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Analysis of Self Similar Motion in the Theory of Stellar Explosion

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Abstract: *In this paper consider a classical model of stellar explosion ,in a conducting gas of self gravitating gas propagating in a non uniform atmosphere. Similarity principle has been used to reduced the equation governing the flow to ordinary differential equation . Analytical solution of this classical model have been investigated .*

I. Introduction

Carrus etal [1] and Sedov [2] were first to discuss model of stellar explosion in which a star is considered to be a perfect self gravitating gas. The distribution of gas at any moment of time spherically symmetric. In the present paper a analytical solution of the classical model of stellar explosion has been investigated. A number of new solutions has been obtained in which radial oscillation of gas occur after the shock wave passes. Taking Newtonian gravitation into account a through analytical study of self similar motion of a gas dynamics under the effect of magnetic field is developed in the theory of stellar explosion which was earlier applied by Novikov and Bogoyavlenky. It is suppose that originally the star is in equilibrium state the gas density , the pressure P the mass M of the gas within sphere of radius r, the radial gas velocity u, the magnetic field h have the form

$$
\begin{aligned}
&P_1 = Ar^{-w}, \qquad u_1 = 0, \; M_1 = \frac{4\pi A r^{3-w}}{3-w}, \\
&h_1 = Cr^{-k}, \; 2k = w+1 \\
&P_1 = \frac{2\pi A^2 G}{(3-w)\,(w-1)}\, r^{2(1-w)} + \frac{C^2\,(1-k)}{2K}\, r^{-2K},
\end{aligned}
\tag{1.1}
$$

Where A, w and G are constant. As a result of energy libration at the centre of symmetry r = 0: a shock wave travels out from the centre. The motion of gas behind the shock front self similar and adiabatic.

$$\frac{d}{dt}\left(\frac{P}{p^{\lambda}}\right) = 0\,, \quad \lambda > 1 \tag{1.2}$$

II. Equation Of Motion And Boundary Condition

The fundamental differential equation signifying the conservation laws of spherically symmetric motion in a self gravitating gas, where the magnetic field is significant are summer [3].

$$\frac{\partial\rho}{\partial t} + u\,\frac{\partial\rho}{\partial r} + \frac{\rho}{r^2}\,\frac{\partial}{\partial r}\,(ur^2) = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\,\frac{\partial p}{\partial r} + \frac{h}{\rho}\,\frac{\partial h}{\partial r} + \frac{1}{\rho}\,\frac{h^2}{r} + \frac{GM}{r^2} = 0, \tag{2.2}$$

$$\frac{\partial P}{\partial t} + u\,\frac{\partial P}{\partial r} - \frac{\gamma P}{\rho}\left(\frac{\partial\rho}{\partial t} + u\,\frac{\partial\rho}{\partial r}\right) = 0\,, \tag{2.3}$$

$$\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial r} + h\,\frac{\partial u}{\partial r} + h\,\frac{u}{r} = 0, \tag{2.4}$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \tag{2.5}$$

Where r, t, u, ρ, P,m are radial distance from centre, time, velocity density, pressure and mass contained in a sphere of radius r.

The disturbance is headed by an isothermal shock with condition

$$\rho_2 \left(V - u_2\right) = \rho_1 V = m_s \tag{2.6}$$

$$P_2 + \frac{h_2^2}{2} - P_1 - \frac{h_1^2}{2} = m_s \; u_2, \tag{2.7}$$

$$h_2 \left(V - u_2\right) = h_1 V, \tag{2.8}$$

$$E_2 + \frac{P_2}{\rho_2} + \frac{1}{2} (V - u_2)^2 + \frac{h_2^2}{\rho_2} = E_1 + \frac{P_1}{\rho_1} + \frac{1}{2} V^2 + \frac{h_1^2}{\rho_1} \tag{2.9}$$

$$T_1 = T_2, \tag{2.10}$$

$$m_1 = m_2 \tag{2.11}$$

Where suffix 1 and 2 denotes the flow variables just ahead and just behind the shock and front respectively denotes the mass flux per unit area across shock and V denote velocity of shock, is given by

$$V = \frac{dr}{dt}, \tag{2.12}$$

According to Sedov [2], the total energy of gas between spheres of radii r_1, and r_2 in equilibrium is

$$E = 8\pi^2 \, GA^2 \; \frac{1 - 2(w-1)(\tau-1) r^{5-2w}}{(\tau-1)(w-1)\,(3-w)\,(5-2w)}, \tag{2.13}$$

$$E = \frac{32\pi}{3(\tau-1)} \; \log \frac{r_2}{r_1}$$

If w < 5/2, the total energy E enclosed within the sphere of radii r is finite if w $\geq$ 5/2 it is infinite but E < 0 if $\gamma > 0$

and E $\geq$ 0 if $\gamma \leq \gamma_0$ where γ_0 = 2w-1/2(w-1). The solutions of the problem of stellar explosions for w $\geq$ 5/2 in the class of gas motion considered can only be applied to certain astrophysical solutions because of divergence of E at the lower limit. These solutions are regarded as intermediate asymptotic solutions valid out side a small neighborhood of r = 0, solutions with $\gamma \leq \gamma_0$ (E $\geq$ 0) describe the break down of unstable stellar equilibrium. The law of energy liberation in a self similar solutions has the form

$$E = \alpha G^{(5-w)/w} \; A^{5/w} \; t^{2[5-2w]/w} \tag{2.14}$$

Evidently E is independent of the time for w = 5/2 or $\alpha = 0$. The constant α is calculated from the solution it self and in some case turn out to be infinite. The corresponding solutions then provided asymptotic form for a very intense explosion. The following new results may directly be deduced by

(1) For $\gamma < 4/3$, $w = 5/2$ and also for $\gamma < 2\,(w-1)/3$ there exist no solution with a vacuum forming within the gas

For $\gamma < 4/3$, $w = 5/2$, $M = 1$, damped Oscillation of the gas occur after the shock wave passes which are connected with the limit of dynamic system.

(2) For $\gamma < 4/3$, $w = 5/2$ all solutions have a spherically vacuum of increasing radii which forms about the centre, the gas monotonically spreading out from it.

(3) For $\gamma < 1/4$, $w = 5/2$, $M = 1$, the gas returns to equilibrium after the shock wave passes but in case when

$$\gamma < \gamma_1 = \frac{4\,[3 + (2w-5)^2]^{-1}}{8\,(w-1)}$$

repeated damped oscillation taken place.

III. Similarity Solutions

The similarity variables which reduce the equation governing the flow to ordinary differential equation is taken as

$$P = \frac{1}{Gt^2} R(\eta)$$

$$P = \frac{r^2}{(Gt)^4} P(\eta)$$

$$m = \frac{r^3}{Gt^2} M(\eta) \qquad (3.1)$$

$$V = \frac{r}{t} V(\eta)$$

where $\eta = r\,(AGt)^{-1/w}$

By using relation (3.1) the differential equation are transformed as

$$-V + V^2 + \eta \frac{\partial V}{\partial \eta}\left[V - \frac{2}{W}\right] + \frac{2p}{r} + \frac{\eta}{R}\frac{\partial P}{\partial \eta} + \frac{2N^2}{R} + \frac{N}{R}\frac{\partial N}{\partial \eta} + M = 0 \qquad (3.2)$$

$$-\left[\frac{2\eta}{W}\frac{\partial p}{\partial \eta} + 4p\right] V(\eta)\,[2p + \frac{\partial p}{\partial \eta}] - \frac{TP}{R}[-2\,\{\frac{\eta}{W}\frac{\partial R}{\partial \eta} + R\,\} + \eta V \frac{\partial R}{\partial \eta} = 0 \qquad (3.3)$$

$$\left[-2N - \frac{2\eta}{W}\frac{\partial N}{\partial \eta}\right] + V\eta\frac{\partial N}{\partial \eta} + N\frac{\partial V}{\partial \eta} = 0 \qquad (3.4)$$

$$-2\left[\frac{\eta}{W}\frac{\partial V}{\partial \eta} + R\right] + \eta V\frac{\partial R}{\partial \eta} + \left[3VR + R\frac{\partial V}{\partial \eta}\right] = 0, \qquad (3.5)$$

and appropriate transformed jump condition are

$$V(1) = \frac{2}{3}\,\Omega\,,$$

$$R(1) = \frac{V}{1-\Omega},$$

$$P(1) = \frac{2V}{9}\left[\Omega + \frac{1}{\tau M^2} - \frac{\Omega(2-\Omega)}{2(1-\Omega)^2} M_A^{-2}\right],$$

$$M(1) = \frac{2}{3}(V)^{1/2}\left[\frac{1}{1-\Omega}\right] M_A^{-1}, \qquad (3.6)$$

Where

$$\Omega = \left(1 - \frac{2}{\tau M^2} - \frac{A_A^{-2}}{4}\right) - \left[\left(\frac{1}{2\tau M^2} + \frac{M_A^{-2}}{4}\right)^2 + \frac{M_A^{-2}}{2}\right],$$

and

$$V = \frac{9}{2}\,\gamma M^2 \left[2(1+W) - \frac{\tau M^2}{M_A^2}(1-W)\right]^{-1},$$

to investigate numerical flow solution we write flow variable in a non dimension form

$$\frac{u}{u_2} = \eta \, \frac{V(\eta)}{V(1)} \, ,$$

$$\frac{\rho}{\rho_0} = \frac{R(\eta)}{R(1)}$$

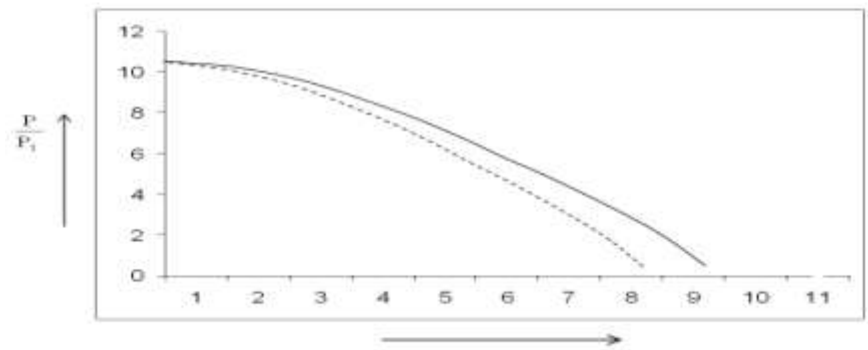
$$\frac{P}{P_2} = \eta^2 \, \frac{P(\eta)}{P(1)}$$

$$\frac{h}{h_2} = \eta^3 \, \frac{M(\eta)}{M(1)}$$

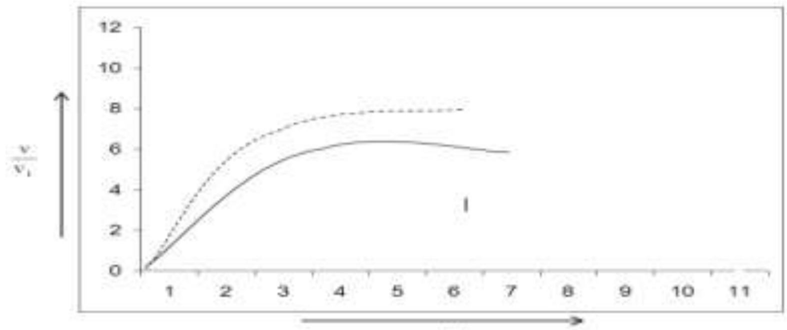
$$\frac{M}{M_2} = \eta \, \frac{3M(\eta)}{M(1)}$$

IV. Result & Discussion :

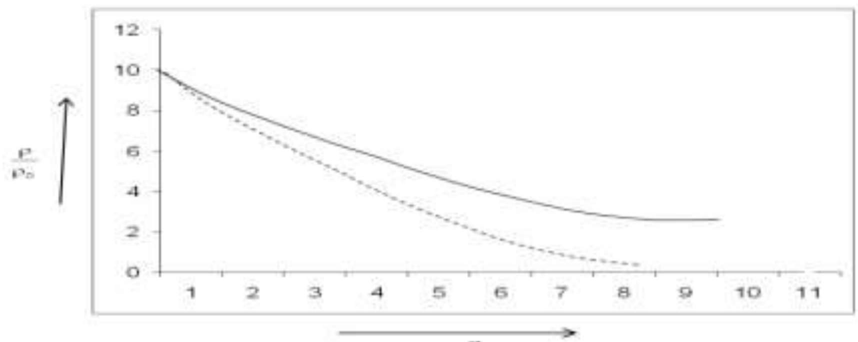
Above relations shows the distribution of velocity, density, pressure, magnetic field and mass distribution in the stellar model, when magnetic field is applied. A numerical approximation may also be obtained which may illustrate the behavior of flow and field variables exactly behind the surface of discontinuity.



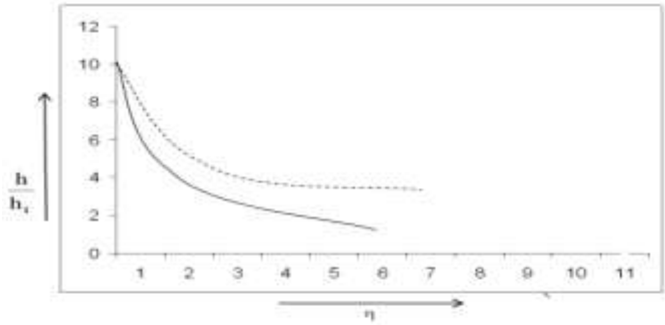
η
variation of Pressure



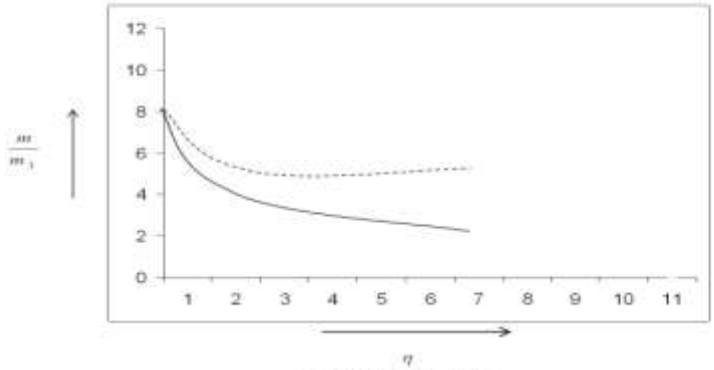
η
variation of velocity



η
variation of Density



variation of magnetic field



variation of Mass

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