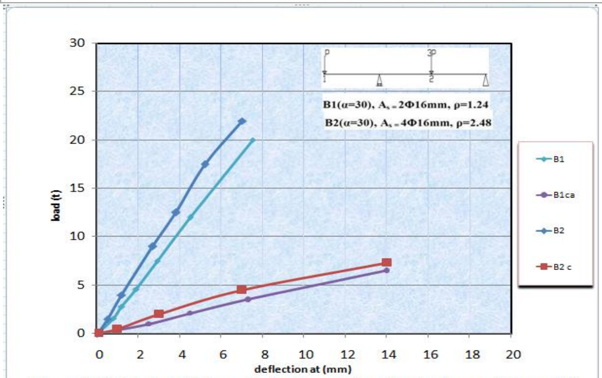


Figure (23) Load - Mid Span Deflection relationship for beam of Group (B)

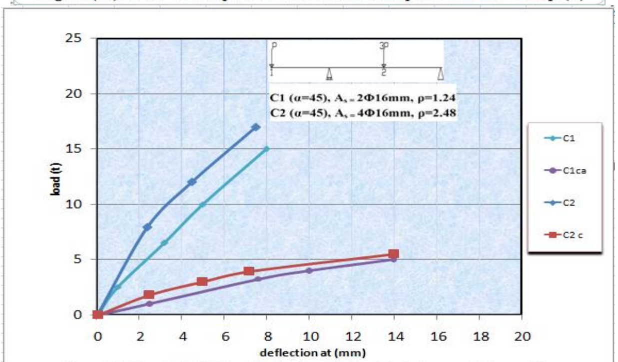


Figure (24) Load - Mid Span Deflection relationship for beam of Group (C)

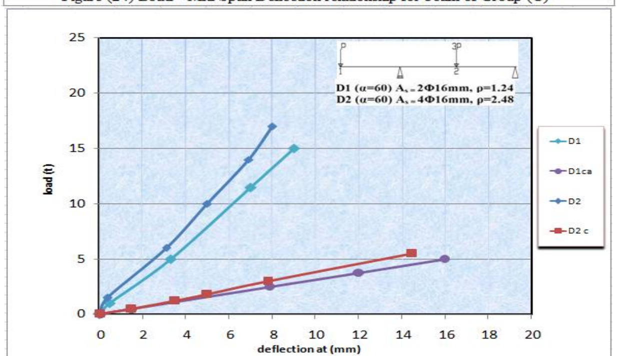


Figure (25) Load - Mid Span Deflection relationship for beam of Group (D

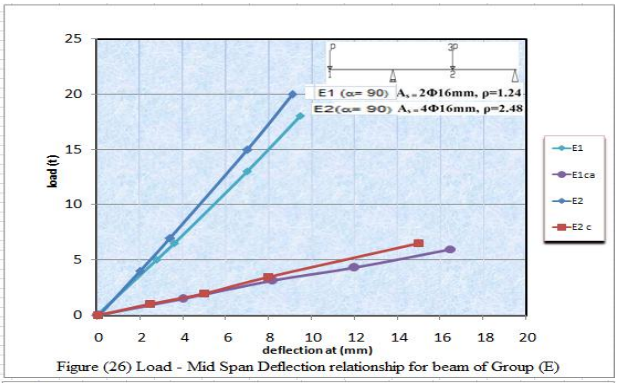


Figure (26) Load - Mid Span Deflection relationship for beam of Group (E)

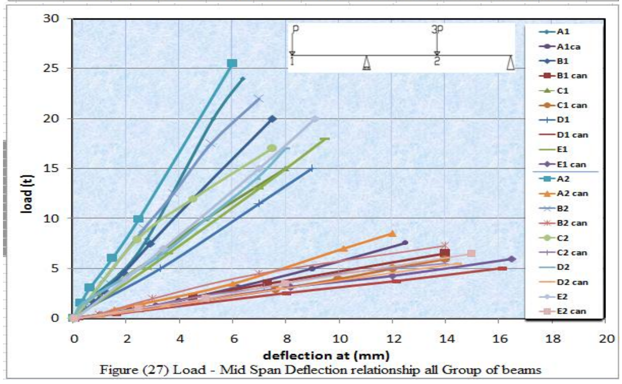


Figure (27) Load - Mid Span Deflection relationship all Group of beams

3.3 Concrete Strain Distribution.

Figures (28) to fig (30), shows the behavior of the concrete strain in compression for all beams. The results indicated that all specimens presented almost have the same trend where the load increased, the strain also increased.

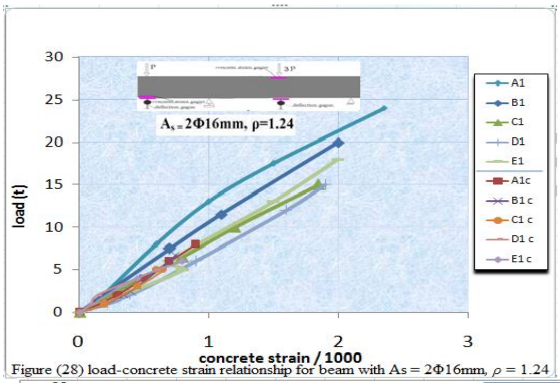


Figure (28) load-concrete strain relationship for beam with As = 2Φ16mm, ρ = 1.24

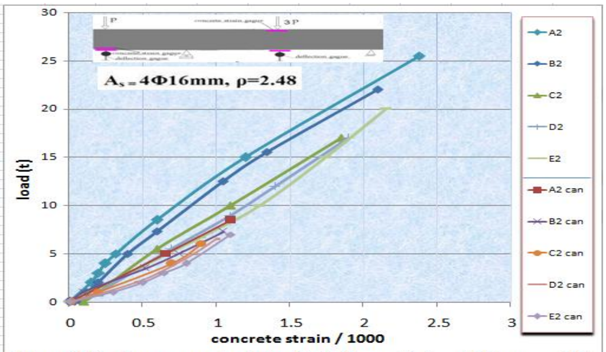


Figure (29) load-concrete strain relationship for beam with As = 4Φ16mm, ρ = 2.48

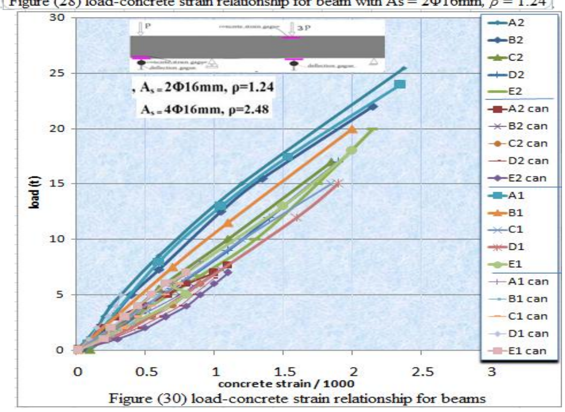


Figure (30) load-concrete strain relationship for beams

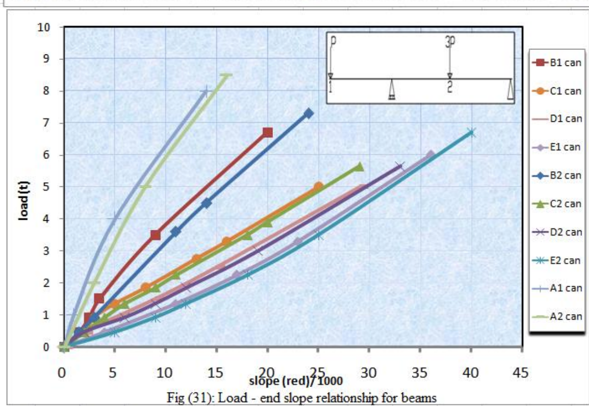


Fig (31): Load - end slope relationship for beams

3.4 Load-Slope characteristics

The maximum measured slope at the center of roller support of the beams is plotted versus the applied load from zero loading up to failure as Shown in Fig. (31). generally the load –slope curve of the tested beams can be divided into three distinct stages As follows:

a- a-The first stage: The un-cracked beams had relatively high flexural Rigidity. Consequently the slope of the load-slope curve in this stage was Steeper than the other stages.
b- b-The second stage: The shear cracks started to form. As the applied Loads was increased, cracks propagated and their width and length Increased. Hence, the slope of the load-slope curve became flatter than the First stage.
c- c-The third stage: The beams started to fail and the slope of the load-slope Curve became flatter than the second stage.

IV. Discussion of Test Result

This item describes and interprets the analysis of the obtained test results of the beams with inclined cantilever. The analysis includes the relationship between the value of cracking and ultimate loads, deflection; and concrete strain for tested beams. The characteristic of tested beams at cracking, ultimate load, deflection and strain are given in tables (3). The values of the experimental measured parameters of beams are shown in figures (12) to (31).From item (3); it is obvious that, all beams failed in shear or shear compression. In the early stages of loading, no flexural cracks were observed in the region of bending moment or shear zone as the applied load increased. With a further increase of load, diagonal cracks formed in the shear span area; and bending moment area and developed towards the loading points.

The failure modes of beams in groups A to E are presented in tables (3) the most common failure for the tested beams is a diagonal tension crack. The shear failure in beams is always initiated by splitting action. The amount of longitudinal steel ratio (ρ) has no effect on final mode of failure. The presence of inclination of angle (α) it has a considerable effect on pattern of cracks and modes of failure. The shear load was observed zone between the mid span load and the roller support. i.e. (through the critical shear zone) equal to double shear load on the cantilever beams. The effect of each parameter individually can be explained as follows:

4.1 Cracking and ultimate load (p_{cr}, p_u).
4.1.1 Effect of cantilever inclination (α): With respect to steel reinforcement As =2Φ16 (ρ= 1.24) and As =4Φ16 (ρ= 2.48) at span of cantilever and beams respectively.
4.1.1. 1-The shear cracking loads (Qcr) when the increasing of angle of inclination(α) has a slight effect on decreasing both the shear cracking load as follows:
From (α= 0.0°to 30°) the shear cracking load decreasing by (12.5, 20, 6.25and11 %)
From (α= 30°to 45°) the shear cracking load decreasing by (7.1, 2.5, 13.3 and1.1%)
From (α= 45°to 60°) the shear cracking load decreasing by (30, 23, 23 and24%)
Finally the increasing inclination of angle from (α= 60°to90°) increasing the shear cracking load by (30, 16.7,30 and16.7) Shown in fig (32 to fig 35).

4.1.1.2- The ultimate loads (Pu) when the increasing of angle of inclination (α) has a slight effect on decreasing both the ultimate load as follows:
From (α= 0.0°to 30°) the ultimate load decreasing by (16.25, 14.1, 16.7 and13.7)
From (α= 30°to 45°) the ultimate load decreasing by (25.3, 22.6, 25 and 22.7)
From (α= 45°to 60°) the ultimate load is constant
Finally the increasing inclination of angle from (α= 60°to90°) increasing ultimate load by (20, 18.6, 20 and17.6) Shown in fig (32 to fig 35).

4.1.1.3- The flexural cracking loads (Pcr) at span of beams when the increasing of angle of inclination (α) has a slight effect on decreasing both the flexural cracking load as follows:
From (α= 0.0°to 30°) the flexural cracking load decreasing by (10 and 5%)
From (α= 30°to 45°) the flexural cracking load decreasing by (27 and 16.8%)
From (α= 45°to 60°) the flexural cracking load decreasing by (7.7 and 11.4%)
Finally the increasing inclination of angle from (α= 60°to90°) increasing the flexural cracking load by (16.6 and 14.3%) Shown in fig (34 and 35).

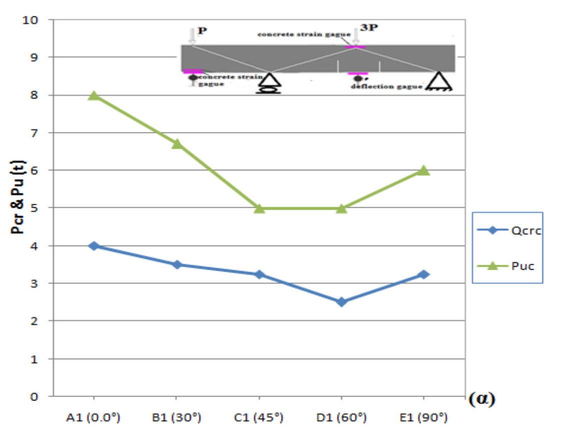


Figure (32): Effect of cantilever inclination (α) on shear cracking and ultimate loads on span of cantilever beams having As =2Φ16 (ρ= 1.24)

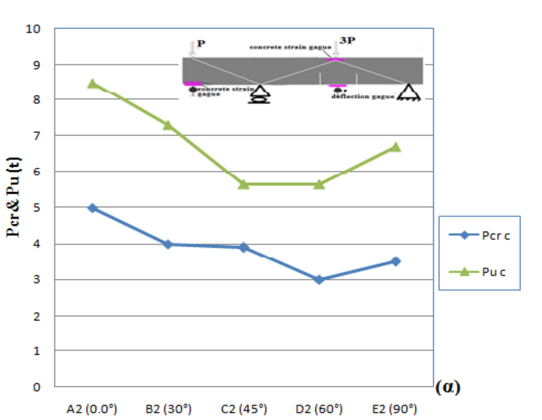


Figure. (33): Effect of cantilever inclination (α) on shear cracking and ultimate loads on span of cantilever beams having As =4Φ16 (ρ= 2.48).

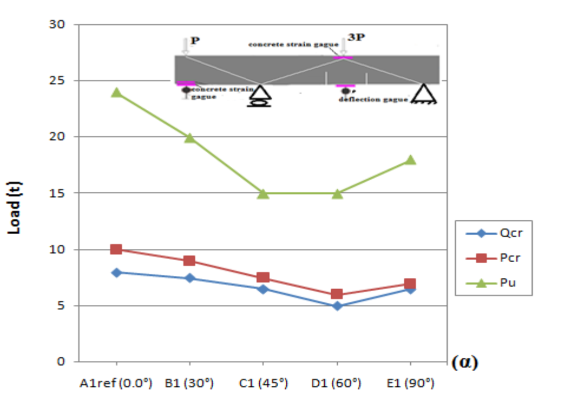


Figure. (34): Effect of cantilever inclination (α) on shear cracking, flexural cracking and ultimate loads on span of beams having As =2Φ16 (ρ= 1.

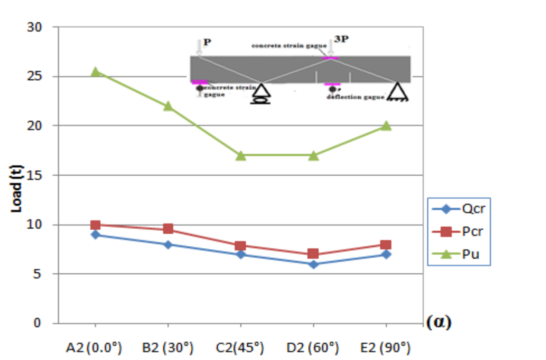


Figure. (35): Effect of cantilever inclination (α) on shear cracking, flexural cracking and ultimate loads on span of beams having As =4Φ16 (ρ= 2.48).

4.1.2 Effect of longitudinal steel ratio (*ρ*)

4.1.2.1 When the increasing of main longitudinal bars on cantilever the diagonal cracking load (Qcrb) and ultimate load (Pu) are increasing respectively as follows:

For (α=0.0°) the values of loads increasing by (25 and 6.2%)
For (α=30°) the values of loads increasing by (14.3and 9%)
For (α=45°) the values of loads increasing by (20and 13%)
For (α=60°) the values of loads increasing by (20and 13 %)
Finally the increasing inclination of angle from (90°) the values of loads increasing by (7.7 and 11.7 %) Shown in fig (36).

4.1.2.2 When the increasing of main longitudinal bars on beams the diagonal cracking load (Qcrb) flexural cracking load (Pcrb) and ultimate load (Pu) are increasing respectively as follows:

For (α=0.0°) the values of loads increasing by (12.5, 12.5 and 2%)
For (α=30°) the values of loads increasing by (6.7, 5.5 and 10%)
For (α=45°) the values of loads increasing by (21.5, 21.5 and 13.3%)
For (α=60°) the values of loads increasing by (20, 16.7 and 13.3 %)
Finally the increasing inclination of angle from (α=90°) the values of loads increasing by (7.7, 14.3 and11.1 %) Shown in fig (37).

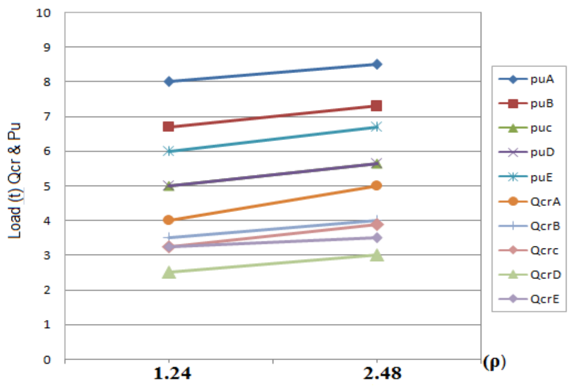


Figure. (36): Effect of steel reinforcement ratio (ρ) on shear cracking and ultimate loads at span of cantilever.

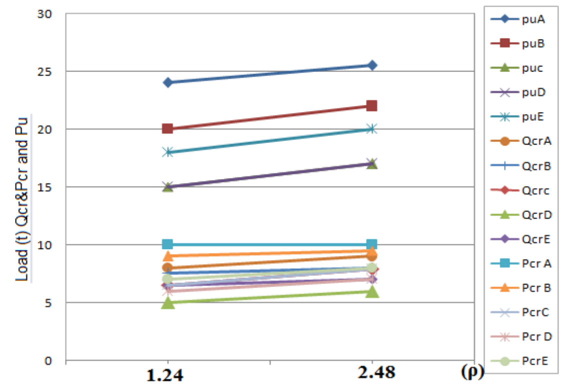


Figure. (37): Effect of steel reinforcement ratio (ρ) on shear cracking, flexural cracking and ultimate loads at span of beams.

4.2 Maximum measured deflection (δ).

4.2.1. Effect of beams cantilever inclination (α) With respect to steel reinforcement As = 2Φ16 (ρ= 1.24) and As 4Φ16 (ρ =2.48) at span of cantilever and beams respectively.

4.2.1.1. The crack deflection (δcr) when the increasing of angle of inclination (α) has a slight effect on increasing the deflection of cracking load as follows:

From (α = 0.0°to 30°) the deflection crack increasing by (17.8, 16.7, 15.4 and 7.7%)
From (α = 30°to 45°) the deflection crack increasing by (4, 2.9, 6.7 and 3.6%)
From (α = 45°to 60°) the deflection crack increasing by (3.9, 8.3, 12.5 and 6.9%)
Finally the increasing inclination of angle from (α = 60°to90°) increasing the deflection cracking load by (3.8, 2.6, 5.5 and 9.7%) Shown in fig (38 to fig 41).

4.2.1. 2-The ultimate deflection (δu) when the increasing of angle of inclination has a slight effect on increasing the deflection ultimate load as follows:

From (α = 0.0°to 30°) the ultimate deflection increasing by (12, 16.7, 15.4 and 16.7%)
From (α = 30°to 45°) the ultimate deflection is (constant, constant, 6.7 and 7.1%)
From (α = 45°to 60°) the ultimate deflection increasing by (14.3, 3.6, 12.5 and 6.7%)
Finally the increasing inclination of angle from (α = 60°to90°) increasing the ultimate deflection by (3, 3.4, 5.5, 13.8%) Shown in fig (38 to fig 41).

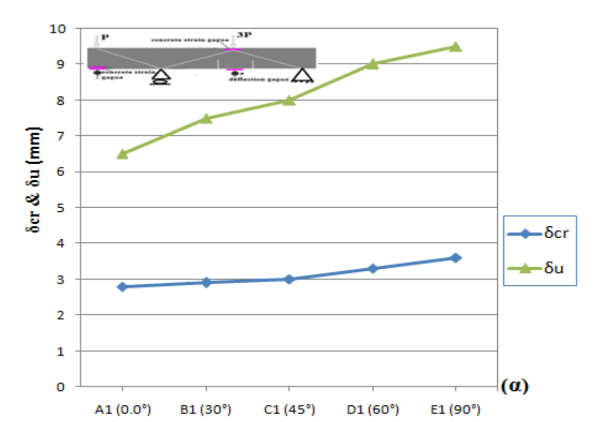


Figure. (38): Effect of cantilever inclination (α) on deflection at shear cracking and ultimate loads on span of beams having (ρ) 1.24 A_s =2Φ16.

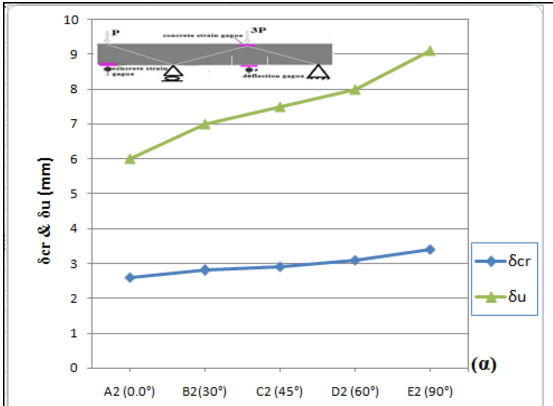


Figure. (39): Effect of cantilever inclination on deflection at shear cracking and ultimate loads on span of beams having (ρ) 2.48 A_s =4Φ16.

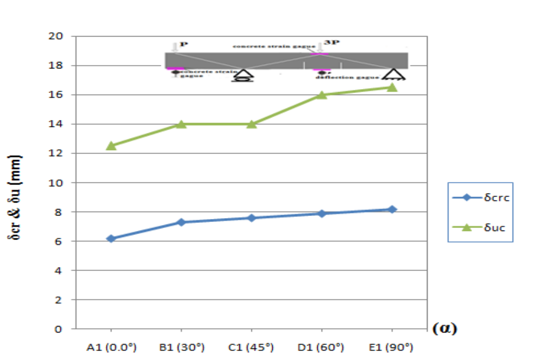


Figure. (40): Effect of cantilever inclination (α) on deflection at shear cracking and ultimate loads on span of cantilever beams having (ρ) 1.24 A_s=2Φ16.

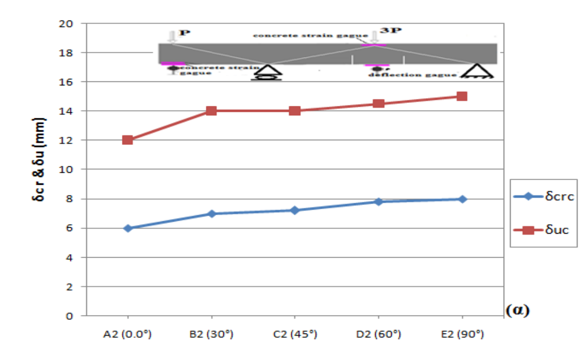


Figure. (41): Effect of cantilever inclination (α) on deflection at shear cracking and ultimate loads on span of cantilever having (ρ) 2.48 A_s=4Φ16.

4.2.2 Effect of longitudinal steel ratio (*ρ*)

4.2.2.1 When the increasing of main longitudinal bars on cantilever the deflection at cracking load (δcr) and deflection at ultimate load (δu) are increasing respectively as follows:
For (α=0.0°) the values of deflection decreasing by (3.2and 4%)
For (α=30°) the values of deflection decreasing by (4.1% and constant)
For (α=45°) the values of deflection decreasing by (5.2% and constant)
For (α=60°) the values of deflection decreasing by (1.25 and 9.4 %)
Finally the increasing inclination of angle from (α= 90°) the values of deflection decreasing by (2.4and 6 %) Shown in fig (42).

4.2.2.2 When the increasing of main longitudinal bars on beams the deflection at cracking load (δcr) and deflection at ultimate load (δu) are increasing respectively as follows:
For (α=0.0°) the values of deflection decreasing by (7and 7.7%)
For (α=30°) the values of deflection decreasing by (3.4and 6.7%)
For (α=45°) the values of deflection decreasing by (3.3 and 6.2%)
For (α=60°) the values of deflection decreasing by (6 and 11 %)
Finally the increasing inclination of angle from (α = 90°) the values of deflection decreasing by (5.5and 4 %) Shown in fig (43).

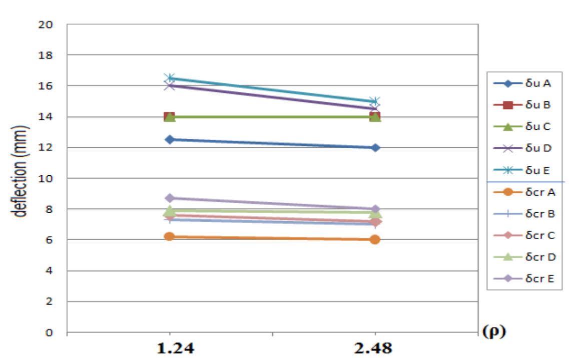


Figure (42): Effect of steel reinforcement ratio (ρ) on deflection at shear cracking, and ultimate loads for cantilever.

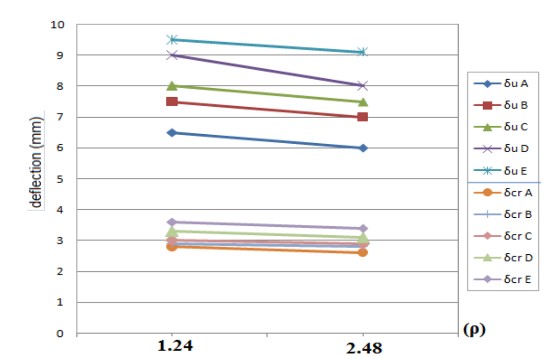


Figure (43): Effect of steel reinforcement ratio (ρ) on deflection at shear cracking and ultimate loads for tested beam.

4.3 Ultimate concrete compressive strains (ξc)

4.3.1 Effect of beams cantilever inclination (α): With respect to steel reinforcement at span of cantilever having As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48)

4.3.1.1 The ultimate concrete compressive strains (ξc): When the increasing of angle of inclination (α) has a slight effect on decreasing the ultimate concrete compressive strains (ξc) as follows:
From (α= 0.0°to 30°) the ultimate concrete compressive strains decreasing by (16.7 and 18.2%)
From (α= 30°to 45°) the ultimate concrete compressive strains decreasing by (20% and constant)
From (α= 45°to 60°) the ultimate concrete compressive strains decreasing by (constant and 5.5%)
Finally the increasing inclination of angle from (α= 60°to90°) increasing the ultimate deflection by (33 and 5.9%) Shown in fig (44).

4.3.1.2 With respect to steel reinforcement at span of beams having As = 2Φ16 (ρ= 1.24) and As 4Φ16

(ρ =2.48)

The ultimate concrete compressive strains (ξc) when the increasing of angle of inclination has a slight effect on decreasing the ultimate concrete compressive strains (ξc) as follows:

From (α= 0.0°to 30°) the ultimate concrete compressive strains decreasing by (10.6 and 9.6%)

From (α= 30°to 45°) the ultimate concrete compressive strains decreasing by (14.3 and 14%)

From (α= 45°to 60°) the ultimate concrete compressive strains decreasing by (2.7 and 2.7%)

Finally the increasing inclination of angle from (α= 60°to90°) increasing the ultimate deflection by (14.3 and 16.6%) Shown in fig (45).

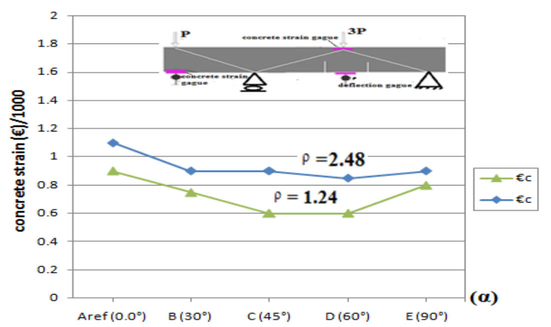


Figure (44):): Effect of the angle of inclination of cantilever(α) on concrete strain at ultimate load for span of cantilever having As = 2Φ16 (ρ= 1.24) and As 4Φ16 (ρ =2.48)

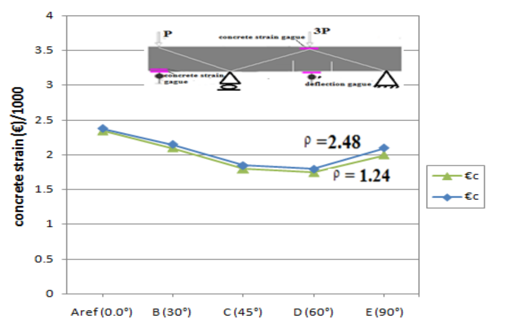


Figure (45): Effect of the angle of inclination of cantilever on concrete strain at ultimate load f or span of beams having As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48)

4.3.2 Effect of longitudinal steel ratio (*ρ*)

4.3.2.1 Effect of longitudinal steel ratio (ρ) at span of cantilever:

For (α=0.0°) the values of concrete compressive strains increasing by (22.2%)

For (α=30°) the values of concrete compressive strains increasing by (16.7%)

For (α=45°) the values of concrete compressive strains increasing by (50%)

For (α=60°) the values of concrete compressive strains increasing by (41.7%)

Finally the increasing inclination of angle from (90°) the values of concrete compressive strains increasing by (12.5 %) shown in fig (46).

4.3.2.2 Effect of longitudinal steel ratio (ρ) at span of beam:

For (α=0.0°) the values of concrete compressive strains increasing by (1.27)

For (α=30°) the values of concrete compressive strains increasing by (2.38%)

For (α=45°) the values of concrete compressive strains increasing by (2.7%)

For (α=60°) the values of concrete compressive strains increasing by (2.85%)

Finally the increasing inclination of angle from (α = 90°) the values of concrete compressive strains increasing by (5 %) shown in fig (47).

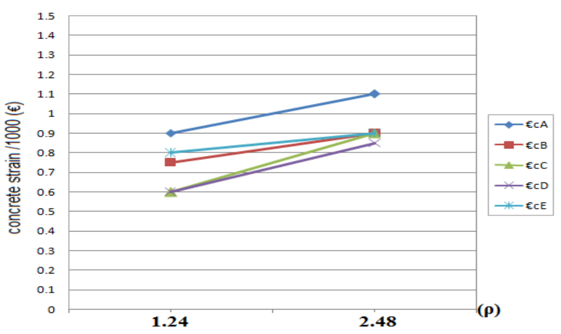


Figure (46):Effect of steel reinforcement ratio (ρ) on the concrete strain (€) at ultimate load on span of cantilever.

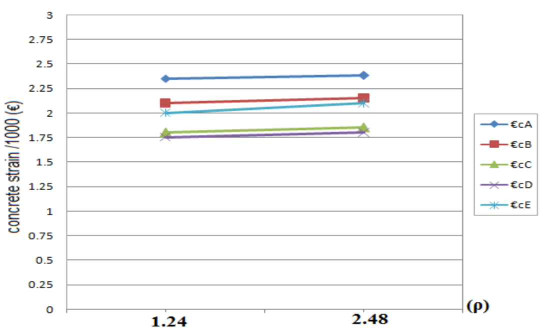


Figure (47): Effect of steel reinforcement ratio (ρ) on the concrete strain (€) at ultimate load on span of beams.

4.4 Maximum Slope of Beams at the Support

4.4.1 Effect of beams cantilever inclination (α)

The values of slope at different loads increase with the increase of the cantilever inclination (α) having As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) the compared values of (θcr) and (θu) respectively:
From (α =0.0° to30°) the compared values were (80 and 75%) (43 and 50 %).
From (α =30° to 45°) the compared values were (44.4 and 42.9%) (25 and 20.8 %).
From (α =45° to 60°) the compared values were (23 and 5%) (16 and 13.8%).
From (α =60° to 90°) the compared values were (43.75and 31.6%) (20.7 and 21.2%) shown in fig (48 and 49).

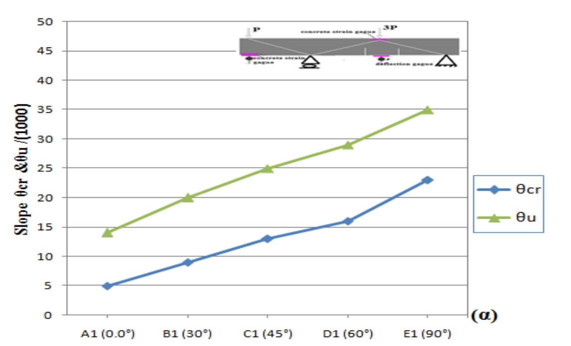


Figure (48): Effect of the angle of inclination of cantilever (α) on Slope at cracking and ultimate loads

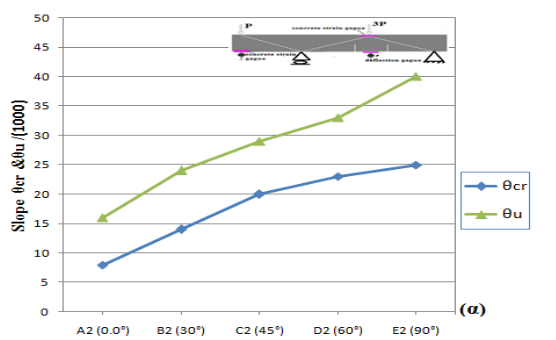


Figure (49): Effect of the angle of inclination of cantilever (α) on Slope at cracking and ultimate loads.

4.4.2Effect of longitudinal steel ratio (ρ)
For (α=0.0°) the values of slope increasing by 60 and 6.25%.
For (α=30°) the values of slope increasing by 8.9 and 20 %.
For (α=45°) the values of slope increasing by 27 and 16 %.
For (α=60°) the values of slope increasing by 25 and 13.8%.
For (α=90°) the values of slope increasing by 8.7 and 14.3% shown in fig (50)

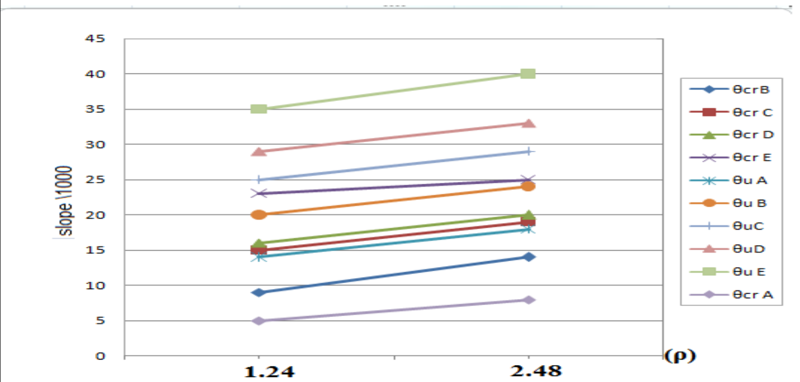


Figure (50): Effect of steel reinforcement ratio (ρ) on Slope at cracking and ultimate loads.

4.5 Comparison between beam A1 (Ref) and tested beams.

Results for beams with cantilever have different in main reinforcement ratio and inclined cantilever (α) in shear region or in flexural zone with respect to results for beam(A1)are included in table (4) and shown in Figs (37)to figs (42).

Table (4): Comparison between A1 (Ref) and tested beams results.

Series No.	speciment			beams						cantilevers				
	No	α	As	Qcr/Qcr (Ref)	Pcr/Pcr (Ref)	Pu/Pu (Ref)	δcr/δcr (Ref)	δu/δu (Ref)	€cx /€c(Ref)	Qcr/Qcr (Ref)	Pu/Pu (Ref)	δcr/δcr (Ref)	δu/δu (Ref)	€cx /€c(Ref)
A	A1	0.0°	2ϕ16	1	1	1	1	1	1	1	1	1	1	1
	A2	0.0°	4ϕ16	1.12	1	1.06	0.93	0.92	1.01	1.25	1.06	0.97	0.96	1.22
B	B1	30°	2ϕ16	0.94	0.90	0.83	1.04	1.15	0.89	0.88	0.84	1.18	1.12	0.83
	B2	30°	4ϕ16	1.00	0.95	0.92	1.00	1.08	0.91	1	0.91	1.13	1.12	1.00
C	C1	45°	2ϕ16	0.81	0.75	0.63	1.07	1.23	0.79	0.81	0.63	1.23	1.12	0.67
	C2	45°	4ϕ16	0.87	0.79	0.71	1.04	1.15	0.77	0.87	0.71	1.16	1.12	1
D	D1	60°	2ϕ16	0.63	0.60	0.63	1.18	1.38	0.74	0.63	0.63	1.27	1.28	0.67
	D2	60°	4ϕ16	0.75	0.70	0.71	1.10	1.23	0.77	0.75	0.71	1.26	1.16	0.94
E	E1	90°	2ϕ16	0.81	0.70	0.75	1.28	1.46	0.85	0.81	0.75	1.32	1.32	0.89
	E2	90°	4ϕ16	0.88	0.80	0.83	1.20	1.40	0.89	0.88	0.84	1.29	1.2	1

4.5.1 Effect of beams cantilever inclination (α)

4.5.1.1 With respect to steel reinforcement As = 2Φ16 (ρ= 1.24) and As 4Φ16 (ρ =2.48) at span of beams.

1-The shear cracking loads (Qcr) when the increasing of angle of inclination (α) has a slight effect on decreasing the shear cracking load were respectively as follows:
From (α= 0.0°to 30°) the compared values were (6% and constant)
From (α= 30°to 45°) the compared values were (19 and 1 %)
From (α= 45°to 60°) the compared values were (37 and 25 %)
Finally the increasing inclination of angle from (α= 60°to90°) the compared values were (19 and 12 %) Shown in fig (50 and 51).

2- The flexural cracking loads (Pcr) when the increasing of angle of inclination (α) has a slight effect on decreasing both the flexural cracking load as follows:
From (α = 0.0°to 30°) the compared values were (10 and 5 %)
From (α = 30°to 45°) the compared values were (35and 21%)
From (α = 45°to 60°) the compared values were (40 and 30%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (30 and 20 %) Shown in fig (50 and 51).

3- The ultimate loads (Pu) when the increasing of angle of inclination (α) has a slight effect on decreasing both the ultimate load as follows:
From (α = 0.0°to 30°) the compared values were (17 and 8 %)
From (α = 30°to 45°) the compared values were (37 and 29%)
From (α = 45°to 60°) the compared values were (37 and 29%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (25 and 17 %) Shown in fig (50 and 51).

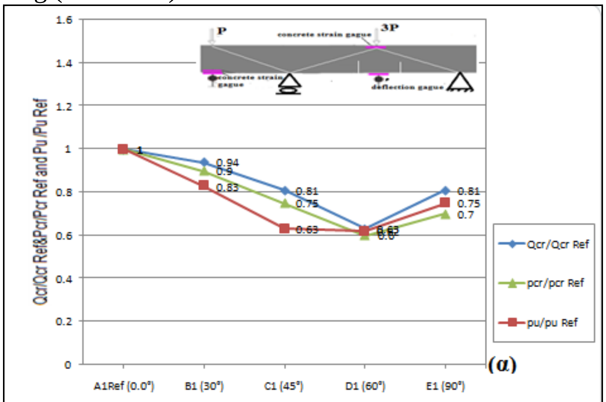


Figure (50): Comparison between results of A1 (Ref) and group of Beams having As = 2Φ16 (ρ= 1.24)

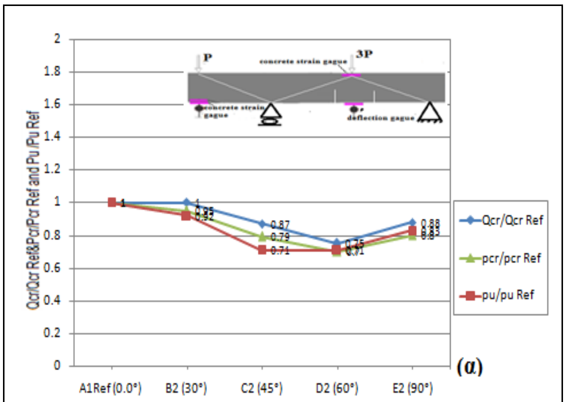
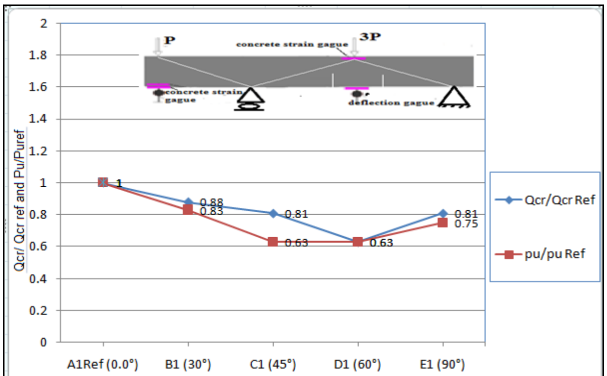


Figure (51): Comparison between results of loads on beam A1 (Ref) and group of Beams having As= 4Φ16 (ρ =2.48)

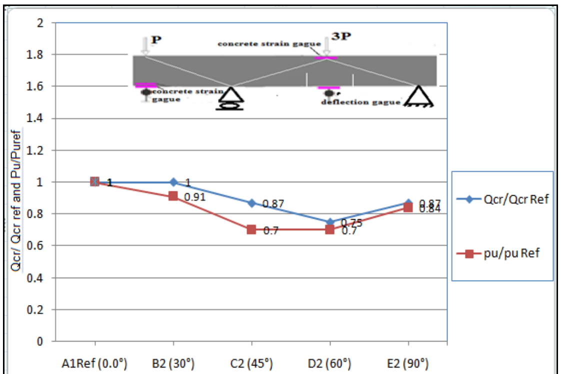
4.5.1.2With respect to steel reinforcement having As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) at span of cantilever beams.

1-The shear cracking loads (Qcr) when the increasing of angle of inclination has a slight effect on decreasing the shear cracking load as follows:
From (α = 0.0°to 30°) the compared values were (12% and constant)
From (α = 30°to 45°) the compared values were (35 and 2%)
From (α = 45°to 60°) the compared values were (37 and 25 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (19 and 12 %) Shown in fig (52 and 53).

2- The ultimate loads (Pu) when the increasing of angle of inclination has a slight effect on decreasing both the ultimate load as follows:
From (α = 0.0°to 30°) the compared values were (16 and 9%)
From (α = 30°to 45°) the compared values were (37 and 29 %)
From (α =45°to 60°) the compared values were (37 and 29 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (25 and 16 %) Shown in fig (52 and 53).



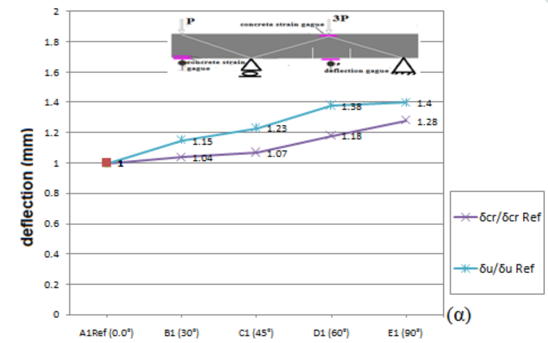
Figure(52): Comparison between A1cantilever (Ref) and cantilever of Beams having ρ= (1.24) As =2Φ16



Figure(53): Comparison between cantilever beams A1 (Ref) and cantilever of Beams having ρ= (2.48) As =4Φ16

4.6 Comparison between cracking and ultimate deflection.
4.6.1With respect to steel reinforcement As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) at span of beams.
4.6.1.1-The deflection at cracking loads (δcr) when the increasing of angle of inclination (α) has a slight effect on increasing the deflection as follows:
From (α = 0.0°to 30°) the compared values were (4%and constant)
From (α = 30°to 45°) the compared values were (7and 4%)
From (α = 45°to 60°) the compared values were (18and 10%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (28and 20%) Shown in fig (54and 55).

4.6.1.2- The deflection at ultimate loads (δu) when the increasing of angle of inclination (α) has a slight effect on increasing both the deflection as follows:
From (α = 0.0°to 30°) the compared values were (15and8 %)
From (α = 30°to 45°) the compared values were (23and15 %)
From (α = 45°to 60°) the compared values were (38and 23%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (46and40 %) Shown in fig (54and 55).



Figure(54): Comparison between cracking and ultimate deflections for A1 (Ref) and tested beams with Steel reinforcement ratio As = 2Φ16 (ρ= 1.24)

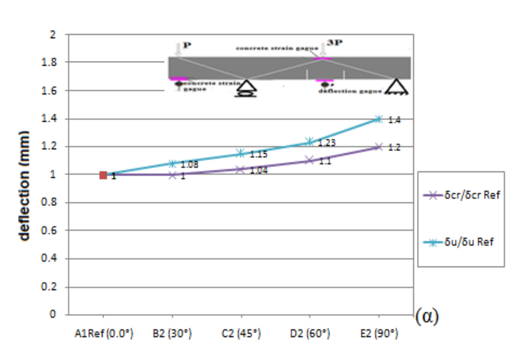


Figure (55): Comparison between cracking and ultimate deflections for A1 (Ref) and tested beams with Steel reinforcement ratio As = 4Φ16 (ρ=2.48)

4.6.2With respect to steel reinforcement (As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) at span of cantilever beams.

4.6.2.1-The deflection at cracking loads (δcr) when the increasing of angle of inclination (α) has a slight effect on decreasing the deflection as follows:

From (α = 0.0°to 30°) the compared values were (18and 13%)
From (α = 30°to 45°) the compared values were (23and16 %)
From (α = 45°to 60°) the compared values were (27and 26%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (32and 29%) Shown in fig (56and 57).

4.6.2.2- The deflection at ultimate loads (δu) when the increasing of angle of inclination (α) has a slight effect on increasing both the deflection as follows:

From (α = 0.0°to 30°) the compared values were (12and 12%)
From (α = 30°to 45°) the compared values were (12and 12%)
From (α = 45°to 60°) the compared values were (28and 16%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (32and20 %) Shown in fig (56and 57).

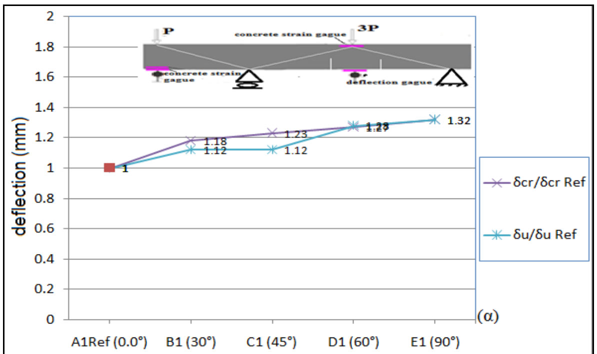


Figure (56) Comparison between cracking and ultimate deflections for A1 (Ref) and tested beams with Steel reinforcement ratio As = 2Φ16 (ρ= 1.24)at span of cantilever.

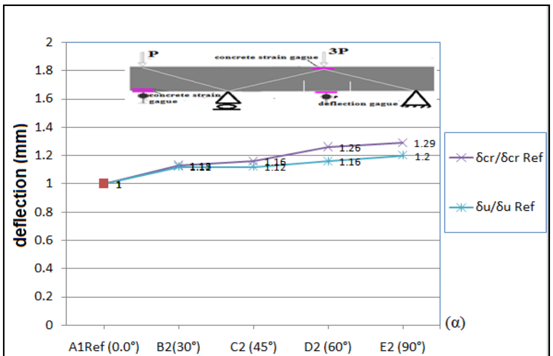


Figure (57): Comparison between cracking and ultimate deflections for A1 (Ref) and tested beams with Steel reinforcement ratio As = 4Φ16 (ρ=2.48) at span of cantilever.

4.7 Comparison between the concrete strains due to ultimate load.

4.7.1With respect to steel reinforcement (As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) at span of cantilever beams.

1- The concrete strains at ultimate loads (£u) when the increasing of angle of inclination (α) has a slight effect on increasing the concrete strains as follows:

From (α = 0.0°to 30°) the compared values were (17%and constant)
From (α = 30°to 45°) the compared values were (33%and constant)
From (α = 45°to 60°) the compared values were (33and6 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (11%and constant) Shown in fig (58).

4.7.2With respect to steel reinforcement (As = 2Φ16 (ρ= 1.24) and As = 4Φ16 (ρ=2.48) at span of beams.
1- The concrete strains at ultimate loads (£u) when the increasing of angle of inclination (α) has a slight effect on increasing the concrete strains as follows:
From (α = 0.0°to 30°) the compared values were (11and 9%)
From (α = 30°to 45°) the compared values were (21and23 %)
From (α = 45°to 60°) the compared values were (26and 23%)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (15and 11%) Shown in fig (59).

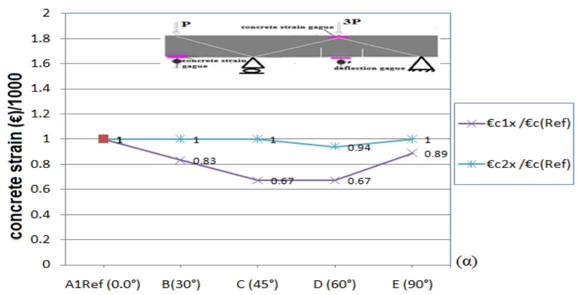
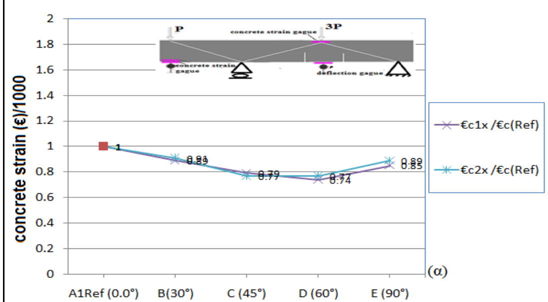


Figure (58): Comparison of the concrete strain due to ultimate load of cantilever beams to the reference beams tested.



Figure(59): Comparison of the concrete strain due to ultimate load of beams to the reference beams tested.

4.8 Effect of stresses due to Mx and Mt.

Beams with cantilever have different in inclined cantilever (α) and main reinforcement ratio (ρ) in shear region or in flexural zone. Included in table (5) and shown in Fig (60) and (61).

Table (5): stresses due to bending moment (M_x) and torsion (Mt).

group	α	M_X	M_t	σcr_c (kg/cm²)	σu_c (kg/cm²)	σcr_b (kg/cm²)	σu_b (kg/cm²)
A1	0.0°	.5P	0	σ_1=137.2	σ_1= 274.3	σ_1= 274.3	σ_1= 823
				σ_2= 0.0	σ_2= 0.0	σ_2= 0.0	σ_2= 0.0
A2	0.0°	.5P	0	σ_1=154.3	σ_1= 291.5	σ_1=308.7	σ_1=874.6
				σ_2= 0.0	σ_2= 0.0	σ_2= 0.0	σ_2= 0.0
B1	30°	.5P	.3P	σ_1= 151.5	σ_1= 290	σ_1=326	σ_1=870
				σ_2=-25	σ_2= -59.6	σ_2= -59.8	σ_2= -183
B2	30°	.5P	.3P	σ_1= 195	σ_1= 317	σ_1= 348	σ_1=956.9
				σ_2= -29	σ_2= -67	σ_2= -82	σ_2= -202
C1	45°	.5P	.5P	σ_1= 172	σ_1= 265	σ_1=344.6	σ_1= 795.3
				σ_2= -60.5	σ_2= -93	σ_2= -121	σ_2= -280
C2	45°	.5P	.5P	σ_1= 167	σ_1=300	σ_1= 418.8	σ_1=901
				σ_2= -59	σ_2= -103	σ_2=-148	σ_2= -318
D1	60°	.5P	.86P	σ_1=180	σ_1= 359.5	σ_1= 359.5	σ_1=1078
				σ_2=-94	σ_2= -188	σ_2= -188	σ_2= -564
D2	60°	.5P	.86P	σ_1= 216	σ_1= 407	σ_1= 431.3	σ_1=1222
				σ_2= -113.3	σ_2= -213.3	σ_2= -226	σ_2= -640
E1	90°	0	.5P	σ_1= 0.0	σ_1= 0.0	σ_1= 0.0	σ_1= 0.0
				σ_2=102.4	σ_2=189	σ_2=205	σ_2= 567
E2	90°	0	.5P	σ_1= 0.0	σ_1= 0.0	σ_1= 0.0	σ_1= 0.0
				σ_2= 109	σ_2=210	σ_2=220.5	σ_2 =630

4.8.1 With respect to steel reinforcement As =2Φ16 (ρ= 1.24) at span of cantilever and beams respectively:
4.8.1.1- The cracking stresses (σcr) when the increasing of angle of inclination (α)
from (α = 0.0°to 30°) the compared values were (7.4 and 22.7%)
From (α = 30°to 45°) the compared values were (17.4 and 21.9%)
From (α = 45°to 60°) the compared values were (45.8and 66.8 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (56.3 and 53.4) Shown in fig (60).
4.8.1.2- The ultimate stresses (σu) when the increasing of angle of inclination (α)

from (α = 0.0°to 30°) the compared values were (7.7 and 5.7%)
From (α = 30°to 45°) the compared values were (5.6 and 8.5%)
From (α = 45°to 60°) the compared values were (35.6 and 35.5 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (47.4 and 47.4) Shown in fig (60).

4.8.2With respect to steel reinforcement As =4Φ16 (ρ= 2.48) at span of cantilever and beams respectively:
4.8.2.1- The cracking stresses (σcr) when the increasing of angle of inclination (α)
From (α = 0.0°to 30°) the compared values were (18.1 and 12.2%)
From (α = 30°to 45°) the compared values were (21.9 and 6.9%)
From (α = 45°to 60°) the compared values were (24.8 and 54.4 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (54.6 and 53.6) Shown in fig (61).

4.8.2.2- The ultimate stresses (σu) when the increasing of angle of inclination (α):
From (α = 0.0°to 30°) the compared values were (8.7 and 9.4%)
From (α = 30°to 45°) the compared values were (8 and 5.8%)
From (α = 45°to 60°) the compared values were (35.45 and 35.6 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (48.1 and 48.4%) Shown in fig (61).

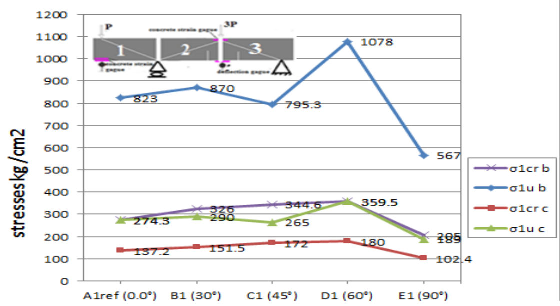


Figure (60): Effect of cantilever inclination (α) on cracking and ultimate stresses having A_s=2Φ16 (ρ) 1.24.

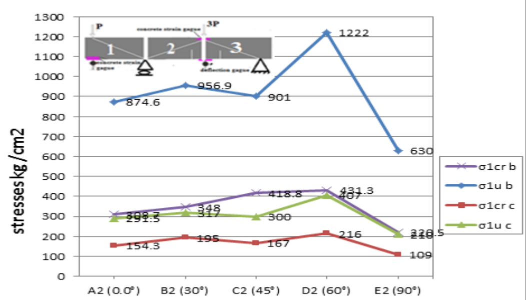


Figure (61): Effect of cantilever inclination (α) on cracking and ultimate stresses having A_s=4Φ16 (ρ) 2.48.

Comparison between the test results and the theoretical values for beam A1according ACI and ECP codes are given in table (6).

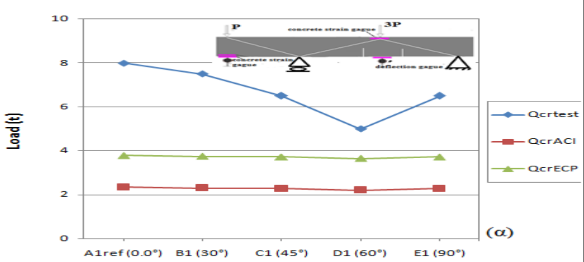
Table (6): Comparison between the test results and the theoretical values for beam A1according ACI and ECP codes.

Series No.	Beam No.	α	As	Qcr b test (t)1	Qcr c test (t)2	Qcr ACI (t) 3	QcrECP (t) 4	Qcr1/Qcr3	Qcr1/Qcr4	Qcr2/Qcr3	Qcr2/Qcr4	pu b test (t)5	pu c test (t)6	pu ACI (t) 7	pu ECP (t) 8	pu 5/pu7	pu5/pu8	pu 6/pu7	pu 6/pu8
A	A1	0.0°	2φ16	8	4	2.35	3.79	3.4	2.11	1.70	1.06	24	8	9.4	11.1	2.6	2.16	0.85	0.72
	A2	0.0°	4φ16	9	4.5	2.35	3.79	3.83	2.37	1.91	1.19	25.5	8.5	9.4	11.1	2.71	2.30	0.90	0.77
B	B1	30°	2φ16	7.5	3.5	2.3	3.75	3.26	2.00	1.52	0.93	20	6.7	9.3	11.0	2.15	1.82	0.72	0.61
	B2	30°	4φ16	8	4	2.3	3.75	3.48	2.13	1.74	1.07	22	7.3	9.3	11	2.37	2.00	0.78	0.66
C	C1	45°	2φ16	6.5	3.25	2.28	3.72	2.85	1.75	1.43	0.87	15	5	9.2	10.9	1.63	1.38	0.54	0.46
	C2	45°	4φ16	7	3.5	2.28	3.72	3.07	1.88	1.54	0.94	17	5.65	9.2	10.9	1.85	1.56	0.61	0.52
D	D1	60°	2φ16	5	2.5	2.2	3.65	2	1.37	1.14	0.68	15	5	9	10.7	1.67	1.40	0.56	0.47
	D2	60°	4φ16	6	3	2.2	3.65	2.73	1.64	1.36	0.82	17	5.65	9	10.7	1.89	1.59	0.63	0.53
E	E1	90°	2φ16	6.5	3.25	2.28	3.72	2.85	1.75	1.43	0.87	18	6	9.2	10.9	1.96	1.65	0.65	0.55
	E2	90°	4φ16	7	3.5	2.28	3.72	3.07	1.88	1.54	0.94	20	6.7	9.2	10.9	2.17	1.83	0.73	0.61

4.9 Effect of beams cantilever inclination (α):

4.9.1 With respect to steel reinforcement at span of beams having As =2Φ16 (ρ= 1.24)
4.9.1.1- The cracking loads (Qcr) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (6.25, 2.12 and 1.05%)
From (α = 30°to 45°) the compared values were (13.3, 0.87 and 0.8%)
From (α = 45°to 60°) the compared values were (23, 3.5 and 1.9 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (30, 3.65 and 1. 9%) Shown in fig (62).

4.9.1.2- The ultimate loads (Pu) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI. Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (16.7, 1.06 and 0.9%)
From (α = 30°to 45°) the compared values were (25, 1.07 and 0.9%)
From (α = 45°to 60°) the compared values were (25, 2.18 and 1.84 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (20, 2.2 and 1.87%) Shown in fig (63).



Figure(62): Comparison between the experimental values and that predicted by other equations for beams with cantilever inclination (α) on the cracking load having As =2Φ16 (ρ= 1.24)

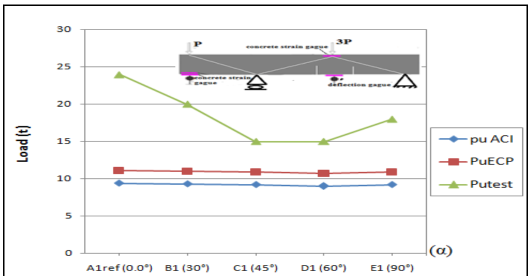


Figure (63): Comparison between the experimental values and that predicted by other equations for beams with cantilever inclination (α) on the cracking load having As =2Φ16 (ρ= 1.24)

4.9.2 With respect to steel reinforcement at span of beams having As =4Φ16 (ρ= 2.48)
1- The cracking loads (Qcr) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI. Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (11.1, 2.12 and 1.05%)
From (α = 30°to 45°) the compared values were (12.5, 0.87 and 0.8%)
From (α = 45°to 60°) the compared values were (14.3, 3.5 and 1.9 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (16.7, 3.65 and 1. 9%) Shown in fig (64).

2- The ultimate loads (Pu) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (14, 1.06 and 0.9%)
From (α = 30°to 45°) the compared values were (29, 1.07 and 0.9%)
From (α = 45°to 60°) the compared values were (constant, 2.18 and 1.84 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (17.7, 2.2 and 1.87%) Shown in fig (65).

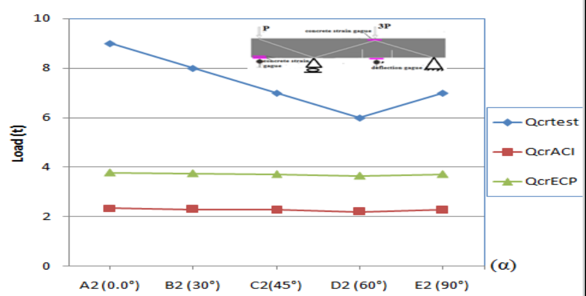
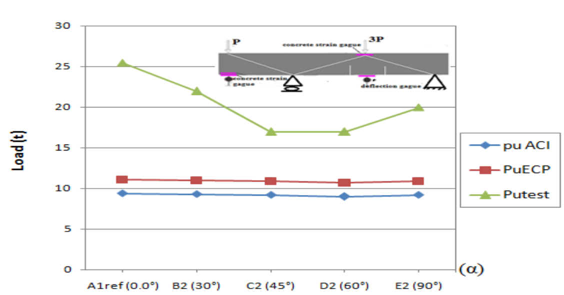


Figure (64): Comparison between the experimental values and that predicted by other equations for beams with cantilever inclination (α) on the cracking load having As =4Φ16 (ρ= 2.48).



Figure(65): Comparison between the experimental values and that predicted by other equations for beams with cantilever inclination (α) on the cracking load having As =4Φ16 (ρ= 2.48)

4.9.3 With respect to steel reinforcement at span of cantilever having As =2Φ16 (ρ= 1.24)

4.9.3.1- The cracking loads (Qcr) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI. Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (12.5, 2.12 and 1.05%)
From (α = 30°to 45°) the compared values were (7.1, 0.87 and 0.8%)
From (α = 45°to 60°) the compared values were (23, 3.5 and 1.9 %)
Finally the increasing inclination of angle from α = (60°to90°) the compared values were (30, 3.65 and 1. 9%) Shown in fig (66).

4.9.3.2- The ultimate loads (Pu) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI. Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (16.25, 1.06 and 0.9%)
From (α = 30°to 45°) the compared values were (25.4, 1.07 and 0.9%)
From (α = 45°to 60°) the compared values were (constant, 2.18 and 1.84 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (20, 2.2 and 1.87%) Shown in fig (67).

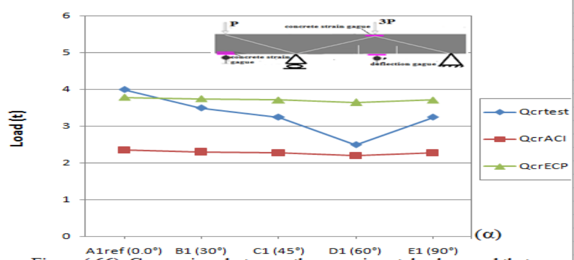
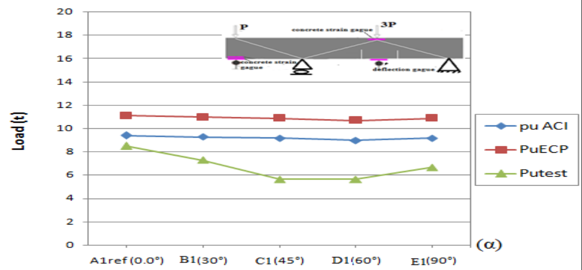


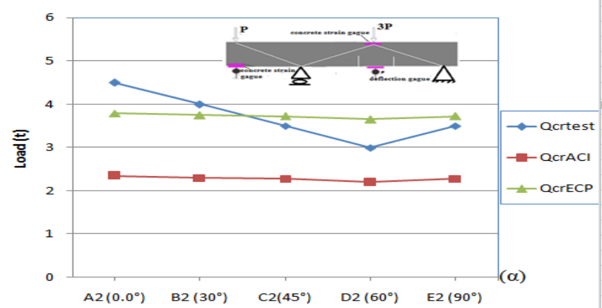
Figure (66): Comparison between the experimental values and that predicted by other equations for cantilevers with cantilever inclination (α) on the cracking load having As =2Φ16 (ρ=1.24)



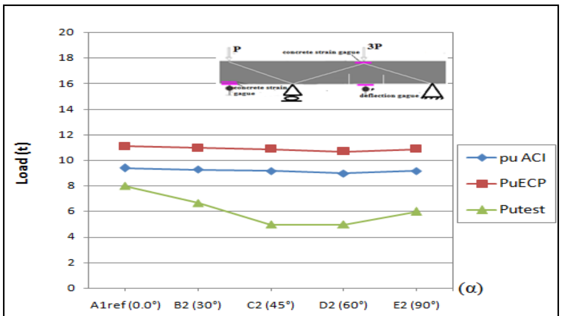
Figure(67): Comparison between the experimental values and that predicted by other equations for cantilevers with cantilever inclination (α) on the cracking load having As =2Φ16 (ρ= 1.24)

4.9.4 With respect to steel reinforcement at span of cantilever having As =4Φ16 (ρ= 2.48)
4.9.4. 1-The cracking loads (Q_{cr}) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (11.1, 2.12 and 1.05%)
From (α = 30°to 45°) the compared values were (12.5, 0.87 and 0.8%)
From (α = 45°to 60°) the compared values were (14.28, 3.5 and 1.9 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (16.7, 3.65 and 1. 9%) Shown in fig (68).

2- The ultimate loads (Pu) when the increasing of angle of inclination (α) from the test results and the predicted values by ACI. Code and ECP equations respectively.
From (α = 0.0°to 30°) the compared values were (14.1, 1.06 and 0.9%)
From (α = 30°to 45°) the compared values were (22.6, 1.07 and 0.9%)
From (α = 45°to 60°) the compared values were (constant, 2.18 and 1.84 %)
Finally the increasing inclination of angle from (α = 60°to90°) the compared values were (18.6, 2.2 and 1.87%) Shown in fig (69).



Figure(68): Comparison between the experimental values and that predicted by other equations for cantilevers with cantilever inclination (α) on the cracking load having As =4Φ16 (ρ= 2.48).



Figure(69): Comparison between the experimental values and that predicted by other equations for cantilevers with cantilever inclination (α) on the cracking load having As =4Φ16 (ρ= 2.48).

V. Conclusions

An experimental work was under taken to investigate the effectiveness of cantilever Inclination (α) and longitudinal steel ratio (*p*) on the shear strength of reinforced concrete beams with cantilever under static loading. The following conclusions can be made from the experimental results.

(1) When the angle of cantilever beam (α) equal to (0.0) has a slight effect on increasing both the cracking and ultimate loads.
(2) The increasing of angle of inclination from (30°to 45°) has a slight effect on decreasing both the cracking and ultimate loads but have an important effect on the maximum deflections, maximum strains and the over all stiffness of beams without any noticeable change in its mode of failure.
(3) The increasing of angle of inclination from (45°to 60°) has a slight effect on decreasing both the cracking and ultimate loads but have an important effect on the maximum deflections, maximum strains and the over all stiffness of beams without any noticeable change in its mode of failure.
(4) The increasing of angle of inclination from (60 to 90) has a slight effect on increasing both the cracking and ultimate loads but have an important effect on the maximum deflections, maximum strains and the over all stiffness of beams without any noticeable change in its mode of failure.
(5) The maximum measured deflection increases, with the increase of the angle of inclination (α) Also the cracking and ultimate deflection decrease.
(6) The addition of main longitudinal bars had a significant effect on the diagonal cracking load, and ultimate load.
(7) The ultimate shear strength is increased by the increase of main longitudinal bars.
(8) Horizontal reinforcement ratio has a pronounced effect in controlling torsion.
(9) The shear load (Qcr) was observed at zone (2) between the mid span load and the roller support. i.e. (through the critical shear zone) equal to double shear load on the cantilever beams.
(10) The best values of results occurred at angle of cantilever inclination (α) for (α=45°) than other cases of the tested beams.
(11) The values of the cracking shear strength of the tested beams show more save in comparison with the corresponding recommended values given in ACI code equation and Egyptian code equation

References

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[2]. ACI (318-83) Building Code Requirements for Reinforced concrete.
[3]. Frank Lloyd Wright preserving significance an Architect, his Innovation, and achieving the organic.
[4]. Mashhour Ahmed Gohneim, design of reinforced concrete structures (volume 1) professor of concrete structures Cairo University. Torsion CB110 and ACI318 codes compared.

سلوك الكمرات الخرسانية المسلحة ذات الكوابيل المائلة

الملخص العربي

تتناول هذه الرسالة دراسة معملية لعشر كمرات ذات كوابيل مستقيمة (0.0) وكوابيل مائلة بدرجة (90,60,45,30) وذلك لدراسة مدي تأثير ميول الكابولي علي الكمرة مع اختلاف زاوية الميل.

الهدف الرئيسي من هذا البحث هو دراسة السلوك الخاص بالكمرات ذات الكوابيل المائلة وذلك تحت تأثير الاحمال الاستاتيكية الكبيرة بداية من المرحلة المرنه منتهية بحد الانهيار. ومن اهم المتغيرات التي تم دراستها في هذا البحث هي : نسبة حديد التسليح الرئيسي في الكمرات وزاوية ميل الكابولي.

جميع الكمرات تم تصنيعها وتجهيزها للاختبار تحت تأثير حملين الاول اعلي منتصف الكمرة والثاني اعلي طرف الكابولي وتصل قيمتة ثلث قيمة الحمل علي الكمرة. جميع الكمرات تم تصنيعها من الخرسانة ذات مقاومة 250 كجم/ سم^2.

وقد اظهرت النتائج ما يلي :

(1) في حالة زاوية ميل الكابولي تساوي (0.0) فان ذلك لة تاثير واضح علي زيادة اقصي حمل في الكمرة والكابولي.

(2) في حالة زيادة زاوية ميل الكابولي من (0.0 الي 60) فان زلك لة تاثير واضح علي نقص قيمة الاحمال علي الكمرة علي الرغم من زيادة قيم الترخيم والانفعال في الكمرة والكابولي.

(3) في حالة زيادة زاوية ميل الكابولي من (60 الي 90) بدات قيمة الاحمال علي الكمرة والكابولي في الزيادة مرة اخري .

(4) اضافة حديد التسليح الرئيسي في الكمرات لة تاثير واضح علي الشروخفي الكمرات وعلي قيم اقصي حمل .

(5) اضافة حديد التسليح الجانبي في الكمرات لة تاثير واضح علي مقاومة عزوم اللي.

(6) اقصي اجهاد للكمرات يزداد بزيادة حديد التسليح الرئيسي.

توصيات البحث والدراسة:

1 - دراسة مدى تأثير بعض العوامل الأخرى على سلوك القص للكمرات الخرسانية المسلحةات الكوابيل المائلة عالية المقاومة مثل تأثير الالياف و درجة الحرارة والرطوبة.

2- **دراسة نفس العوامل الحالية مع استخدام أنواع أخرى من الخرسانة عالية المقاومة والتى تصل مقاومتها إلى 1200 كجم/سم^2 .**