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Note on Intuitionistic N-Closed Sets

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Abstract: *In this paper we introduce and investigate intuitionistic N -closed sets and Intuitionistic almost regular space in a intuitionistic topological spaces.*
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I. Introduction

First, D.Coker et al [2] introduced intuitionistic fuzzy topological spaces, intuitionistic topological spaces and the concept of Compactness on Intuitionistic topological spaces.In this paper we introduce and investigate intuitionistic Almost regular spaces and intuitionistic N-closed sets on intuitionistic topological spaces.Also we investigate their properties via intutionistic $T_2(i)$ -spaces.

II. Preliminaries

Throughout this paper $(\tilde{X},\tilde{\tau})$ (or briefly $\tilde{X}$) represent intuitionistic topological space on which no separation axioms are assumed unless explicitly stated.
Let us recall the following definitions, which are useful in the sequel.

Definition II.1. [1] Let X be a nonempty set.An intuitionistic set A is an object of the form $A=(A_1,A_2)$ where A_1 and A_2 are disjoint subsets of X . The set A_1 is the set of all members of A and A_2 is the set of all non-members of A.

Definition II.2. [1] Let X be a nonempty set, $a\in X$ and $A=(A_1,A_2)$ be an intuitionistic subset of X. Intuitionistic set $\tilde{a}=(\{a\},\{a\}^c)$ is called an intuitionistic point in X. The intuitionistic point $\tilde{a}\in A$ iff $a\in A_1$.

Definition II.3. [1] Let X be a nonempty set. $A=(A_1,A_2)$, $B=(B_1,B_2)$ and $\{A_i=(A_i^{(1)},A_i^{(2)})/i\in I\}$ are intuitionistic subsets of X. Then

(i) $A\subseteq B$ iff $A_1\subseteq B_1$ and $B_2\subseteq A_2$.
(ii) $A=B$ iff $A\subseteq B$ and $B\subseteq A$.
(iii) $A^c=(A_2,A_1)$.
(iv) $\bigcap A_i=(\bigcap A_i^{(1)},\bigcup A_i^{(2)})$.
(v) $\bigcup A_i=(\bigcup A_i^{(1)},\bigcap A_i^{(2)})$.
(vi) $\tilde{\emptyset}=(\emptyset,X)$.
(vii) $\tilde{X}=(X,\emptyset)$.

Definition II.4. [2] An intuitionistic topology on a nonempty set X is a family $\tilde{\tau}$ of intuitionistic sets in X containing Φ, $\tilde{X}$ and closed under finite infima and arbitrary suprema.
Then the pair $(\tilde{X},\tilde{\tau})$ is called an intuitionistic topological space.Every member of $\tilde{\tau}$ is known as an intuitionistic open set in $\tilde{X}$ The complement A^c of an intuitionistic open set A is called intuitionistic closed set in $\tilde{X}$.

Definition II.5. [2] Let X be a nonempty set and let A be an intuitionistic subset of X. Then the intuitionistic interior and intuitionistic closure of A is defined by

(i). $\text{int}(A) = \bigcup\{U : U$ is an intuitionistic open set of $\tilde{X}$ and $U \subseteq A\}$.

(ii). cl (A) = $\bigcap$ {F : F is an intutionistic closed set of $\tilde{X}$ and A $\subseteq$ F}.

For any intutionistic subset A of X, int (A^c) = [cl (A)]^c and cl (A^c)= [int (A)]^c.

Definition II.6.[2] Let f : $(\tilde{X},\tilde{\tau}) \to (\tilde{Y},\tilde{\sigma})$ be a function .If A = (A_1, A_2) is an intutionistic subset of X, then the image of A under f, denoted by f(A), is an intutionistic subset of Y and defined by f(A) = (f(A_1),f-(A_2)), where f-(A_2) = (f((A_2)^c))^c.

Definition II.7.[2] Let f : $(\tilde{X},\tilde{\tau}) \to (\tilde{Y},\tilde{\sigma})$ be a function.If B = (B_1, B_2) is an intutionistic subset of Y, then the pre-image of B under f, denoted by f^{-1}(B) is an intutionistic subset of X and defined by f^{-1}(B) = (f^{-1}(B_1),f^{-1}(B_2)).

Definition II.8.[2] Let $(\tilde{X},\tilde{\tau})$ be a intutionistic topological space.

(i) If a family {G_i = < G^1_i ,G^2_i > : i $\in$ J} of intutionistic open sets in X satisfies the condition $\bigcup_{i\in J} G_i = \tilde{X}$, then it is called an **intutionistic open cover** of $\tilde{X}$. A finite subfamily of intutionistic open cover {G_i = < G^1_i ,G^2_i > : i $\in$ J} of $\tilde{X}$, which is also an open cover of $\tilde{X}$, is called an **intutionistic finite sub cover** of $\tilde{X}$.

(ii) A family {F_i = < F^1_i ,F^2_i > : i $\in$ J} of intuitionistic closed sets in X satisfies the finite intersection property (briefly FIP) iff every finite subfamily { F_i : i = 1,2,...n} of {F_i = < F^1_i ,F^2_i > : i $\in$ J} satisfies the condition $\bigcap_{i=1}^{n} F_i \neq \Phi$.

Definition II.9.[2] An intutionistic topological space $(\tilde{X},\tilde{\tau})$ is said to be intutionistic compact iff every intutionistic open cover {G_i = < G^1_i ,G^2_i > : i $\in$ J} of $\tilde{X}$ has a intutionistic finite subcover.

Definition II.10.[2]An intutionistic topological space $(\tilde{X},\tilde{\tau})$ is said to be intutionistic nearly compact iff every {G_i = < G^1_i ,G^2_i > : i $\in$ J}of $\tilde{X}$,there exists a finite set J_0 of J such that $\bigcup_{i\in J_0} \text{int}(cl(G_i)) = \tilde{X}$.

III. Intuitionistic N -Closed Sets.

Proposition III.1. [2] Let $(\tilde{X},\tilde{\tau})$ be a intuitionistic topological space.

(i). If a family $\{G_i =< G_i^1, G_i^2 >: i \in J\}$ of intuitionistic open sets in $(\tilde{X},\tilde{\tau})$ satisfies the condition $A \subseteq \bigcup_{i\in J} G_i$ then it is an **intuitionistic open cover** of A.

(ii).Let $\{G_i =< G_i^1, G_i^2 >: i \in J\}$ be a family of intuitionistic open sets in $(\tilde{X},\tilde{\tau})$,which covers A. If there exists a finite subset J_0 of J such that $A \subseteq \bigcup_{i\in J_0} G_i$ then $\{G_i =< G_i^1, G_i^2 >: i \in J_0\}$ is a **intuitionistic finite subcover of** A.

Definition III.2. An intuitionistic subset A in a intuitionistic topological space $(\tilde{X},\tilde{\tau})$ is a intuitionistic N -closed in $(\tilde{X},\tilde{\tau})$ iff for each intuitionistic open cover $\{G_i =< G_i^1, G_i^2 >: i \in J\}$ of A, there exists a finite set J_0 of J such that $A \subseteq \bigcup_{i\in J_0} \text{int}(cl(G_i))$.

Example III.3. Let $X = \mathrm{P}$ and $\tilde{\tau}$ is as follows.(i) $\tilde{\varnothing}$, (ii) $\tilde{\mathrm{P}}$,

(iii) $< \cup(a_i, b_i), (-\infty, c] >$ where $a_i, b_i \in \mathrm{P}$ and $\{a_i : i \in J\}$ is bounded below and $c \in inf\{a_i : i \in J\}$.

(iv) $\left\langle \bigcup\left(a_i, b_i\right), \Phi \right\rangle$ where $a_i, b_i \in \mathrm{P}$ and $\{a_i : i \in J\}$ is not bounded below.

$A =< [o,1], (-\infty, 0) >$ is an intuitionistic N -closed set in $(\tilde{X},\tilde{\tau})$.

Definition III.4. An intuitionistic topological space $(\tilde{X},\tilde{\tau})$ is said to be intuitionistic almost regular if for each intuitionistic regular closed set A and any intuitionistic point $\tilde{x}$ not in A, there exists disjoint intuitionistic open sets U and V such that $A \subseteq U$ and $\tilde{x} \in V$.

The following theorem characterizes intuitionistic N-Closed spaces.

Theorem III.5. An intuitionistic subset A of a intuitionistic topological space $(\tilde{X}, \tilde{\tau})$ is intuitionistic N-closed if and only if for each intuitionistic regular open cover of A has a intuitionistic finite sub cover.

Proof. Necessity- Let A be a intuitionistic N-closed set in a intuitionistic topological space $(\tilde{X}, \tilde{\tau})$ and $\{G_i : i \in J\}$ be any intuitionistic regular open cover of A. Therefore, $A \subseteq \bigcup_{i \in J} G_i = \bigcup_{i \in J} \mathrm{int}(cl(G_i))$ and hence $\{\mathrm{int}(cl(G_i)) : i \in J\}$ is an intuitionistic open cover of A. By hypothesis,there exists a finite set J_0 of J such that $A \subseteq \bigcup_{i \in J_0} \mathrm{int}(cl(\mathrm{int}(cl(G_i)))) \subseteq \bigcup_{i \in J_0} \mathrm{int}(cl(G_i)) = \bigcup_{i \in J_0} G_i$.Thus, $A \subseteq \bigcup_{i \in J_0} \mathrm{int}(cl(G_i))$.

Sufficiency- Suppose that $\{G_i : i \in J\}$ be an open cover of A.By theorem 3.5 [3], $\mathrm{int}(cl(G_i))$ is intuitionistic regular open set for each i.Also, $\{\mathrm{int}(cl(G_i)) : i \in J\}$ is an intuitionistic regular open cover of A.By hypothesis,there existys a finite set J_0 of J such that $A \subseteq \bigcup_{i \in J_0} \mathrm{int}(cl(G_i))$.Therefore. A is intuitionistic N-closed set.

Theorem III.6. An intuitionistic space $(\tilde{X}, \tilde{\tau})$ is nearly compact if and only if it is intuitionistic N-closed.

Proof. It follows from the definitions.

Theorem III.7. In an intuitionistic topological space $(\tilde{X}, \tilde{\tau})$,the intersection of a intuitionistic N-closed set and a intuitionistic regular closed set is always a intuitionistic N-closed set.

Proof. Let A be any intuitionistic N-closed and B be intuitionistic regular closed subset of a intuitionistic topological space $(\tilde{X}, \tilde{\tau})$.Suppose that $\{G_i : i \in J\}$ is any intuitonistic regular open cover of $A \bigcap B$. Since $A = (A \setminus B) \bigcup (A \bigcap B)$, we have $A \subseteq (X \setminus B) \bigcup_{i \in J} G_i$.Therefore, $\{(X \setminus B), \bigcup \{G_i : i \in J\}\}$ is a intuitionistic regular open cover of a intuitionistic N-closed set A in $(\tilde{X}, \tilde{\tau})$. By hypothesis,there exists a finite subset J_0 of J such that $A \subseteq (\bigcup_{i \in J_0} G_i) \bigcup (X \setminus B)$. Therefore ,there exists a finite subset J_0 of J such that $A \bigcap B \subseteq \bigcup_{i \in J_0} G_i$. Thus, $A \bigcap B$ is a intuitionistic N-closed set in $(\tilde{X}, \tilde{\tau})$.

Theorem III.8. Let A and B be intuitionistic subsets of a intuitionistic topological space $(\tilde{X}, \tilde{\tau})$ such that $B \subseteq A$. Then the following statements hold.

(i).If A is intuitionistic N-closed and B is intuitionistic regular closed set in $(\tilde{X}, \tilde{\tau})$, then B is intuitionistic N-closed set in $(\tilde{X}, \tilde{\tau})$.

(ii).If A is intuitionistic N-closed and B is intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$, then $A \setminus B$ is intuitionistic N-closed set in $(\tilde{X}, \tilde{\tau})$.

(iii).If A is intuitionistic regular closed and $\tilde{X}$ is intuitionistic nearly compact,then A is intuitionistic N-closed set in $\tilde{X}$.

proof-(i). Since $B \subseteq A$, we have $A \cap B = B$. Thus, by theorem III.7, B is intuitionistic N-closed set in $(\tilde{X}, \tilde{\tau})$.

(ii). Let A be a intuitionistic N-closed and B be intuitionistic regular open subsets of a intuitionistic topological space $(\widetilde{X},\widetilde{\tau})$. Suppose that $\{G_i : i \in J\}$ is any intuitionistic regular open cover of $A \setminus B$. Since $A = (A \setminus B) \bigcup (A \bigcap B) = (A \setminus B) \bigcup B,$ we have $A \subseteq \left(\bigcup_{i \in J} G_i \right) \bigcup B.$ Therefore, $\{G_i : i \in J\} \bigcup B$ is a intuitionistic regular open cover of intuitionistic N-closed set A in $(\widetilde{X},\widetilde{\tau})$. By hypothesis,there exists a finite subset J_0 of J such that $A \setminus B \subseteq \bigcup_{i \in J_0} G_i$. Thus, $A \setminus B$ is a intuitionistic N-closed set.

(iii).By theorem III.6, $(\widetilde{X},\widetilde{\tau})$ is intuitionistic N-closed. Since $A \subseteq X,$ we have $A \bigcap X = A.$ By (i), A is intuitionistic N-closed set in $(\widetilde{X},\widetilde{\tau})$.

Definition III.9.A function f: $(\widetilde{X},\widetilde{\tau}) \to (\widetilde{Y},\widetilde{\sigma})$ is said to be intuitionistic almost continuous iff $f^{-1}(V)$ is intuitionistic open in $(\widetilde{X},\widetilde{\tau})$ for every intuitionistic regular open set V of $(\widetilde{Y},\widetilde{\sigma})$.

Theorem III.10. Let f : $(\widetilde{X},\widetilde{\tau}) \to (\widetilde{Y},\widetilde{\sigma})$ be a intuitionistic almost continuous function.Then any intuitionistic almost continuous image of a intuitionistic compact set in $(\widetilde{X},\widetilde{\tau})$ is intuitionistic N-closed set in a $(\widetilde{Y},\widetilde{\sigma})$.

Proof. Given that f: $(\widetilde{X},\widetilde{\tau}) \to (\widetilde{Y},\widetilde{\sigma})$ is intuitionistic almost continuous.Let A be intuitionistic compact set of $(\widetilde{X},\widetilde{\tau})$. Let $\{G_i : i \in J\}$ be any intuitionistic regular open cover of $f(A)$. Since f is intuitionistic almost continuous, for each $i \in J$, $f^{-1}(G_i)$ is intuitionistic open in $(\widetilde{X},\widetilde{\tau})$.Also,

$A \subseteq f^{-1}(f(A)) \subseteq f^{-1}\left(\bigcup_{i \in J} G_i \right) = \bigcup_{i \in J} f^{-1}(G_i).$ Therefore $\{f^{-1}(G_i) : i \in J\}$ is a intuitionistic open cover of a intuitionistic compact set A.Therefore, there exists a finite number of intuitionistic regular open sets $G_1, G_2, ... G_n$ in $(\widetilde{X},\widetilde{\tau})$ such that $A \subseteq \bigcup_{i=1}^{i=n} f^{-1}(G_i).$

Thus, $f(A) \subseteq f\left(\bigcup_{i=1}^{n} f^{-1}(G_i) \right) = \left(\bigcup_{i=1}^{n} f(f^{-1}(G_i)) \right) \subseteq \bigcup_{i=1}^{n} G_i$ .Thus, $f(A)$ is intuitionistic N-closed set in $(\widetilde{Y},\widetilde{\sigma})$.

IV. Between intuitionistic N-Closed Sets and T_2(i).

Definition IV.1.[6] A intuitionistic topological space is said to be $T_2(i)$ iff for all distinct pair of points $x, y \in X,$ there exists a pair of disjoint intuitionistic open sets U and V such that $\widetilde{x} \in U$ and $\widetilde{y} \in V.$

Proposition IV.2. In a intuitionistic topological space $(\widetilde{X},\widetilde{\tau})$,the intuitionistic sets U and A are such that $U \bigcap A = \Phi$ if and only if $A \subseteq U^c$.

Proof. Obvious.

Proposition IV.3. Let A be any intuitionistic subset of a intuitionistic topological space $(\widetilde{X},\widetilde{\tau})$ and $\widetilde{x}$ be any intuitionistic point of X . Then $\widetilde{x} \in cl(A)$ iff every intuitionistic open set containing $\widetilde{x},$ intersect $A.$

Proof. **Necessity-** Let $\widetilde{x} \in cl(A)$ and let U be any intuitionistic open set containing $\widetilde{x}$ such that $U \bigcap A = \Phi$ Then by theorem 4.2, $A \subseteq U^c$ and hence U^c is intuitionistic closed set containing $A.$ But $cl(A)$ is the smallest intuitionistic closed set containing $A.$ Therefore $cl(A) \subseteq U^c$ Now $\widetilde{x} \notin U^c$ implies that $\widetilde{x} \notin cl(A)$, a contradiction.

Sufficiency- Suppose that $\widetilde{x} \notin cl(A)$. By definition,there exists an intuitionistic closed set F containing A does not containing $\widetilde{x}.$ Thus $\widetilde{x} \in F^c$ such that $F^c \subseteq A^c$ and hence $F^c \bigcap A = \Phi$. Now there exists a intuitionistic open set F^c containing $\tilde{x}$, does not intersect A, a contradiction.Therefore $\tilde{x} \in cl(A)$.

Lemma IV.4. Let A and B be any two intuitionistic sets in intuitionistic topological space $(\tilde{X}, \tilde{\tau})$. Then $cl(A \cap B) \subseteq cl(A) \cap cl(B)$.

Proof. [2] By proposition 3.16, $A \subseteq cl(A)$ and $B \subseteq cl(B)$. Therefore , $A \cap B \subseteq cl(A) \cap cl(B)$ and . $cl(A) \cap cl(B)$ is a intuitionistic closed set containing $A \cap B.$ But $cl(A \cap B)$ is the smallest intuitionistic closed set containing $A \cap B.$ Thus , $cl(A \cap B) \subseteq cl(A) \cap cl(B)$.

Remark IV.5. The reversible inclusion is not true in general from the following example.

ExampleIV.6. Let $X = \{a,b\}$ and $\tilde{\tau} = \{\tilde{\Phi}, \tilde{X}, <\{a,\varnothing\}>, <\{b,\varnothing\}>, <\{a,b\}>, <\varnothing,\varnothing>, <\varnothing,b>\}$. If $A = <\varnothing,b>$ and $B = <\{b,a\}>$, then $A \cap B = \tilde{\Phi} = c\tilde{l}(A \cap B)$ whereas $cl(A) \cap cl(B) = B$.

Theorem IV.7. Let $(\tilde{X}, \tilde{\tau})$ be a $T_2(i)$ space.Then for every intuitionistic N-closed set A in $(\tilde{X}, \tilde{\tau})$ and every point $\tilde{y} \in \tilde{X} \setminus A,$ there exists a pair of disjoint intuitionistic regularly open sets U and V such that $\tilde{y} \in U$ and $A \subseteq V.$

Proof. Given that is a $T_2(i)$ space and A is any intuitionistic N-closed set and $\tilde{y} \in \tilde{X} \setminus A,$ is an arbitrary intuitionistic point in $(\tilde{X}, \tilde{\tau})$.Let $\tilde{x} \in A$ be arbitrary.Then $\tilde{x} \neq \tilde{y}.$ By hypothesis,there exists disjoint intuitionistic open sets $U_{\tilde{x}}$ and $V_{\tilde{x}}$ such that $\tilde{x} \in V_{\tilde{x}}$ and $\tilde{y} \in U_{\tilde{x}}.$ By theorem 3.5 [3], $\text{int}(cl(V_{\tilde{x}}))$ is an intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$.Hence $\left\{\text{int}(cl(V_{\tilde{x}})) : \tilde{x} \in A\right\}$ is an intuitionistic regular open cover of a intuitionistic N-closed set A. Therefore, there exists a finite number of intuitionistic points $\left\{\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n\right\}$ in A such that $A \subseteq \bigcup_{i=1}^{n} \text{int}(cl(V_{\tilde{x}_i}))$ .Let $U_0 = \bigcap_{i=1}^{n} U_{\tilde{x}_i}$ and $V_0 = \bigcup_{i=1}^{n} \text{int}(cl(V_{\tilde{x}_i}))$ then U_0 and V_0 are intuitionistic open sets such that $\tilde{y} \in U_0$ and $A \subseteq V_0.$ To prove $U_0 \cap V_0 = \varnothing,$ we take a point $\tilde{z} \in U_0 \cap V_0.$ Then $\tilde{z} \in U_{\tilde{x}_i}$ and $\tilde{z} \in \text{int}(cl)(V_{\tilde{x}_i}) \subseteq cl(V_{\tilde{x}_i})$ for some $i.$ By Proposition IV.3, every intuitionistic open subset containing $\tilde{z}$ intersects $V_{\tilde{x}_i}$. Therefore $U_{\tilde{x}_i} \cap V_{\tilde{x}_i} \neq \tilde{\varnothing}$ for some $i,$ a contradiction. Therefore $U_0 \cap V_0 = \tilde{\varnothing}.$ Let $U = \text{int}(cl)(U_0)$ and $V = \text{int}(cl)(V_0)$ Then U and V are a pair of disjoint intuitionistic regularly open sets such that $\tilde{y} \in U$ and $A \subseteq V.$

Corollary IV.8.Every intuitionistic nearly compact, $T_2(i)$ space is almost regular.

Proof. By (iii) of theorem III.8, every intuitionistic regular closed set of a intuitionistic nearly compact space is intuitionistic N-closed and hence by theorem 4.7, $(\tilde{X}, \tilde{\tau})$ is intuitionistic almost regular space.

Lemma IV.9. In a intuitionistic topological space $(\tilde{X}, \tilde{\tau})$, intersection of any two intuitionistic regular open sets is a intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$.

Proof. Let A and B be any two intuitionistic regular open sets in a intuitionistic space $(\tilde{X}, \tilde{\tau})$.

Therefore,they are intuitionistic open sets in $(\tilde{X}, \tilde{\tau})$ and so their intersections.

Thus , $A \cap B = \text{int}(A \cap B) \subseteq \text{int}(cl(A \cap B))$ .On the other hand, by lemma IV.4,

$cl(A \cap B) \subseteq cl(A) \cap cl(B)$.

Therefore, $\text{int}(cl(A \cap B)) \subseteq \text{int}(cl(A) \cap cl(B)) = \text{int}(cl(A)) \cap \text{int}(cl(B)) = A \cap B$.

Hence $\text{int}(cl(A \cap B)) = A \cap B$ .Thus, intersection of any two intuitionistic regular open sets is a intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$.

Theorem IV.10. Let $(\tilde{X},\tilde{\tau})$ be a $T_2(i)$ space.Then for any two disjoint intuitionistic N-closed sets A and $B,$ there exists a pair of disjoint intuitionistic regular open sets U and V such that $A\subseteq U$ and $B\subseteq V.$

Proof. Given that A and B are any two disjoint intuitionistic N -closed sets in $(\tilde{X},\tilde{\tau})$ .Let $\tilde{y}$ be any point of B .Then $\tilde{y}\in X\setminus A.$ By theorem IV.7,there exists disjoint intuitionistic regularly open sets $U_{\tilde{y}}$ and $V_{\tilde{y}}$ such that $\tilde{y}\in V_{\tilde{y}}$ and $A\subseteq U_{\tilde{y}}.$ Then the family $\{V_{\tilde{y}}:\tilde{y}\in B\}$ is an intuitionistic regular open cover for a intuitionistic N -closed set B in $(\tilde{X},\tilde{\tau})$. Therefore,there exists a finite number of intuitionistic points $\tilde{y}_1,\tilde{y}_2,...\tilde{y}_n$ in B such that $B\subseteq\bigcup_{i=1}^{n}V_{\tilde{y}_i}.$ Let $U=\bigcap_{i=1}^{n}U_{\tilde{y}_i}$ and $V_0=\bigcup_{i=}^{n}V_{\tilde{y}_i}.$ Then $A\subseteq U$ and $B\subseteq V_0$ such that $U\bigcap V_0=\tilde{\varnothing}.$ If we define $V=\text{int}(cl)(V_0)$,then U and V are disjoint regular open sets such that $A\subseteq U$ and $B\subseteq V.$

Theorem IV.11. Every intuitionistic singleton set $\{\tilde{x}\}$ of $(\tilde{X},\tilde{\tau})$ is intuitionistic N -closed subset in $(\tilde{X},\tilde{\tau})$.

Proof. Let $\{\tilde{x}\}$ be any intuitionistic point of $(\tilde{X},\tilde{\tau})$ and suppose that $\{G_i\in\tilde{\tau}:i\in J\}$ is any intuitionistic open cover of $\{\tilde{x}\}.$ Therefore $\{\tilde{x}\}\subseteq\bigcup_{i\in J}G_i$ and hence $\{\tilde{x}\}\in G_i$ for some $i.$ Thus, $\{\tilde{x}\}\in G_i=\text{int}(G_i)\subseteq\text{int}(cl(G_i))$ for some i and hence $\{\tilde{x}\}$ is intuitionistic N -closed subset of $(\tilde{X},\tilde{\tau})$.

Corollary IV.12.A intuitionistic topological space $(\tilde{X},\tilde{\tau})$ is said to be $T_2(i)$ iff for any two disjoint intuitionistic N -closed sets A and $B,$ there exists a pair of disjoint intuitionistic regular open sets U and V such that $A\subseteq U$ and $B\subseteq V.$

Proof. **Necessity-**It follows from theorem 4.10.

Sufficiency- Let $\{\tilde{x}\}$ and $\{\tilde{y}\}$ be any two intuitionistic points of $(\tilde{X},\tilde{\tau})$ such that $\{\tilde{x}\}\neq\{\tilde{y}\}.$ By theorem 4.11, $\{\tilde{x}\}$ and $\{\tilde{y}\}$ are two disjoint intuitionistic N -closed subsets of $(\tilde{X},\tilde{\tau})$ .By hypothesis,there exists disjoint intuitionistic regular open sets U and V in $(\tilde{X},\tilde{\tau})$ such that $\{\tilde{x}\}\subseteq U$ and $\{\tilde{y}\}\subseteq V.$ Thus $(\tilde{X},\tilde{\tau})$ is $T_2(i).$

Theorem IV.13. In a intuitionistic topological space the following statements are equivalent.

(i). $(\tilde{X},\tilde{\tau})$ is intuitionistic almost regular space.

(ii). For each intuitionistic point $\tilde{x}\in\tilde{X},$ and each intuitionistic regular-open set V in $(\tilde{X},\tilde{\tau})$ containing $\tilde{x}$,there exists a intuitionistic regular open set U in such that $\tilde{x}\in U\subseteq cl(U)\subseteq V$.

(iii). For each intuitionistic point $\tilde{x}\in\tilde{X},$and each intuitionistic open set U in $(\tilde{X},\tilde{\tau})$ containing $\tilde{x}$, there exists a intuitionistic regular open set V in $(\tilde{X},\tilde{\tau})$ such that $\tilde{x}\in V\subseteq cl(V)\subseteq\text{int}(cl(U))$.

(iv).For each intuitionistic point $\tilde{x}\in\tilde{X},$ and each intuitionistic open set U in $(\tilde{X},\tilde{\tau})$ containing $\tilde{x},$ there exists a intuitionistic open set V in such that $\tilde{x}\in V\subseteq cl(V)\subseteq\text{int}(cl(U))$.

(v). For every intuitionistic regular-closed set A in $(\tilde{X},\tilde{\tau})$ and each intuitionistic point $\tilde{x}\notin A,$ there exists intuitionistic open sets U and V such that $\tilde{x}\in U, A\subseteq V$ and $cl(U)\bigcap cl(V)=\tilde{\Phi}$.

proof.(i) $\Rightarrow$ **(ii)**.Let $\tilde{x}\in\tilde{X},$ be any intuitionistic point and V be any intuitionistic regular-open set in containing $\tilde{x}.$ Then V^c is an intuitionistic regular-closed set in $(\tilde{X},\tilde{\tau})$ does not containing $\tilde{x}.$ By hypothesis,there exists disjoint intuitionistic open sets U_1 and U_2 in $(\tilde{X},\tilde{\tau})$ such that $\{\tilde{x}\}\subseteq U_1$ and $V^c\subseteq U_2.$ Therefore, $cl(U_1)\bigcap U_2=\tilde{\Phi}$ implies that $\tilde{cl}(U_1)\subseteq U_2^c\subseteq V.$

Moreover, $U_1 = \text{int}(U_1) \subseteq \text{int}(cl(U_1)) \subseteq cl(U_1) \subseteq V$. If we define $U = \text{int}(cl(U_1))$, then U is a intuitionistic regular-open set such that $\tilde{x} \in U \subseteq cl(U) \subseteq V$.

(ii) $\Rightarrow$ (iii). Let $\tilde{x}$ be any intuitionistic point in $(\tilde{X}, \tilde{\tau})$ and U be any intuitionistic open set in $(\tilde{X}, \tilde{\tau})$ containing $\tilde{x}$. By theorem 3.5 [3], $\text{int}(cl(U))$ is intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$ containing $\tilde{x}$. By hypothesis,there exists a intuitionistic regular open set V in such that $\tilde{x} \in V \subseteq cl(V) \subseteq \text{int}(cl(U))$.

(iii) $\Rightarrow$ (iv). Obvious.

(iv) $\Rightarrow$ (v). Let $\tilde{x}$ be any intuitionistic point in $(\tilde{X}, \tilde{\tau})$ and A be any intuitionistic regular closed set in $(\tilde{X}, \tilde{\tau})$ does not containing $\tilde{x}$. Then A^c is a intuitionistic regular open set in $(\tilde{X}, \tilde{\tau})$ and hence intuitionistic open set in $(\tilde{X}, \tilde{\tau})$ containing $\tilde{x}$. By hypothesis, there exists an intuitionistic open set V_1 containing $\tilde{x}$ such that $V_1 \subseteq cl(V_1) \subseteq \text{int}(cl(A^c)) = A^c$. Therefore, $A \subseteq \left[cl(V_1)\right]^c$. In a similar way,if we apply hypothesis to the intuitionistic open set V_1,then there exists an intuitionistic open set V_2 containing $\tilde{x}$ such that $cl(V_2) \subseteq \text{int}(cl(V_1))$. If we define $U = V_2$ and V = $\left[cl(V_1)\right]^c$,then U and V are intuitionistic open sets such that $\tilde{x} \in U, A \subseteq V$ and $cl(U) \cap cl(V) = \tilde{\Phi}$.

(v) $\Rightarrow$ (i). Let $\tilde{x}$ be any intuitionistic point in $(\tilde{X}, \tilde{\tau})$ and A be any intuitionistic regular closed set does not containing $\tilde{x}$. By hypothesis,there exists intuitionistic open sets U and V in $(\tilde{X}, \tilde{\tau})$ such that $\tilde{x} \in U$ and $A \subseteq V$, $cl(U) \cap cl(V) = \tilde{\Phi}$. Hence $U \cap V = \Phi$ Thus, $(\tilde{X}, \tilde{\tau})$ is a intuitionistic almost regular space.

Theorem IV.14. An intuitionistic topological space $(\tilde{X}, \tilde{\tau})$ is almost regular if and only if for any intuitionistic N-closed set A and intuitionistic regular closed set B in $(\tilde{X}, \tilde{\tau})$ such that $A \cap B = \tilde{\Phi}$, there exists intuitionistic open sets U and V in $(\tilde{X}, \tilde{\tau})$ such that $A \subseteq U, B \subseteq V$ and $cl(U) \cap cl(V) = \tilde{\Phi}$.

Proof. Necessity- Let A be any intuitionistic N-closed set and B be any intuitionistic regular closed set in $(\tilde{X}, \tilde{\tau})$.By hypothesis and by theorem IV.13(v), for each intuitionistic point $\tilde{x} \in A$, (and hence $\tilde{x} \notin B$,) there exists intuitionistic open sets $U_{\tilde{x}}$ and $V_{\tilde{x}}$ such that $\tilde{x} \in U_{\tilde{x}}$ and $B \subseteq V_{\tilde{x}}$ and $cl(U_{\tilde{x}}) \cap cl(V_{\tilde{x}}) = \tilde{\Phi}$. Then the family $\{U_{\tilde{x}} : \tilde{x} \in A\}$ covers A. Since A is intuitionistic N-closed set in $(\tilde{X}, \tilde{\tau})$, there exists a finite number of intuitionistic points $\tilde{x}_1, \tilde{x}_2, \ldots \tilde{x}_n$ in A such that $A \subseteq \bigcup_{i=1}^{n} \text{int}(cl(U_{\tilde{x}_i}))$. If we define $U = \bigcup_{i=1}^{n} \text{int}(cl(U_{\tilde{x}_i}))$ and $V = \bigcap_{i=1}^{n} V_{x_i}$. Then U and V are intuitionistic open sets such that $A \subseteq U, B \subseteq V$ and $cl(U) \cap cl(V) = \tilde{\Phi}$.

Sufficiency-It follows from theorem 4.13 (v) $\Rightarrow$ (i).

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