IOSR Journal of Mathematics (IOSRJM)
ISSN: 2278-5728 Volume 1, Issue 4 (July-Aug 2012), PP 37-40
www.iosrjournals.org

A Stochastic Model with Scbz Property in Global Warming

S.Vijaya,
Faculty of Mathematics, Annamalai university, Annamalai nagar, Chidambaram.

Abstract: *In the study of Greenhouse effect one of the important aspect of global warming is relating to increase of temperature. The factors like CO2,CO and Nitrogen etc plays a vital role to hasten the process of increase in global temperature. The only source of global warming is CO2 emission . Stochastic models are widely used in the study of global warming and its consequences. There are many aspects which are taken for study, the time to green house effect depending upon the increase of global temperature. If the global temperature crosses the threshold level which in turn leads to greenhouse effect. The threshold itself is considered to be a random variable. In this paper assuming that threshold satisfies the property known as Setting the Clock Back to Zero (SCBZ) property, the expected time to seroconversion and its variance are derived. The results are augmented with suitable numerical illustrations.*
***Key words**: Greenhouse effect, Global warming, threshold. The AMS classification number is 92C60.*

I. Introduction

In the study of global warming, the estimation of expected time to Greenhouse effect and its variance arc considered to be indicative of the event progression. A global warming is exposed to global temperature at random time points and it is due to CO2 emission at random time intervals. The temperature of various places is also a random variable and it is mainly responsible for CO2 emission. As and when the total global temperature crosses the level called the threshold, the Greenhouse effect takes place. Under these assumptions, a mathematical model is developed by [3], in which the expected time to scroconversion and its variance are obtained by using the shock model and cumulative damage process by [1]. In doing so, they have assumed that the threshold level is a random variable is distributed as exponential.

A random variable Y has a distribution which satisfies the SCBZ property, if $\dfrac{S(y+y_0,\theta^*)}{S(y_0,\theta)}$ = S (y, θ_0) where

$\theta^* = \theta^*(y_o, \theta) \in \Omega$ where Ω is the parameter space where S (y, θ_0) = the survivor function
= P_r $(Y > y)$
and y_0 is called the truncation point. Where y_0 is taken to be a random variable

II. Assumptions Of The Model

1. By burning of fossil and other fuels, a random amount of CO2 emmission occurs.
2. CO2 emission is the only source of global warming.
3. CO2 emission is a damage process which is linear and cumulative
4. Increase of global temperature is caused by CO2 emission are assumed to be identically independent random variable
5. If the global temperature exeeds threshold level Y which is itself is a random variable, then greenhouse effect takes place.
6. The process which generate the CO2 emission, the sequence of increase in global temperature and threshold are mutually independent.
7. From the large number of CO2 emissions between successive events, a random sample of K observations are taken

III. Notations

X_i - A random variable representing the increase of the global temperature due to CO2 emission in the i^{th} event. X_i's are i.i.d with p.d.f g(.) and distribution function G(.).
Y- A random variable representing the global warming threshold with p.d.f h(.) and distribution function H(.).
U_i- A random variable representing the interarrival times between successive events, i = 1, 2, ... k, with p.d.f f(.).
$g_k(.)$- The p.d.f. of the random variable $\sum x_i$, i= 1, 2, . . .k.
$F_k(.)$- The kth convolution of F(.)
T- The continuous random variable denoting the time to

Greenhouse effect.

$f^*(s)$ - Laplace transform of f(.)

$f_{u(1)}^*(s), f^*_{u(k)}(s)$ - Laplace transform of $f_{u(1)}(.)$ and $f_{u(k)}(.)$ respectively.

IV. The Model

In this paper it is assumed that the threshold distribution satisfies the so called Setting the Clock Back to Zero property (SCBZ). According to this properly the threshold distribution has a parametric change after a particular value of the random variable which is called the truncation point. For a detailed study of SCBZ property one can refer [2]. The assumption of the SCBZ property for the threshold distribution is justified by the fact that the threshold undergoes a change with the passage of time due to the progression of the temperature after the initial exposure.

Here we consider the case where the truncation point y_0 is a random variable then

$$h(y) = \begin{cases} \theta_1 e^{-\theta_1 y}; & y \le y_0 \\ \theta_2 e^{-\theta_2 y} e^{\tau_0(\theta_2-\theta_1)}; & y > y_0 \end{cases}$$

Assume $y_0 \sim \exp(\lambda)$

The survivor function of y_0 ($y_0 \geq y$) is $e^{-\lambda y}$

The distribution function of y_0 ($y_0 < y$) is $\int_0^y \lambda e^{-\lambda y_0} dy_0$

When y_0 is a random variable

$$\text{Then} \quad h(y) = \theta_1 e^{-\theta_1 y} e^{-\lambda y} + \theta_2 e^{-\theta_2 y} \int_0^y e^{y_0(\theta_2-\theta_1)} \lambda e^{-\lambda y_0} dy_0$$

$$= \theta_1 e^{-(\theta_1+\lambda)y} + \theta_2 e^{-\theta_2 y} \lambda \int_0^y e^{-y_0(\theta_1-\theta_2+\lambda)} dy_0$$

$$= \theta_1 e^{-(\theta_1+\lambda)y} + \lambda\theta_2 e^{-\theta_2 y}\left[\frac{1}{\theta_1-\theta_2+\lambda} - \frac{e^{-y(\theta_1-\theta_2+\lambda)}}{\theta_1-\theta_2+\lambda}\right]$$

$$= \theta_1 e^{-y(\lambda+\theta_1)} + \frac{\lambda\theta_2 e^{-\theta_2 y}}{\theta_1-\theta_2+\lambda} - \frac{\lambda\theta_2 e^{-y(\theta_1+\lambda)}}{\theta_1-\theta_2+\lambda}$$

$$= e^{-(\theta_1+\lambda)y}\left[\theta_1 - \frac{\lambda\theta_2}{\theta_1-\theta_2+\lambda}\right] + \frac{\lambda\theta_2 e^{-\theta_2 y}}{\theta_1-\theta_2+\lambda}$$

$$= \frac{(\theta_1-\theta_2)(\lambda+\theta_1)}{\theta_1-\theta_2+\lambda} e^{-(\theta_1+\lambda)y} + \frac{\lambda\theta_2}{\theta_1-\theta_2+\lambda} e^{-\theta_2 y}$$

$$\therefore H(y) = \frac{(\theta_1-\theta_2)(\lambda+\theta_1)}{\theta_1-\theta_2+\lambda}\int_0^y e^{-(\theta_1+\lambda)t} dt + \frac{\lambda\theta_2}{\theta_1-\theta_2+\lambda}\int_0^y e^{-\theta_2 t} dt$$

$$= \frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda} - \frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda} e^{-(\theta_1+\lambda)y} + \frac{\lambda}{\theta_1-\theta_2+\lambda} - \frac{\lambda e^{-\theta_2 y}}{\theta_1-\theta_2+\lambda}$$

$$\therefore \overline{H}(y) = \frac{\theta_1-\theta_2}{\theta_1-\theta_2+\lambda} - e^{-(\theta_1+\lambda)y} + \frac{\lambda}{\theta_1-\theta_2+\lambda} e^{-\theta_2 y}$$

$$\text{Now, } P[T > t] = \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\int_0^{\infty} g_k(x)\overline{H}(x) dx$$

$$\therefore P[T > t] = \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\int_0^{\infty} g_k(x)\left[\frac{(\theta_1-\theta_2)e^{-(\theta_1+\lambda)y} + \lambda_2 e^{-\theta_2 y}}{\theta_1-\theta_2+\lambda}\right]$$

$$= \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\frac{1}{\theta_1-\theta_2+\lambda}\left[(\theta_1-\theta_2)\int_0^{\infty} g_k(x) e^{-(\theta_1+\lambda)y} dy + \lambda\int_0^{\infty} g_k(x) e^{-\theta_2 y} dy\right]$$

$$= \frac{1}{\theta_1-\theta_2+\lambda}\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left\{(\theta_1-\theta_2)g_k^*(\theta_1+\lambda)+\lambda g_k^*(\theta_2)\right\}$$

$$= p\left\{1-\left(1-g^*(\theta_1+\lambda)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_1+\lambda)\right]^{k-1}\right\}$$

$$+q\left\{1-\left(1-g^*(\theta_2)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_2)\right]^{k-1}\right\}$$

$$= 1-p\left(1-g^*(\theta_1+\lambda)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_1+\lambda)\right]^{k-1} - q\left(1-g^*(\theta_2)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_2)\right]^{k-1}$$

Here p+q =1, $p = \frac{\theta_1-\theta_2}{\lambda+\theta_1-\theta_2}$, $q = \frac{\lambda}{\lambda+\theta_1-\theta_2}$

Now,

$$L(t) = P[T < t] = 1- p[T \geq t]$$

$$= p\left(1-g^*(\theta_1+\lambda)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_1+\lambda)\right]^{k-1} + q\left(1-g^*(\theta_2)\right)\sum_{k=1}^{\infty}F_k(t)\left[g^*(\theta_2)\right]^{k-1} \tag{1}$$

Taking Laplace transform

$$l^*(s) = p\left(1-g^*(\theta_1+\lambda)\right)\sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[g^*(\theta_1+\lambda)\right]^{k-1}\Big\}$$

$$+q\left(1-g^*(\theta_2)\right)\sum_{k=1}^{\infty}\left[f^*(s)\right]^k\left[g^*(\theta_2)\right]^{k-1}\Big\}$$

$$= p\left[1-\frac{g^*(\theta_1+\lambda)f^*(s)}{1-f^*(s)g^*(\theta_1+\lambda)}\right]+q\left[1-\frac{g^*(\theta_2)f^*(s)}{1-f^*(s)g^*(\theta_2)}\right]$$

$$= l^*(s_1) + l^*(s_2) \tag{2}$$

Now,

$$l^*(s_1) = p\left(1-\frac{g^*(\theta_1+\lambda)f^*(s)}{1-f^*(s)g^*(\theta_1+\lambda)}\right)$$

If g(·)~exp c; f(·)~exp α

$$\therefore l^*(s_1) = p\left(\frac{\alpha(\theta_1+\lambda)}{(\theta_1+\lambda)+s(\theta_1+\lambda+c)}\right)$$

$$\therefore \frac{d}{ds}(l^*(s_1)) = p\left\{\frac{-\alpha(\theta_1+\lambda)(\theta_1+\lambda+c)}{\left[(\alpha(\theta_1+\lambda)+s(\theta_1+\lambda+c)\right]^2}\right.$$

$$\therefore -\frac{d}{ds}\boldsymbol{\ell}^*(s_1)\Big|_{s=0} = p\left\{\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right\}$$

$$\frac{d^2}{ds^2}\left(\boldsymbol{\ell}^*(s_1)\right)\Big|_{s=0} = 2p\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)^2$$

Similarly

$$\boldsymbol{\ell}^*(s_2) = q\frac{\left(1-g^*(\theta_2)\right)f^*(s)}{1-g^*(\theta_2)f^*(s)}$$

$$= q\left\{\frac{\alpha\theta_2}{\alpha\theta_2+s(c+\theta_2)}\right\}$$

$$\therefore \left. \frac{d}{ds^2} \boldsymbol{\ell}^*(s_2) \right|_{s=0} = q\left(\frac{c+\theta_2}{\theta_2\,\alpha}\right)$$

$$\left. \frac{d^2\boldsymbol{\ell}^*(s_2)}{ds^2} \right|_{s=0} = q\left\{2\left(\frac{c+\theta_2}{\theta_2\,\alpha}\right)^2\right\}$$

Therefore

$$E(T)\big|_{s=0} = p\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right) + q\left(\frac{c+\theta_2}{\theta_2\alpha}\right) \qquad (3)$$

$$V(T) = E(T^2) - (E(T))^2$$

$$= 2p\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)^2 + 2q\left(\frac{c+\theta_2}{\theta_2\alpha}\right)^2 - p^2\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)^2 - q^2\left(\frac{c+\theta_2}{\theta_2\alpha}\right)^2$$

$$-2pq\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)\left(\frac{c+\theta_2}{\theta_2\alpha}\right)$$

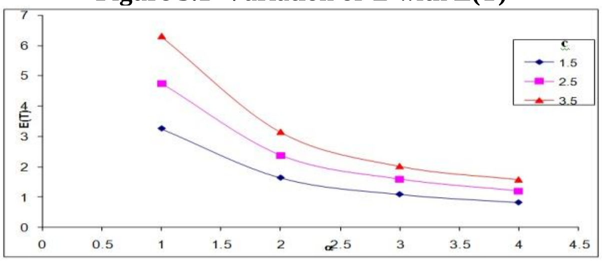
$$= p\,(2-p)\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)^2 + q(2-q)\left(\frac{c+\theta_2}{\theta_2\alpha}\right)^2 - 2pq\left(\frac{\theta_1+\lambda+c}{\alpha(\theta_1+\lambda)}\right)\left(\frac{c+\theta_2}{\theta_2\alpha}\right) \qquad (4)$$

V. Numerical Illustration

Table 5.1

α	□ = 1.0, c = 1.5, θ_1 = 0.5, θ_2 = 1.0		□ = 2.0, c = 2.5, θ_1 = 0.5, θ_2 = 1.0		□ = 3.0, c = 3.5, θ_1 = 0.5, θ_2 = 1.0	
	E (T)	V(T)	E (T)	V(T)	E (T)	V(T)
1	3.26	2.27	4.76	6.25	6.3	11.75
2	1.63	0.90	2.38	1.52	3.14	3.05
3	1.08	0.26	1.59	0.68	2.01	1.73
4	0.81	0.14	1.2	0.36	1.57	0.75

Figure 5.1 Variation of □ with E(T)



VI. Conclusion

In Table 5.1 the changes of E(T) and V(T) consequent to the increase the values of α, namely the parameter of the distribution of the interarrival times in considered three different cases namely c = 1.5, c = 2.5 and c = 3.5 are compared where λ is the parameter of the change point. For c =1.5, it may be noted that expectation of T and variance both decreases as α increases. This is due to the fact that $\theta_1 < \theta_2$, c is the parameter of the random variable X_i denoting the contribution to CO2 emission is more and so the expected time to cross the threshold namely E(T) and its variance decreases.

References

[1]. Esary, J. D., Marshall, A. W. and Proschan, I'., Shock models and wear
[2]. processes. Ann. Probability, 627-649 (1973).
[3]. Raja Rao, B., Life Expectancy for a Class of Life Distributions having the "Setting the Clock Back to Zero" property, Mathematical Biosciences 98, 251-271 (1990).
[4]. Sathiyamoorlhi, R. and Kannan, R., Stochastic model for the time to scro-conversion of HIV transmission. Journal of Kerala Statistical Association Vol. 12, Dec. 2001, 23-28 (2001).