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Beyond Equations: Surprising Applications Of Algebra In Diverse Mathematical Fields

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Abstract
must have knowledge of the practical applications of algebra and how it is applied. Due to the fact that mathematics is used in day-to-day life, we may now examine the ideas of algebra and the applications of algebra in day-to-day life. At this point in time, we have developed a modest division of the objects that are carried about in a bag when shopping. When we eventually get to the algebra part of pre-algebra, it's called Basic Algebra. The ideas that are presented here will be used in every subsequent math course that you enroll in after this one. We'll get you started on some fun activities like plotting graphs and figuring out solutions to difficult equations. In light of the fact that we are now studying algebra, however, there is a real-world application of algebra that may be found in geometry. The social media platforms available these days have significantly advanced. Because we are unable to answer such figured riddles, we will need to resort to employing algebraic equations.
Keywords *Variables, Equation, Variable Exponent, Algebra*

I. INTRODUCTION

Mathematically speaking, algebra is a subfield of mathematics that focuses on relations, operations, and the constructs of those three. It is a fundamental concept in mathematics and has a wide range of uses in our everyday lives. It is also one of the building blocks of mathematics. Algebra, in addition to its importance as a foundational topic in mathematics, is of great assistance to students of all ages in acquiring a comprehensive grasp of other advanced subfields of mathematics, such as calculus, geometry, arithmetic, and so on. A field of mathematics that involves the generalization of arithmetic operations and connections via the use of alphabetic symbols to represent unknown numbers or members of defined groups of numbers. The subfield of mathematics that deals with more abstract formal structures, such sets and groups, among other examples.

I began compiling interesting linear algebra applications quite a few years ago, and the collection that resulted from that effort is presented here. The applications primarily pertain to the primary areas of mathematical study that I am interested in, which are combinatorics, geometry, and computer science respectively. The vast majority of them are mathematical, in the sense that they prove theorems, and some of them incorporate ingenious techniques of calculating things, sometimes known as algorithms. It is not uncommon for one to be caught off guard when linear algebraic techniques make an appearance. After a certain point in time, I began referring to the things in the collection as "miniatures." After that, I came to the conclusion that in order to be eligible for a miniature, a whole presentation of a result, including the context and everything else, shouldn't be longer than four typeset pages (in A4 size). This rule is completely arbitrary, as many rules are, but it does have some foundation in reason. That foundation is the fact that this extent can typically be covered efficiently in a lecture that is ninety minutes long, which is the regular duration at the institutions where I have had the opportunity to teach. Then, of course, there are certain exceptions to the norm, miniatures that are just six pages long and which I simply couldn't bring myself to leave out. I believed that thirty-three was a decent enough number and a reasonable spot to stop at, despite the fact that the collection could certainly be expanded endlessly. The exposition is written mostly for teachers (I've taught virtually all of the sections at some point in my career), but it is also meant for students who are interested in beautiful mathematical concepts even if they demand some level of mental effort. It is hoped that the content is ready for use in class, in which case any specifics that are left to the reader should not include any temptations. I will assume that you have some prior knowledge of elementary linear algebra, a passing acquaintance with polynomials, and some knowledge of the nomenclature associated with graph theory and geometry.

The difficulties of the parts range from easy to moderate to severe, and I have organized them in this from what I consider to be the least difficult to the most challenging. It was important to me that each segment be able to stand on its own. If you have a solid foundation from your undergraduate studies, you may begin reading the book from. This is somewhat in contrast to the organization of content found in a standard mathematics textbook, in which new concepts are introduced gradually. in order to comprehend anything presented on page, one typically has to be familiar with the information presented on pages that came before it, or, with any luck, the that are most suited. Naturally, the anti-textbook structure results in some tedious repeats, and perhaps more importantly, it places a cap on the level of intricacy that may be achieved. On the other side, I think there are benefits as well, including the following: After realizing that I couldn't retain the crucial terminology between the typically brief reading periods, I decided to give up reading numerous textbooks well before page People who have young children will understand what I'm talking about. The reader may, after going through numerous parts, notice some similar patterns in the supplied proofs.

These patterns may be addressed at great length, but I have chosen not to include any general explanations on linearalgebraic techniques in this article. Nothing in this work is unique, and many of the examples are rather widely known and can be found in a wide variety of sources (in some instances, including other books I've written). The following is a list of many different general reference books. In the cases where I was able to locate the original sources, I have also given references to them. However, I have tried to keep the historical comments to a bare minimum, and I have only made a limited attempt to trace the roots of the concepts (many apologies to writers whose work is mentioned incorrectly or not at all; I would be pleased to know about such instances). I would be grateful if you could point out any errors and provide thoughts on how the exposition may be improved.

Mathematical Induction

Let's say we want to demonstrate that.

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

where n is any natural number. This formula can be checked quite simply for relatively small numbers such as n = 1, 2, 3, or 4, but it is hard to check it for all of the natural numbers on an individual basis. In order to demonstrate that the formula is correct in all circumstances, a more comprehensive approach is necessary. Assume that the equation has been checked for accuracy for the first n instances. From the information that we have, we are going to make an effort to demonstrate that we can derive the formula for the (n+1)th example.

$$1 = \frac{1(1+1)}{2}.$$

Providing that the first n instances have been validated, then

$$\begin{aligned} 1 + 2 + \cdots + n + (n+1) &= \frac{n(n+1)}{2} + n + 1 \\ &= \frac{n^2 + 3n + 2}{2} \\ &= \frac{(n+1)[(n+1)+1]}{2}. \end{aligned}$$

Induction via mathematics. We give a specific proof for the smallest integer being considered, followed by a generic argument showing that if the statement holds for a given case, then it must also hold for the next case in the sequence. This allows us to avoid the impossible task of attempting to verify a statement about some subset S of the positive integers N on a case-by-case basis, which would be the case if S is an infinite set. Instead of doing this, we give a specific proof for the smallest integer being.

If $n = 1$, The binomial theorem is straightforward to demonstrate. Assume for the moment that the answer is correct for any value of n that is larger than or equal to Then.

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n \\
&= (a+b)\left(\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}\right) \\
&= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}b^{n-k} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n+1-k} \\
&= a^{n+1} + \sum_{k=1}^{n}\binom{n}{k-1}a^k b^{n+1-k} + \sum_{k=1}^{n}\binom{n}{k}a^k b^{n+1-k} + b^{n+1} \\
&= a^{n+1} + \sum_{k=1}^{n}\left[\binom{n}{k-1} + \binom{n}{k}\right]a^k b^{n+1-k} + b^{n+1} \\
&= \sum_{k=0}^{n+1}\binom{n+1}{k}a^k b^{n+1-k}.
\end{aligned}$$

We have a statement that is comparable to the mathematical induction principle, and it may be extremely helpful in many situations.

OBJEACTIVES

1. The Study Applications of Algebra in Mathematics.
2. The Study Real Life Application Where Algebra Is Used.

The Division Algorithm

The method for division is one of the applications of the principle of well-ordering that we shall use rather often.

$$a = bq + r$$

This is an excellent illustration of the form of evidence known as "existence and uniqueness." Before moving on, we have to establish that the numbers q and r are in fact real.

$$S = \{a - bk : k \in \mathbb{Z} \text{ and } a - bk \geq 0\}$$

The Euclidean Algorithm

In addition to its many other applications, the Theorem enables us to calculate the greatest common divisor of two numbers.

Let's find the 945 and 2415 that have the greatest common divisor by computing it. Before anything further, note that

$$\begin{aligned}
2415 &= 945 \cdot 2 + 525 \\
945 &= 525 \cdot 1 + 420 \\
525 &= 420 \cdot 1 + 105 \\
420 &= 105 \cdot 4 + 0.
\end{aligned}$$

Reversing our steps, 105 divides 420, 105 divides 525, 105 divides 945, and 105 divides 2415. As a result, 105 is the perfect divisor for both 945 and 2415. If d were still another factor that 945 and 2415 had in common, then it would also need to be able to divide the number 105. As a result, the solution to this equation is 105.

When completing algebra issues, students are often required to think in a more abstract manner than they do when working on simple mathematical tasks such as arithmetic. Reasoning based on algebra demands pupils to process numerous pieces of complicated information simultaneously, which might restrict students' ability to acquire new information and expand their knowledge base. (This kind of thinking is frequently characterized as exerting a high cognitive load or as demanding working memory, both of which might interfere with the capacity of pupils to learn.4) Solved issues may lighten the load of abstract thinking for students by presenting them with the problem and several parts of the solution all at once, therefore facilitating more effective learning and reducing the time required to complete individual steps.

Row Reduction and Echelon Forms

In this lesson, not only will you receive additional experience with row reduction, but you'll also learn about two key kinds of matrix forms. In this section, we will also analyze the circumstances under which a linear system either has a single solution, an unlimited number of solutions, or no solution at all. In the last step, we will discuss a practical parameter that is referred to as the rank of a matrix.

Consider the system in a linear fashion.

$$\begin{aligned} x_1 + 5x_2 - 2x_4 - x_5 + 7x_6 &= -4 \\ 2x_2 - 2x_3 + 3x_6 &= 0 \\ -9x_4 - x_5 + x_6 &= -1 \\ 5x_5 + x_6 &= 5 \\ 0 &= 0 \end{aligned}$$

possessing an enhanced matrix.

$$\begin{bmatrix} 1 & 5 & 0 & -2 & -1 & 7 & -4 \\ 0 & 2 & -2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -9 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 5 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The augmented matrix described above has the following properties:

A matrix is said to be in row echelon form (REF) if and only if it satisfies the conditions P^1 and P^2. In REF, a leading entry is the non-zero item that is located to the left of the row's beginning.

$$\begin{bmatrix} \mathbf{1} & 5 & 0 & -2 & -1 & 7 & -4 \\ 0 & \mathbf{2} & -2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \mathbf{-9} & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & \mathbf{5} & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The fact that all entries below a leading entry are equal to zero is one of the consequences of the property P^2:

$$\begin{bmatrix} \mathbf{1} & 5 & 0 & -2 & -4 & -1 & -7 \\ \mathbf{0} & \mathbf{2} & -2 & 0 & 0 & 3 & 0 \\ \mathbf{0} & \mathbf{0} & 0 & \mathbf{-9} & -1 & 1 & -1 \\ \mathbf{0} & \mathbf{0} & 0 & \mathbf{0} & \mathbf{5} & 1 & 5 \\ \mathbf{0} & \mathbf{0} & 0 & \mathbf{0} & \mathbf{0} & 0 & 0 \end{bmatrix}$$

Vector Equations

Following that, we will discuss vectors and vector equations. In particular, we discuss the linear combination issue, which poses the question of whether or not it is feasible to represent one vector in terms of other vectors; we will go into more detail in the sections that follow. As we are about to see, the solution to the issue of linear combinations is just the solution to a linear system of equations.

$$\mathbf{v} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Here is a vector in

$$\mathbf{v} = \begin{bmatrix} -3 \\ 0 \\ 11 \end{bmatrix}$$

Here is a vector in

$$\mathbf{v} = \begin{bmatrix} 9 \\ 0 \\ -3 \\ 6 \\ 0 \\ 3 \end{bmatrix}$$

We are able to do operations such as adding and subtracting vectors, as well as multiplying vectors by integers or scalars. As an example, the addition of two vectors is shown here:

$$\begin{bmatrix} 0 \\ -5 \\ 9 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ 9 \\ 3 \end{bmatrix}$$

And finally, the product of a scalar and a vector when multiplied together:

$$3 \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \\ 15 \end{bmatrix}$$

And now we will do both procedures simultaneously:

$$-2 \begin{bmatrix} 4 \\ -8 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} -2 \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \\ -6 \end{bmatrix} + \begin{bmatrix} -6 \\ 27 \\ 12 \end{bmatrix} = \begin{bmatrix} -14 \\ 43 \\ 6 \end{bmatrix}$$

These mathematical processes are collectively referred to as "the algebra" of vectors. Following is an example that demonstrates how vectors may be utilized in a manner that is quite natural to express the answer to a linear equation.

II. CONCLUSION

The work that the Australian Bureau of Statistics does including problem-solving, inquiry, testing, design, and analysis relies heavily on mathematics. It enables the development of a complete data base of information in a manner that is efficient with regard to cost. It helps us to get value from the data by exploring the patterns that are contained within it and estimating the level of confidence that can be fairly placed in the conclusions that are formed from the data. One of the most interesting facets of the use of mathematics in business is the fact that, in the real world, no one solution ever appears to immediately apply. Rather, the concepts behind previous results need to be understood as a foundation for the development of subsequent solutions, which will lead to the enhancement and development of new theories to match the actual world. Researchers and graduate practitioners who have a very deep comprehension of the principles underlying the known theory and the foundation on which it has been established are required to be able to expand and correctly apply that theory in the context of practice. This understanding is necessary in order to fulfill this requirement.

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