

Inventory Model for Ramp type items with Trade Credit under Extra ordinary purchase

Suchismita Pradhan ¹, P.K. Tripathy²

¹(Dept. of Mathematics, Orissa Engineering college, Bhubaneswar-751007, India)

²(P.G. Department of Statistics, Utkal University, Bhubaneswar-751004, India)

Abstract: We have endeavoured here to depict an interactive inventory model between producer (supplier) and prospective buyer (vendor) in the presence of an extraordinary purchase and credit period with two parameter weibull deterioration for items with ramp type demand. The case of shortages is also studied. Additionally, we deal with the problem of a supplier disposing a large inventory level through an offer to a prospective buyer in the form of a credit period. From our model we confirm that the behaviour of replenishment period and trade credit period and change of the total variable cost observed from numerical evidence should be interpreted with caution in any business organisation.

Keywords: credit period, ramp type demand, shortages, Weibull deterioration

I. Introduction

The trade credit period offered by the supplier to the retailer, encourages retailer to buy more and it is a powerful promotional tool that attracts vendor to go for extra ordinary purchases. An inventory model is developed in which inventory are replenished in ramp type admitting a weibull deterioration and the supplier offers the progressive trade credit to the buyer. A progressive trade credit means that trade forwarding is carried out as follows: if the retailer pays the outstanding amount the permissible delay period the supplier does not charge any interest. If the payment is done after this period, the retailer will have to pay interest charges. Ramp type demand is one in which demand increase up to a certain time after which it stabilizes and become constant. Ramp type demand precisely depicts the demand of the items, such as newly launched fashion goods and cosmetics, garments, automobiles etc., for which the demand increases as they are launched into the market and some time after it becomes constant.

The model is quite easy as a negotiating tool especially, since no reaction is anticipated from other suppliers and/or other buyers to this type of offer. This is so because such practices are acceptable within the logistic industry because the once-and-for all nature of the offer itself precludes other firms from reacting on time. In fact, the ability to discriminate across prospective wholesalers with minimum disruption of accepted business practices, quite often vendors, the credit period is more attractive payment reduction mode when confronted with large levels of inventory of a particular commodity, a suppliers approach towards the availability of the extra stock for which demands from wholesaler behave ramp but deteriorate weibull, can be averred as an offer to the prospective buyer for an extra ordinary purchase under defined credit period within which no payment is required, in exchanges for the purchase of an additional units offer over and above the regular order.

The work of the researches who used ramptype demand as demand function for the formulation of economic order quantity inventory model is summarized in the following table :

Reference	Objectives	Constraints	Contributions	Limitations
Mandal & Pal (13) 1998	Finding EOQ	Ramp type demand, const, rate of deteriorations. Shortage not allowed	An approximate Solution for EOQ is obtained	Approximate Solution, Const. rate of deterioration
Kun-Shan & ouyang (12) 2000	Finding EOQ	Ramp type demand, Const, rate of deterioration shortage allowed	An exact solution for EOQ is obtained	Constant rate of deterioration
Jalan, Giri S.Chaudhuri (11) 2001	Finding EOQ	Ramp type demand weibull deteriorations, shortage allowed	EOQ given by Numerical technique	EOQ can't be obtained analytic-cally
Kun-Shan [1], 2001	Finding EOQ	Ramp type demand weibull deteriorations, shortage allowed, Partial back logging	EOQ obtained for 3 numerical examples	Method explained by numerical examples
Jain & Mukesh [2], 2007	Finding EOQ	Ramp type demand weibull deterioration starting with shortage	An exact solution for EOQ model	Method explained by numerical examples

A glance at the above table reveals that different researchers developed EOQ models by taking ramp type demand, deterioration (constant/weibull distribution) and shortage (allowed/not allowed/in starting). Out of these researchers only (12) and (2) obtained an exact solution for EOQ.

The motivation behind developing an EOQ model in the present paper is to introduce time dependent rate of deterioration (two parameter weibull distribution deterioration) incorporating ramp type demand with the conditions of permissible delay in payments. Main emphasis is laid on working out an exact solution for the desired model. An example is provided which stands in support of the developed model. In section 2, assumption and notations are presented. Section 3, the mathematical model is formulated and in section 4, numerical examples are cited to illustrate the model and sensitivity analysis of the optimal solution with respect to trade credit period is carried out.

II. Assumptions And Notations

Assumptions

The following assumptions are used to develop the mathematical model:

1. The inventory system deals with single item only.
2. Replenishment occurs instantaneously on ordering (i.e. lead time is zero).
3. No repair or replacement of deteriorated units.
4. Demand function $R(t)$ is taken to be ramp type function of time.

$$R(t) = Ae^{b[t-(t-\mu)H(t-\mu)]}$$

where $H(1-\mu)$ is the well known Heaviside's step function defined as:

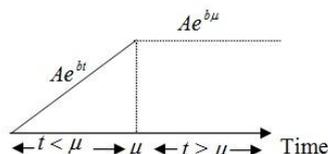
$$H(t-\mu) = \begin{cases} 1 & t \geq \mu \\ 0 & t < \mu \end{cases}$$

And

A = initial demand rate

b = a constant governing the exponential demand rate.

The pictorial representation of demand function $R(t)$ is shown in figure



5. Shortages are allowed and backlogged.

Notations:

- $I(t)$ = Inventory level at time t .
- $\phi(t) = \alpha \beta t^{\beta-1}$ = Deterioration follows weibull distribution where $0 < \alpha < 1, \beta > 0$.
- K, P, h, S = Ordering cost of inventory \$ per order, purchase cost \$ per unit, holding cost excluding interest charges \$/unit / year, shortage cost \$/unit/year respectively.
- I_e, I_r = interest earned, interest charges \$/year respectively where $I_r \geq I_e$.
- M = permissible delay in settling the accounts, $0 < M < 1$ year.
- T, T_1 = length of the replenishment cycle, time when shortages occur ($0 \leq T_1 < T$) respectively.
- $TVC_1(T_1, T), TVC_2(T_1, T)$ = Average total inventory cost per unit time for $M \leq T_1$ and for $M > T_1$ respectively.

III. Model Formulation

The planning horizon is divided into sub-intervals of length T units. Orders are at time point $0, T, 2T, 3T, \dots$ being the order quantity being sufficient to bring the stock height to a certain maximum level.

Depletion of inventory occurs due to combined effects of demand and deterioration in the interval $0 < t < T_1$. Demand is completely backlogged in the interval $T_1 < t < T$. Variation of inventory level $I(t)$ with respect to time is given by

$$\frac{dI(t)}{dt} + \phi(t)I(t) = -R(t)$$

or $\frac{dI(t)}{dt} + \alpha \beta t^{\beta-1} I(t) = -Ae^{b[t-(t-\mu)H(t-\mu)]}$ (1)

$0 < t < T_1$

For shortage the equation becomes

$$\frac{dI(t)}{dt} = -Ae^{b[t-(t-\mu)H(t-\mu)]} \dots\dots\dots(2)$$

$$T_1 < t < T$$

For $t \geq \mu$ the demand $R(t)$ becomes $Ae^{b\mu}$ as $H(t - \mu) = 1$.

Hence, equations (1) and (2) can be expressed as

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -Ae^{b\mu} \dots\dots\dots(3)$$

and $\frac{dI(t)}{dt} = -Ae^{b\mu} \dots\dots\dots(4)$

with boundary condition $I(T_1)=0$

The solution of equation (3) is given by

$$I(t) = \frac{Ae^{b\mu}}{\alpha^2 \beta^2} \left[T_1^{1-2\beta} (\alpha\beta T_1^\beta + \beta - 1) e^{\alpha(T_1^\beta - t^\beta)} - t^{1-2\beta} (\alpha\beta t^\beta + \beta - 1) \right] \dots\dots\dots(5)$$

for $0 < t < T_1$ only for inventory

and the solution of (4) is given by

$$I(t) = Ae^{b\mu} (T_1 - t) \text{ for } T_1 < t < T, \text{ only for shortage.}$$

Expected holding cost per unit per unit time is

$$HC = h \int_0^{T_1} I(t) dt$$

So,

$$HC = \frac{hAe^{b\mu}}{-\alpha^2 \beta^2} \left[\frac{T_1^{1-2\beta} (\alpha\beta T_1^\beta + \beta - 1) \left\{ (T_1^{1-\beta} - (\frac{1}{\beta} - 1)T_1^{1-2\beta} \cdot \alpha^{-1}) \right\}}{\alpha\beta} - \frac{\alpha\beta T_1^{2-\beta}}{2-\beta} - \frac{\beta T_1^{2-2\beta}}{2-2\beta} + \frac{T_1^{2-2\beta}}{2-2\beta} \right] \dots\dots\dots(7)$$

$$\text{Average number of deteriorated items} = I(0) \cdot \int_0^{T_1} Ae^{b\mu} dt$$

$$= \frac{Ae^{b\mu}}{\alpha^2 \beta^2} \left[T_1^{1-2\beta} (\alpha\beta T_1^\beta + \beta - 1) e^{\alpha T_1^\beta} \right] - Ae^{b\mu} T_1$$

Hence the expected deterioration cost per unit time is

$$DC = P \left[\frac{Ae^{b\mu}}{\alpha^2 \beta^2} \left\{ T_1^{1-2\beta} (\alpha\beta T_1^\beta + \beta - 1) e^{\alpha T_1^\beta} \right\} - Ae^{b\mu} T_1 \right] \dots\dots\dots(8)$$

Over the interval (T_1, T) , expected shortage cost per unit per unit time

$$SC = S \int_{T_1}^T Ae^{b\mu} (T_1 - t) dt$$

So,

$$SC = SAe^{b\mu} \left[T \left(T_1 - \frac{T}{2} \right) - \frac{T_1^2}{2} \right] \dots\dots\dots(9)$$

Panel-1: $M \leq T_1$

For $M \leq T_1$, the buyer has stock on hand beyond M and so he can use the sale revenue to earn interest at an annual rate I_e up to T_1 . The interest earned, denoted by IE_1 , is therefore,

$$IE_1 = PI_e \int_0^{T_1} (T_1 - t) Ae^{b\mu} dt = \frac{PI_e Ae^{b\mu} T_1^2}{2} \dots\dots\dots(10)$$

However, beyond the fixed credit period M , the unsold stock is assumed to be financed with an annual rate I_f and the interest payable, denoted by IP , is given by

$$+ \frac{SAe^{b\mu}}{T} \left[T \left(T_1 - \frac{T}{2} \right) - \frac{T_1^2}{2} \right] + \frac{PI_e Ae^{b\mu}}{T} \left[MT_1 - \frac{T_1^2}{2} \right]$$

The optimal values of T_1 and T (say T_1^* and T^*), which minimize total average cost per unit per unit time, can be obtained by solving the following equations simultaneously.

$$\frac{\partial TVC_2(T_1, T)}{\partial T_1} = 0 \quad \text{and} \quad \frac{\partial TVC_2(T_1, T)}{\partial T} = 0 \quad \dots\dots\dots (14)$$

Provided these satisfy the sufficient conditions

$$\frac{\partial^2 TVC_2(T_1, T)}{\partial T_1^2} > 0 \quad \text{and} \quad \frac{\partial^2 TVC_2(T_1, T)}{\partial T^2} > 0$$

and

$$\left[\left\{ \frac{\partial^2 TVC_2(T_1, T)}{\partial T_1^2} \right\} \left\{ \frac{\partial^2 TVC_2(T_1, T)}{\partial T^2} \right\} - \left\{ \frac{\partial^2 TVC_2(T_1, T)}{\partial T_1 \partial T} \right\}^2 \right] > 0$$

Compute the optimal values of T_1 and T (say T_1^* and T^*) for a given value of M such that these values must satisfy both the conditions of equations (12) and (14) respectively.

IV. Numerical Example

To illustrate the preceding theory, the following example is considered. Let $K=100, h=12, A=100, P=200, S=30, b=0.08, \mu=0.12, \alpha=0.002, \beta=1.5, I_r=0.15, I_e=0.13$.

TABLE -1

Panel-1 $M \leq T_1$

M	T_1	T	TVC_1
5	95.76369	117.795334	101454.257812
10	90.82537	107.173447	92978.382812
15	88.88950	100.211868	85601.265625
20	72.45267	74.584206	54197.382812**
25	52.43768	53.186420	64794.816406
30	48.23120	49.149250	74697.890625
35	46.01540	46.054947	85235.234375
40	43.97472	44.035244	96776.687500
45	40.87325	36.323273	107567.923300

TABLE -2

Panel-2 $M > T_1$

M	T_1	T	TVC_1
5	4.8528610	2239.550293	6814908.50000
10	9.9910000	586.812317	1794847.50000
15	13.5000000	348.279816	1062690.62500
20	16.8457336	241.840912	730912.875000
25	23.4326740	145.568481	420037.593700
30	27.9364800	113.839600	309742.000000
35	32.9955273	92.791710	228593.750000
40	37.4598600	81.973366	178945.687500
45	42.8763490	75.265831	136440.578120
50	48.6598300	73.447083	106138.750000
55	54.3686500	75.254250	87315.468750
60	59.9962543	79.283096	76616.335938
65	64.9540520	84.016418	71917.140625
70	69.9952670	89.544540	70441.625000**
75	74.8652000	95.323685	71385.093750
80	78.9943760	100.444275	73608.523438

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85	82.8765300	105.369896	76649.312500
90	89.0194800	113.239166	82909.484375

Nature of Problem Studied:

Numerical analysis suggests several conclusions. First an increase in ramp demand riskiness measured by variance of demand with market return leads to lower reorder point and lower lot size when the replenishment time is lower than the trade credit period. When replenishment time is greater, an increase in demand riskiness decreases the reorder point but may result in greater lot size. Second, the average inventory in each panel is a strictly decreasing function of the risk of demand. Then it is reasonable to infer that this value of the fixed opportunity cost of capital is a good approximation of the true risk of inventory investment in the given scenario. On the other hand, if the difference in replenishment and trade credit period is large, then it is not approximate to use this fixed cost of the capital in the cost minimization model for the given scenario.

Panel-1 and Panel-2 together indicate that the total cost is relatively insensitive to adjustment for small risk.

Sensitivity Analysis of decision variables:

TABLE -3

Case – I

Parameters		T1	T	TVC1
k	+50%	72.426946	74.553215	54139.300781
	+40%	72.4364328	74.565796	54163.171875
	+30%	72.438624	74.567268	54165.71875
	+20%	72.439679	74.568581	54167.984375
	+10%	72.44845	74.579109	54187.89062
	-10%	72.45857	74.591309	54210.73437
	-20%	72.46232	74.59584	54219.136719
	-30%	72.46623	74.600571	54227.91015
	-40%	72.46659	74.601044	54228.57031
-50%	72.469582	74.604668	54235.2343	
h	+50%	95.45692	95.470215	120169.7031
	+40%	93.7895	96.292313	106331.84375
	+30%	92.3689945	95.902901	99138.9375
	+20%	90.89457	95.409019	91993.24218
	+10%	88.7683	94.029984	84323.648488
	-10%	72.9857	77.399048	47065.51953
	-20%	88.76593	99.342888	62854.6284
	-30%	81.4678	90.829727	50835.296875
	-40%	77.89745	87.181267	42073.9570
-50%	75.011295	84.348061	34159.80468	
a	+50%	88.8774	95.529678	118333.625
	+40%	81.76598	86.260284	99151.07031
	+30%	82.99876	87.856865	94187.8125
	+20%	86.978	93.044434	92435.29687
	+10%	88.789	95.413902	86687.78906
	-10%	120.96248	135.104996	80330.4375
	-20%	128.8794	143.230179	69007.5390
	-30%	120.6578	134.772934	62524.7929
	-40%	125.67835	140.068878	52675.5585
-50%	127.9853	142.365585	43367.8828	
P	+50%	83.19765	94.409050	39183.277344
	+40%	84.0128645	94.35424	39240.0625
	+30%	84.678946	94.020721	39125.625
	+20%	86.1287	94.700768	39042.25
	+10%	86.9875	94.477814	39222.722656
	-10%	78.9545	81.497696	32758.792969
	-20%	91.99768	96.392921	39055.1875
	-30%	92.98564	95.948715	38431.378906

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Parameters		T1	T	TVC1
	-40%	96.0178366	97.857109	38220.242188
	-50%	97.43971	97.690475	37357.125
S	+50%	83.09487	88.82383	71454.59375
	+40%	84.78639	90.800217	73764.226562
	+30%	86.99687	93.455887	76448.8125
	+20%	87.3784	93.833466	77030.234375
	+10%	79.0167	83.036201	66449.867188
	-10%	89.001728	95.555222	79214.828125
	-20%	89.78563	96.453239	80127.828125
	-30%	90.0012	96.566238	80556.296875
	-40%	93.98672	102.111519	83508.945312
	-50%	95.01275	103.616875	81470.140625
b	+50%	87.001956	92.700577	77810.890625
	+40%	87.29847	93.162437	77964.796875
	+30%	87.96839785	94.112541	78483.554688
	+20%	87.99462	94.222603	78355.773438
	+10%	89.00176	95.615669	79160.5625
	-10%	87.9365	94.374039	77839.328125
	-20%	88.6592004	95.396774	78376.625
	-30%	89.87245	97.063889	79318.4375
	-40%	87.86529	94.508125	77312.125
-50%	87.2372	93.75972	76539.40625	
μ	+50%	71.2567	78.007821	47393.5274
	+40%	71.45	77.821274	48354.757812
	+30%	71.6897	77.004845	49812.042969
	+20%	71.93	76.185272	51256.5
	+10%	72.299987	75.518417	52981.769531
	-10%	72.73	73.795502	55681.714844
	-20%	72.999568	72.992348	57132.417969
	-30%	73.3089	72.23017	58656.109375
	-40%	73.6234	71.467606	60178.781250
-50%	---	----	---	
α	+50%	63.2865	60.449309	40901.75390
	+40%	69.76293	69.292589	50130.132812
	+30%	70.01983	69.988945	51950.382812
	+20%	75.39685	77.405922	59834.714844
	+10%	81.956928	86.401176	68821.90625
	-10%	81.87845	87.422493	69691.046875
	-20%	104.578274	117.539215	99305.460938
	-30%	116.7265	134.145294	114489.445312
	-40%	134.87583	158.875504	135315.8125
-50%	142.002842	168.873138	149468.75	
β	+20%	---	---	---
	+10%	56.448975	44.460365	58014.921875
	-10%	105.006752	119.548264	90408.484375
	-20%	450.996847	592.010925	373028.78125
	-30%	---	---	---
I _r	+50%	84.4216	89.706955	74203.34375
	+40%	84.97182	90.424088	74845.109375
	+30%	85.52378	91.144257	75469.992188
	+20%	86.62572	92.583778	76663.046875
	+10%	88.893	95.549980	78904.03125
	-10%	93.7328	101.874413	82840.601562
	-20%	92.4893	100.252679	81930.078125
	-30%	91.3873	98.813095	81066.875
-40%	89.26398	96.0355	79244.960938	

Parameters		T1	T	TVC1
	-50%	87.4239	93.627579	77483.75
I _e	+50%	87.90182	105.105629	92516.1875
	+40%	87.29856	102.141068	89306.42187
	+30%	91.5328	106.093788	91035.015625
	+20%	87.58753	98.300255	84091.023438
	+10%	87.786592	96.365517	81200.625
	-10%	78.653912	80.345985	62995.816406
	-20%	73.984537	72.977371	51344.105469
	-30%	81.98348	80.176331	60190.6875
	-40%	87.97836	84.660141	61320.644531
	-50%	88.01002587	82.092384	56089.675781

Table-3 presents the results for the panel-1 when interest charge is large. A consistent observation is that both the total variable cost and the replenishment time period increases as the variance demand with inventory increases. Variance of demand with inventory increases, the lot size increases initially and then decreased. It seems that lot size is flexible so that it can be greater/lower depending on the cost structure of the firm and the variance of market demand with the on hand inventory level. The table demonstrates that the benefit of using risk adjusted inventory policies in total variable cost can be significant only when α increases less than 30%. Our result shows, how an increase in total inventory cost may lead to an increased risk of k, S, b, μ and I_r . Firms with large time replenishment should have use a lower value of parameters h, a, p and I_e of capital input compensate for the increased risk of cash flows. This compliments the contentions of many practitioners about demand process, deterioration, sales and shortage and thereby the determination of the behaviour of the inventory process. This structure seems to be useful in providing insight into the problem as a whole and clarifying the different interactions within the spectrum of model variations.

This is not only accounts for the higher riskiness of cash flows, but also gives incentives to reduce the cost and control their inventory levels more meaningfully. Such an effort could also provide a number of other indirect benefits like improved quality, and less deterioration.

The sensitivity of the behaviour of decision variables trade credit and time replenishment is some what erratic in Panel-2 if the time for replenishment exceeds the trade credit period.

V. Scope and Limitations

An economic order quantity model has been developed considering ramp type demand, incorporating some realistic features such as deterioration, supplies credits and shortages. The credit approach permits a proper recognition of the financial implication of the opportunity cost in inventory analysis . Some items such as fashionable clothes, domestic goods (newly launched), luxury items, electronics products are easily identifiable with such kind of setup. The model is very practical since often the retailer has the capital with him, but business sense makes him to take advantage of the supplier credit.

The extent to which the credit period has affected the business world is clearly elucidated through the result, where the effect of credit period is visible shown over the total system cost.

VI. Concluding remarks

Firms with a large replenishment time and high variance of demand with the market return can benefit subcutaneously by using risk adjustment policy as indicated by reduction of total variable cost. We reiterate that the behaviour should be interpreted with caution for both replenishment period and trade credit period and change in total variable cost observed from numerical evidence should be interpreted with caution. However, the penalties are relatively small when the opportunity cost of capital is small. This suggests that the penalty in present cost from using a large value of opportunity cost of capital can be as high as 20%. The cost minimization model should be lower for firms with shorter replenishment times than for firms with large replenishment times. On the other hand when demand is high the penalty of using a low value of opportunity cost of capital can be as high as 10%. This suggests that the opportunity cost of capital in the cost minimization model should be higher for firms with large replenishment time and large variance of demand.

Some extensions to this model appear feasible. One possibility is to consider the effect of indicating the special order when the on-hand inventory is not zero. The incorporation of default risk may provide additional insights. We restrict our consideration to single stage manufacturing items of ramp type demand. In addition, our results might also be useful for multistage items where different items are demanded variously. The developed model may be generalised by assuming a multiperiod version and its variations. However, these extensions perhaps require different mathematical methods. The study of these issues justifies additional

research. Researchers on developing performance evaluation measures index could also profit from anchoring the concept of maximizing the value of the firm.

There is ample scope for further extension for research and study. For example, we may extend the model by introducing the inflation dependent demand to more generalized demand pattern that fluctuates with inflation and ramp type demand rate. Also, we could extend the model to incorporate some more features, such as quantity discount, two ware houses with multi-items in the system and permissible delay in payment with cash discount.

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