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A Note on a New Form of Weak Continuous Multifunction

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***Abstract**. The aim of this paper is to introduce the study of ultra - upper and lower- slightly continuous ultra multifunctions.*

I. Introduction and Preliminaries

The concept of bitopological spaces was introduced by Kelly [2] in the year 1963. When all the works in bitopological spaces were depending on pair-wise concept, the definition of (1, 2)α-open sets introduced by Thivagar [3], in the year 1991,opened a new era of research in bitopology. He also defined (1, 2)α-continuous function and its weaker and stronger forms between two bitopological spaces. Continuity and multifunctions are two basic properties in general topology and set valued analysis. By multifunctions, we mean amapping from a point to a set. In the year 1978, Popa [7] introduced upper and lower weakly continuous functions and studied their properties. This concept of multifunctions was extended to bitopological spaces by defining ultra- multifunctions [5].

The main purpose of this article is to define and to generalize the ultra-upper and ultra-lower slightly continuous ultra- multifunctions in bitopological spaces. Throughout this paper, X means (X,τ), where X is a non empty set and τ is the topology defined on it. By Y, we mean the bitopological space (Y, σ_1, σ_2), where Y is a non empty set with two topologies σ_1 and σ_2 defined on it. 2000 Math. Subject Classification: 54C10, 54C08.

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Definition 1.1. Let A be a subset of a bitopological space (Y, σ_1, σ_2). Then A is said to be [3] (i) $\sigma_1\sigma_2$-open if $A \in \sigma_1\cup\sigma_2$, (ii)$\sigma_1\sigma_2$-closed if $A^c \in \sigma_1\cup\sigma_2$, (iii)(1,2)α-open or ultra-open if $A \subseteq \sigma_1\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\text{-Int}(A)))$, where σ_1-Int(A) is the interior of A with respect to the topology σ_1 and $\sigma_1\sigma_2$-Cl(A) is the intersection of all $\sigma_1\sigma_2$-closed sets containing A. Also A is said to be (1, 2)α-closed iff A^c is (1, 2)α-open. (iv) $\text{Int}_{(1,2)\alpha}(A)$ is the union of all (1, 2)α-open sets contained in A. The set of all (1,2)α-open sets are denoted as (1,2)αO(X) and if this set forms a topology, then X is called as an ultra space.

Definition 1.2. An ultra multifunction [5] $F_u: (X,\tau) \rightarrow (Y,\sigma_1,\sigma_2)$ is a point to a set correspondence and is assumed that $F_u(x)= \varphi$ for all $x \in X$. Definition 1.3. The image set $U \subset X$ of the multifunction $F_u : X \rightarrow Y$ is defined [5] by $F_u(U) = \cup\{F_u(x) / x \in U \}$. Definition 1.4. For an ultra multifunction F_u, the upper and lower inverse [3]of F_u is defined for any set $V \subseteq Y$, as $F_u^+(V)= \{ x \in X / F_u(x) \subseteq V \}$ and $F_u^-(V)= \{ x \in X / F_u(x)\cap V = \varphi\}$. Lemma 1.5. For any ultra multifunction $F_{u+}(V)\subseteq F_{u-}(V)$. This result is proved in [5].

2. Ultra-Upper(Lower)-Slightly Continuous FunctionsDefinition 2.1. An ultra multifunction $F_u : X \rightarrow Y$ is said to be ultra-upper- slightly continuous if for each $x \in X$ and each (1, 2)α-clopen set V of Y containing $F_u(x)$, there exists an open set U of X containing x such that $F_u(U) \subset V$.Definition 2.2. An ultra multifunction $F_u : X \rightarrow Y$ is said to be ultra-lower-slightly continuous if for each $x \in X$ and each (1, 2)α -clopen set V of Y such that $F_u(x)\cap V = \varphi$, there exists an open set U of X containing x such that $F_u(x)\cap V = \varphi$ for all $x \in U$. 3

Example 2.3. Let X ={1, 2, 3} with the topology τ = {φ, , X, {1, 2}} and Y = {a, b, c, d} with two topologies σ_1 ={φ, Y, {a}, {b}, {a, b}, {b, c, d}, {a, c, d}, {c, d}} and σ_2 = {φ, Y, {a, c, d}}.Define $F_u : X \rightarrow Y$ as F_u (1)={a}, F_u (2)={c},F_u(3) = {a, d, c}.Here F_u is ultra-upper-slightly continuous and ultra-lower-slightly continuous. Definition 2.4. A topological space $(Y, \sigma_1\ \sigma_2)$ is said to be ultra-externally-disconnected (U.E.D) if the (1, 2)α-closure of each (1, 2)α-open set of Y is (1, 2)α- open. 3. Comparisions Remark 3.1. Every ultra - slightly Continuous multifunctions is ultra almost continuous and ultra weakly contionuous multifunctions. Example 3.2. Let X ={1, 2, 3} with the topology τ = {φ, , X, {1, 2}} and Y = {a, b, c, d} with two topologies σ_1 ={φ, Y, {a, b}, {c, d}} and σ_2 = {φ, Y }.Define $F_u : X \rightarrow Y$ as F_u (1)={a, b}, F_u (2)={b}, F_u(3) = {a}. It is ultra-slightly continuous, ultra almost continuous and ultra weakly contionuous multifunc-tions. Example 3.3. The following example shows that the converse need not be true. Let X ={1, 2, 3} with the topology τ = {φ, X, {1}} and Y = {a, b, c} with two topologies σ_1 ={φ, Y, {a}} and σ_2 = {φ, Y, {a}, {b, c}}.Define $F_u : X \rightarrow Y$ as F_u (1)={a}, F_u (2)={a, c}, F_u(3) = {a, b}. It is ultra weakly contionuous but not ultra-slightly continuous multifunctionsExample 3.4. Let X ={1, 2, 3} with the topology τ = {φ, X, {1, 2}} and Y = {a, b, c, d} with two topologies σ_1 ={φ, Y, {d}, {a, d}, {b, d}, {a, b, d}} and σ_2 = {φ, Y, {a}}.Define $F_u : X \rightarrow Y$ as F_u (1)={a}, F_u (2)={b, d}, F_u(3) = {b, d, c}. It is ultra almost contionuous but not ultra-slightly continuous multifunctions.

II. Characterization Of Ultra-Slightly Multifunctions

Theorem 4.1. For an ultra multifunction $F_u : X \rightarrow Y$, the following are equivalent.
(i) F_u is ultra upper slightly continuous.
(ii) $F_u^{+}(V)$ is open in X for every (1, 2)α-clopen set V of Y.
(iii) $F_u^{-}(V)$ is closed in X for every (1, 2)α-clopen set V of Y.

Proof : (i)⇒ (ii) Let V be a (1, 2)α-clopen set of Y and $x \in F_u^{+}(V)$ .Then$F_u(x) \in V$ and so there exists an open set U of X containing x such thatF_u (U)⊂ V. Hence we have $x \in U \subset F_u^{+}(V)$. And so, $x \in U \subset \text{Int}(F_u^{+}(V))$,which implies $F_u^{+}(V) \subset \text{Int}(F_u^{+}(V))$. Hence the result.

(ii)⇒ (i) Let x∈ X and V be any (1, 2)α -clopen set of Y containing $F_u(x)$.Then $x \in F_u^{+}(V)$ and $F_u^{+}(V)$ is open. So $x \in \text{Int}(F_u^{+}(V))$. Therefore thereexists an open set U of X containing x such that $x \in U \subset F_u^{+}(V)$, whichimplies F (U)⊂ V. Hence F_u is ultra-upper-slightly continuous.(ii) ⇒ (iii) Let K be a (1, 2)α -clopen set of Y. Then Y-K is also a clopenset of Y. Also X- $F_u^{-}(K) = F_u + (Y\text{-}K)$.Then Int $(F_u^{+}(Y\text{-}K)) = X - \text{Cl}(F_u^{-}(K))$.So $F_u^{-}(K)$ is a closed set in X.

(iii) ⇒ (ii) The fact that $F_u(Y - V) = X - F_u^{+}(V)$ gives the result.Theorem 4.2. For an ultra multifunction $F_u : X \rightarrow Y$, the following areequivalent.(i) F_u is ultra-lower-slightly continuous.(ii) $F_u^{-}(V)$ is an open set of X for every (1, 2)α -clopen set V of Y.(iii) $F_u^{+}(V)$ is a closed set of X for every (1, 2)α -clopen set V of Y.

Proof: (i) ⇒ (ii) Let V be a (1, 2)α -clopen set of Y and $x \in F_u^{-}(V)$. ThenF_u (x)⊂ V. By the definition, there exists an open set U of X containing xsuch that $F_u(x) \cap V = \varphi$ for all $x \in U$. Therefore, $F_u(U) \subset V$ and $U \subset F_u^{-}(V)$.As U is open, $x \in U \subset \text{Int}(F_u^{-}(V))$. Hence $F_u^{-}(V) \subset \text{Int}(F_u^{-}(V))$.A NOTE ON A NEW FORM OF WEAK CONTINUOUS MULTIFUNCTION 5(ii) ⇒ (iii) Let V be a (1, 2)α -clopen set of Y. Then Y-V is also a (1, 2)α-clopen set of Y. Also $X - (F_u^{+}(V)) = F_u^{-}(Y - V) = \text{Int}(F_u^{-}(Y - V)) = X\text{-Cl}$
$(F_u^{+}(V))$. Therefore $F_u^{+}(V) = \text{Cl}(F_u^{+}(V))$.(iii) ⇒ (i) Let x be any point of X and V be any (1, 2)α -clopen set of Y such that $F_u(x) \cap V = \varphi$. Therefore $x \in F_u^{-}(V)$ and $X - F_u^{-}(V) = F_u^{+}(Y - V))$.By assumption $F_u^{+}(Y - V)$ is closed implies $x \in \text{Cl}(F_u^{+}(Y - V))$. Hence thereexists an open set U of X containing x such that $U \cap F_u^{+}(Y - V) = \varphi$. That is$U \cap X - F_u^{-}(V) = \varphi$ and so, $U \subset F_u^{-}(V)$. Thus $F_u^{-}(U) \subset V$ and we get that$F_u(x) \cap V = \varphi$ for all $x \in U$.

Theorem 4.3. Let (Y, σ_1, σ_2) be a U.E.D. For a multifunction $F_u : X \rightarrow Y$,the following are equivalent.
(i) F_u is ultra-upper-slightly continuous.(ii) $\text{Cl}(F_u^{-}(V)) \subset F_u^{-}(\text{Cl}(V))$ for every (1, 2)α -open set V of Y.
(iii) $F_u^{+}(\text{Int}(1,2)\alpha\ (C)) \subset \text{Int}(F_u^{+}(C))$ for every (1, 2)α-closed set C of Y.Proof : (i) ⇒ (ii) Let V be a (1, 2)α-open set of Y. Since Y is U.E.D, $\text{Cl}_{(1,2)\alpha}(V))$ is a clopen set of Y. By Theorem 5.2.1, $F_u^{-}(\text{Cl}(1,2)\alpha\ (V))$ is a closed set ofX. So, $F_u^{-}(\text{Cl}(1,2)\alpha\ (V)) = \text{Cl}(F_u^{-}(\text{Cl}(1,2)\alpha\ (V)))$. Again $F_u(V) \subset F_u^{-}(\text{Cl}(1,2)\alpha(V))$. Therefore, $\text{Cl}(F_u^{-}(V)) \subset \text{Cl}(F_u^{-}(\text{Cl}(1,2)\alpha^{(V))} = F_u^{-}(\text{Cl}(1,2)\alpha(V))$. Hence$\text{Cl}(F_u^{-}(V)) \subset F_u^{-}(\text{Cl}(1,2)\alpha(V))$.(ii)⇒ (iii) Let C be any (1, 2)α- closed set of Y. Take V = Y-C, whichis(1, 2)α -open. Now $X - \text{Int}(F_u^{+}(C)) = \text{Cl}(X - F_u^{+}(C)) = \text{Cl}(F_u^{-}(Y\text{-}C)) = F_u^{-}(\text{Cl}_{(1,2)\alpha}(Y\text{-}C))$. Now, $X - \text{Int}(F_u^{+}(C)) \subset F_u^{-}(\text{Cl}(1,2)\alpha(Y\text{-}C)) = F_u^{-}(Y - \text{Int}$
$(1,2)\alpha(C)) = X - F_u^{-}(\text{Int}(1,2)\alpha(C))$. Hence we can say that $F_u^{+}(\text{Int}(1,2)\alpha(C)) \subset \text{Int}(F_u^{+}(C))$.

(iii)⇒ (i) Let $x \in X$ and V be a (1, 2)α -open set containing $F_u(x)$. Then by (iii),we have $x \in F_u^{+}(V) = F_u^{+}(\text{Int}(1,2)\alpha(V)) \subset \text{Int}(F_u^{+}(V))$.Therefore there exists an open set U of x such that $x \in U \subset F_u^{+}(V)$. Now, we have $F_u^{-}(U) \subset V$ and hence F_u is ultra-upper-slightly continuous.

6 Theorem 4.4. Let (Y, σ_1, σ_2) be a U.E.D. For a multifunction $F_u : X \rightarrow Y$,
the following are equivalent.(i) F_u is ultra-lower-slightly continuous.
(ii) $\text{Cl}(F_u^{+}(V)) \subset F_u^{+}(\text{Cl}_{(1,2)\alpha}\ (V))$ for every (1, 2)α -open set V of Y.(iii) $F_u^{-}(\text{Int}_{(1,2)\alpha}\ (C)) \in \text{Int}(F_u^{-}(C))$ for every (1, 2)α-closed set C of Y Proof : Similar to the Theorem 5.2.3.Definition 4.5. A bitopological space (Y, σ_1, σ_2) is called an ultra connected space [9] if there exists two disjoint non empty (1, 2)α -open sets U and V such that Y=U ∩ V.

Theorem 4.6. Let $F_u : X \rightarrow Y$ be an ultra-upper-slightly continuous function. If X is a connected space, then Y is an ultra connected space. Proof : Suppose Y is not ultra connected. Then there exists two disjoint non empty (1, 2)α-open sets U and V such that Y= U ∩ V. Also U and V are(1, 2)α -clopen sets. Given F_u is ultra-upper-slightly continuous. So $F_u^{+}(U)$and $F_u^{+}(V)$ are (1, 2)α-open sets in X. Now $F_u^{+}(U)$ and $F_u^{+}(V)$ are non emptyand disjoint and $X = F_u^{+}(U) \cup F_u^{+}(V)$ .This proves that X is not connected. Acontradiction.

Theorem 4.7. Let $F_u : X \rightarrow Y$ be an ultra-upper-slightly continuous surjection. If Y is ultra normal and U.E.D then X is T_2. Proof : Suppose Y is ultra normal. For any two distinct (1, 2)α -closed sets H and K there exists two (1, 2)α-open sets U and V such that H ⊂ U and K ⊂ V. $H \subset \text{Cl}_{(1,2)\alpha}(U)$ and $K \subset \text{Cl}_{(1,2\alpha}(V)$. As F_u is ra-upper-slightly continuous and Y is U.E.D. $F_u^{+}(V)\ (\text{Cl}(1,2)\alpha^{(U))\ \text{and}}\ F_u^{+}(\text{Cl}(1,2)\alpha\ (V))$ are open
in X. Since F_u is ultra-upper-slightly continuous, for x∈ X and for each (1, 2)α -clopen set say $\text{Cl}_{(1,2)\alpha}$ (U) containing $F_u^{+}(x)$, there exists an open set G of X containing x such that $F_u(G) \subset \text{Cl}_{(1,2)\alpha}(U)$. For y ∈ X , we get a (1, 2)α -clopen set $\text{Cl}_{(1,2)\alpha}$ (V) containing $F_u^{+}(y)$, there exists an open set H of X containing y such that $F_u(H) \subset \text{Cl}_{(1,2)\alpha}(V)$.Clearly $\text{Cl}_{(1,2)\alpha}(U)$ and $\text{Cl}_{(1,2)\alpha}$ (V) are disjoint and non empty. Hence X is T_2. 7

III. Conclusion

Topology, which until recently was a conglomeration of loosely related the-orems, become a systematic science and topological methods penetrated in to many other domain of sciences,like image processing , DNA structures etc. Since then several thousands of works have been dedicated to the investigation of bitopology , but a very few are with multi functions. It is to be noticed in a recent study the over expression of P-gycoprotein (Pgp) in breast and other cancer is thought to be largely involved in the development of multidrug resis-tance to chemotherapy. This Pgp has been reported to have multiple topology and multiple functions. Hence a research on multifunctions in bitopological spaces may definitely has its own place in medical science and also in other fields of sciences.

References

[1] M.C. Dutta ., Contribution to theory of bitopological spaces, Ph.D Thesis, Pilani, 1971.

[2] J.C.Kelli., Bitopological Spaces, Pro.London Math.Soc,13(1963),71-89.

[3] M.Lellis Thivagar., Generalization of (1,2)α-continuous functions, Pure and Applied Mathematika Sciences 33 (1991),55-63.

[4] G.Navalagi, M.Lellis Thivagar and R.RajaRajeswari., (1, 2)α- Hyperconnected spaces, International Journal of Mathematics and Analysis, Vol.3 (2006), 120 -129.

[5] G.Navalagi, M.Leelis Thivagar and R.RajaRajeswari., On ultra multifunctios in bitopo-logical spaces, International Journal of Mathematics, Coputer Science and Information Technology Vol.1,No.1(2008),69-74.

[6] 6. V.Popa., A note on weakly and almost continuous multifunctions,Univ.u Novom sadu, Zb.Rad.Priorod. - Mat.Fak.Ser.Mat.21, 2(1991), 31-38.

[7] 7. V.Popa and T.Noiri., on upper and lower weakly- - continuous multifunctions,Novi.Sad.J.Math ,Vol.32, No.1 (2002), 7-24.

[8] 8. R.RajaRajeswari,Bitopological concepts of some separation properties, Ph.D Thesis, Madurai Kamaraj University, Madurai, India, 2009

[9] 9. M.S.Sarask, N.Gowrisankar and N.Rajesh., on upper and lower γ- continuous mul-tifunctions, Int.J.contemp.Math.Sciences, Vol.5 (2010), No.6, 281 - 288.