Heat Transfer in the Steady Flow of a Second-Order Fluid between Two Enclosed Discs Rotating With Different Angular Velocities

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Abstract: The problem of heat transfer in the steady flow of an incompressible second-order fluid between two enclosed discs rotating with different angular velocities has been discussed. The effect of elasto-viscosity, cross-viscosity parameters and ratio of the angular velocity on the temperature profile and Nusselt number for the cases of radial outflow and inflow have been investigated in the regions of no-recirculation and recirculation.

Key words: Heat transfer, steady flow, second-order fluid, two enclosed discs, rotating with different angular velocities.

I. Introduction

The phenomenon of flow and heat transfer of the fluid between two enclosed rotating discs (enclosed in a cylindrical casing) or shrouded discs has important engineering application as its generalization could be helpful in the study of heat transfer analysis of air cooling of turbine discs[1] and the determination of oil film temperature of pedestal bearing with central feeding of lubricant[2]. Soo et al[3] has investigated the nature of heat transfer from an enclosed rotating disc for viscous flow. Sharma and Agarwal[4] reconsidered this problem using an improved formulation suggested by Sharma[5]. Sharma and Singh[6] have studied the problem of heat transfer in the flow of a second-order fluid between two enclosed rotating discs.

In the present investigations we are concerned with the heat transfer in the steady incompressible flow of a second-order fluid between two enclosed discs rotating with different angular velocities in the following two cases:

(i) When the discs rotate in the same sense with equal angular velocity.
(ii) When both discs rotate with different angular velocities.

II. Formulation Of The Problem

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll[7] can be written as

\[ \tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \]  

where

\[ d_{ij} = \frac{1}{2}(u_i, j + u_j, i), \]
\[ e_{ij} = \frac{1}{2}(a_i, j + a_j, i) + u^m, i u^m, j \]
\[ c_{ij} = d_i^m d_m^j \]

where \( p \) is the hydrostatic pressure, \( \tau_{ij} \) is the stress tensor, \( u_i \) and \( a_i \) are velocity and acceleration vector.

The equation (1) together with the momentum equation for no extraneous force

\[ \rho \left( \frac{\partial u_i}{\partial t} + u^m u_i, m \right) = \tau_{i, m} \]  

and the equation of continuity for incompressible fluid

\[ \rho \left( \frac{\partial u_i}{\partial t} + u^m u_i, m \right) = \tau_{i, m} \]
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\[ u_i^i = 0 \]  \hspace{1cm} \text{(4)}

where \( \rho \) is the density of the fluid and (, ) represents covariant differentiation, forms the set of governing equations. The material constants \( \mu_1, \mu_2 \) and \( \mu_3 \) are known as coefficients of Newtonian-viscosity, elastico-viscosity and cross-viscosity respectively, whose values are given by \( \mu_1 = 18.5, \mu_2 = -0.2 \) and \( \mu_3 = 1.0 \) (all expressed in CGS units for 5.46% solutions of poly-isobutylene in cetane at 30°C as suggested by Markowitz[8]).

In a three dimensional cylindrical set of co-ordinate \((r, \Theta, z)\), the system consists of two finite discs of radius \( r_s \) (coinciding with the plane \( z = 0 \) and \( z = z_0 < r_s \)) rotating with different velocities. The lower disc is rotating with angular velocity \( \Omega \) whenever the upper disc is rotating with different angular velocity \( N \Omega \) in an incompressible second-order fluid forming the part of a co-axial cylindrical casing. The symmetrical radial steady out flow has a small mass rate \( \dot{m} \) for radial outflow and \( -\dot{m} \) for radial inflow.

The boundary conditions on the velocity profile are:

\[
\begin{align*}
    z = 0 & : u = 0, \quad v = r \Omega, \quad w = 0, \quad T = T_a \\
    z = z_0 & : u = 0, \quad v = N r \Omega, \quad w = 0, \quad T = T_b
\end{align*}
\]  \hspace{1cm} \text{(7)}

where \((u, v, w)\) are the velocity components along the cylindrical system of axis \((r, \Theta, z)\). The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [6].

\[
\begin{align*}
    U &= -\xi H'(\zeta) + \xi^{-1} \left( \frac{R_m}{R_z} \right) M'(\zeta) \\
    V &= \xi G(\zeta) + \xi^{-1} \left( \frac{R_l}{R_z} \right) L(\zeta) \\
    W &= 2H(\zeta)
\end{align*}
\]  \hspace{1cm} \text{(8)}

where \( U = \frac{u}{\Omega z_0}, V = \frac{v}{\Omega z_0}, W = \frac{w}{\Omega z_0} \), \( T^* = \left( \frac{c_v}{v_1 \Omega} \right) T, \Phi^* = \frac{\Phi}{\mu_1 \Omega^2}, \xi = \frac{r}{z_0}, \)

\( \zeta = \frac{z}{z_0} \) are dimensionless quantities and \( H(\zeta), G(\zeta), L(\zeta), M'(\zeta) \) are dimensionless functions of the dimensionless variable \( \zeta \). \( R_m \left( = \frac{m}{(2 \pi \rho z_0 v_1)} \right), R_l \left( = \frac{l}{(2 \pi \rho z_0 v_1)} \right) \) are dimensionless numbers to be
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called Reynolds number of net radial outflow and circulatory flow respectively \( R_m \) is negative for a net radial inflow. \( R_z = \frac{\Omega z_0^2}{v_1} \) be the flow Reynold’s number. The small mass rate ‘\( m' \) of the radial outflow is represented by
\[
m = 2\pi p \int_0^{z_0} ru\,dz \quad \ldots(9)
\]
Using expression (8), the boundary conditions transform for \( G, L \) and \( H \) into the following form:
\[
\begin{align*}
\zeta = 0 & : \quad G = 1, \quad L = 0, \quad H = 0, \quad H' = 0 \\
\zeta = 1 & : \quad G = N, \quad L = 0, \quad H = 0, \quad H' = 0 \quad \ldots(10)
\end{align*}
\]
The condition on \( M \) on the boundaries are obtainable from the equation (9) for \( m' \) as follows
\[
M(1) - M(0) = 1, \quad M(1) = 1, \quad M(0) = 0 \quad \ldots(12)
\]
Using equation (1) and expression (8) in equation (3) and neglecting the squares and higher powers of \( \frac{R_m}{R_z} \) (assumed small) the dimensionless equation of motion for transverse component of the velocity is-
\[
0 = -2\Omega^2 z_0 \xi (HG' - H'G) - \Omega^2 z_0 \left( \frac{R_m}{R_z} \right) \left( \frac{2M'G}{\xi} \right) - \Omega^2 z_0 \left( \frac{R_l}{R_z} \right) \left( \frac{2HL'}{\xi} \right) \\
+ \frac{\Omega}{z_0} \left[ \xi G'' + \left( \frac{R_l}{R_z} \right) \left( \frac{L''}{\xi} \right) \right] + \left( \frac{2v_2}{z_0} \right) \left[ \left( \frac{R_l}{R_z} \right) \left( \frac{\Omega^2}{\xi} \right) \right] (H''L' + H'''L) \\
+ HL'' + H'L'\prime) + (\Omega^2 \xi) (HG'' - H''G) + \left( \frac{R_m}{R_z} \right) \left( \frac{2\Omega^2}{\xi} \right) (M'G'') \\
+ M''G') + \left( \frac{2v_3\Omega^2}{z_0} \right) \left[ \xi (H'G'' - H''G') + \left( \frac{R_l}{R_z} \right) \left( \frac{1}{\xi} \right) (H''L' + H'''L) \\
+ H'L'\prime) + \left( \frac{R_m}{R_z} \right) \left( \frac{1}{\xi} \right) (2M''G' + M'G''\prime) \right] \quad \ldots(13)
\]
After eliminating the pressure \( p \) from the equation of motion of radial and axial component we get–
\[
-2\Omega^2 z_0 \xi (HH'' + GG') + \left( \frac{R_m}{R_z} \right) \left( \frac{2\Omega^2 z_0}{\xi} \right) (H'M'' + HM'''\prime) - \left( \frac{R_l}{R_z} \right) \times \]
\[
\left( \frac{2\Omega^2 z_0}{\xi} \right) (LG' + L'G) - \left( \frac{v_1\Omega}{z_0} \right) \left[ \left( \frac{R_m}{R_z} \right) \left( \frac{M'iv}{\xi} \right) - \xi H_{iv}' \right] - \left( \frac{2v_2}{z_0} \right) \times \]
\[
\left[ -\Omega^2 \xi (2H''H'' + H'H_{iv}' + HH_{iv}' + 4G'G'') + \left( \frac{R_m}{R_z} \right) \left( \frac{\Omega^2}{\xi} \right) (2H''M'' \\
+ H_{iv}'M' + 2H'M'' + 2H'M_{iv}' + HM_{iv}') - \left( \frac{R_l}{R_z} \right) \left( \frac{2\Omega^2}{\xi} \right) (2L'G'' \prime) \right] \right]
\]
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\[ + L''G' + LG'' = \left( \frac{2

\text{...}(14)\text{...}

The energy equation (5) take the dimensionless form as:

\[ U \frac{\partial T^*}{\partial \zeta} + W \frac{\partial T^*}{\partial \zeta} = \frac{1}{PR_c} \left( \frac{\partial T^*}{\partial \zeta^2} + \frac{1}{\zeta} \frac{\partial T^*}{\partial \zeta} + \frac{\partial^2 T^*}{\partial \zeta^2} \right) + \Phi^* \text{...}(15)\text{...}

where \( p = \left( \frac{H_c v}{k} \right) \) is the prandtl number.

III. Solution Of The Problem

On equating the coefficients of \( \frac{\xi}{2} \) and \( \frac{1}{\xi} \) from the equations (13) and (14) and assuming \( R_m \sim R_z \) and expanding the function \( H, G, L, M \) in the ascending powers of \( R_z \) (assumed small such that \( R_z^3 \) and higher power of \( R_z \) are neglected) as:

\[
G(\zeta) = G_0(\zeta) + R_z G_1(\zeta) + R_z^2 G_2(\zeta)
\]

\[
L(\zeta) = L_0(\zeta) + R_z L_1(\zeta) + R_z^2 L_2(\zeta)
\]

\[
H(\zeta) = H_0(\zeta) + R_z H_1(\zeta) + R_z^2 H_2(\zeta)
\]

\[
M(\zeta) = M_0(\zeta) + R_z M_1(\zeta) + R_z^2 M_2(\zeta)
\]

... (16)

We get the set of differential equations in terms of \( G_0, G_1, G_2, L_0, L_1, L_2, H_0, H_1, H_2, M_0, M_1, M_2 \) and their derivatives which when integrated subject to the boundary conditions:

\[
G_0(0) = 1, \quad G_1(0) = 0, \quad G_2(0) = 0 \quad G_0(1) = N, \quad G_1(1) = 0, \quad G_2(1) = 0
\]

\[
H_0(0) = 0, \quad H_1(0) = 0, \quad H_2(0) = 0 \quad H_0(1) = 0, \quad H_1(1) = 0, \quad H_2(1) = 0
\]

\[
H'_0(0) = 0, \quad H'_1(0) = 0, \quad H'_2(0) = 0 \quad H'_0(1) = 0, \quad H'_1(1) = 0, \quad H'_2(1) = 0
\]

\[
L_0(0) = 0, \quad L_1(0) = 0, \quad L_2(0) = 0 \quad L_0(1) = 0, \quad L_1(1) = 0, \quad L_2(1) = 0
\]

\[
M_0(0) = 0, \quad M_1(0) = 0, \quad M_2(0) = 0 \quad M_0(1) = 0, \quad M_1(1) = 0, \quad M_2(1) = 0
\]

\[
M'_0(0) = 0, \quad M'_1(0) = 0, \quad M'_2(0) = 0 \quad M'_0(1) = 0, \quad M'_1(1) = 0, \quad M'_2(1) = 0
\]

... (17)

Gives the values of \( G_0, G_1, G_2, H_0, H_1, H_2, L_0, L_1, L_2, M_0, M_1, M_2 \).

Hence we get the values of the functions \( H, G, L, M \) calculated under the assumption of smallness of \( \left( \frac{R_m}{R_z} \right) \) and equivalence of the order of \( R_m \) and \( R_l \) correct to the square of flow Reynolds number \( R_z \), and first power of \( \left( \frac{R_m}{R_z} \right) \) are:

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\[ H = \left( \frac{R_z}{60} \right) \left[ \zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2 + N^2(\zeta^5 - 3\zeta^3 + 2\zeta^2) \\ -N(2\zeta^5 - 5\zeta^4 + 4\zeta^3 - \zeta^2) \right] \]

\[ G = 1 + (N - 1)\zeta + \left( \frac{R_z^2}{6300} \right) \left[ 20\zeta^7 - 140\zeta^6 + 357\zeta^5 - 420\zeta^4 + 210\zeta^3 - 27\zeta \\ -N^3(20\zeta^7 - 63\zeta^5 + 35\zeta^4 + 8\zeta) - N(60\zeta^7 - 280\zeta^6 + 441\zeta^5 - 280\zeta^4 \\ + 70\zeta^3 - 11\zeta) + N^2(60\zeta^7 - 140\zeta^6 + 21\zeta^5 + 175\zeta^4 - 140\zeta^3 + 24\zeta) \right] \]

\[ + \left( \frac{R_z^2}{30} \right)(\tau_1 + \tau_2) \left[ N^3(\zeta^5 - 3\zeta^3 + 2\zeta^2) - (\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2) \right] \]

\[ + N(3\zeta^5 - 10\zeta^4 + 11\zeta^3 - 4\zeta^2) - N^2(3\zeta^5 - 5\zeta^4 + 3\zeta^3 + \zeta^2) \]

\[ L = \left( \frac{R_m}{R_l} \right)R_z \left[ \frac{1}{5} \left( 3\zeta^5 - 10\zeta^4 + 10\zeta^3 - 3\zeta \right) - N(3\zeta^5 - 5\zeta^4 + 2\zeta) \right. \\
\left. + 20(\tau_1 + \tau_2)(N - 1)(2\zeta^3 - 3\zeta^2 + \zeta) \right] \]

\[ M' = 6(\zeta - \zeta^2) + \left( \frac{R_z^2}{4200} \right) \left[ 60\zeta^8 - 300\zeta^7 + 588\zeta^6 - 294\zeta^5 - 630\zeta^4 + 840\zeta^3 \\ -263\zeta^2 - \zeta + N^2(60\zeta^8 - 180\zeta^7 + 168\zeta^6 - 294\zeta^5 + 420\zeta^4 - 263\zeta^2 \\ + 89\zeta) - 2N(60\zeta^8 - 240\zeta^7 + 238\zeta^6 + 126\zeta^5 - 105\zeta^4 - 280\zeta^3 + 247\zeta^2 \\ - 46\zeta) \right] - \left( \frac{R_z^2}{525} \right)(\tau_1 + \tau_2) \left[ 105\zeta^6 - 595\zeta^5 + 1190\zeta^4 - 939\zeta^2 + 239\zeta \\ + N^2(105\zeta^6 - 630\zeta^5 + 980\zeta^4 - 910\zeta^3 + 636\zeta^2 - 181\zeta) - N(210\zeta^6 \\ - 630\zeta^5 + 385\zeta^4 + 280\zeta^3 - 303\zeta^2 + 58\zeta) \right] \]

\[ + \frac{8}{5} R_z^2(\tau_1 + \tau_2)^2(N - 1)^2(5\zeta^4 - 10\zeta^3 + 6\zeta^2 - \zeta) \] \quad \ldots (18)

where \( v_1 = \frac{\mu_1}{\rho} \), \( v_2 = \frac{\mu_2}{\rho} \) and \( v_3 = \frac{\mu_3}{\rho} \) are the kinematic-viscosity, elasto- viscosity and cross-viscosity respectively, \( \tau_1 = \frac{v_2}{z_0} \) and \( \tau_2 = \frac{v_3}{z_0} \) are the elasto-viscous and cross-viscous dimensionless parameters.
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Using (8) in equation (15) and neglecting terms of order \(\frac{R_m}{R_z}\)^2 and higher, we get

\[
\left[ \left( -\xi H' + \frac{1}{\xi} \left( \frac{R_m}{R_z} \right) M' \right) \frac{\partial T^*}{\partial \xi} + 2H \frac{\partial T^*}{\partial \zeta} \right]
\]

\[
= \frac{1}{PR_z} \left[ \frac{\partial^2 T^*}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial T^*}{\partial \xi} + \frac{\partial^2 T^*}{\partial \zeta^2} \right] + \frac{1}{2} \left[ 24H' + 4 \frac{R_m}{R_z} \left( \frac{R_l}{R_m} G'L' - H''M'' \right) 
+ 48\tau_1R_z (H' + HH'H'') + 2R_m\tau_1 \left\{ \frac{R_l}{R_m} (6H'L'G' + 6LG'H'' + 2HG'') 
+ 2HL'G'') + (-6H'H''M'' - 2HH'M'' - 2M'H''^2 - 2HM'H'' + 4M'G'') \right\} 
+ 48\tau_2R_z H' + 2R_m\tau_2 \left\{ (-3M'H''^2 - 6H'H''M'' + 3M'G'') + \frac{R_l}{R_m} (6H'L'G' 
+ 6LG'H'') \right\} \right] + \frac{1}{2} \xi \left( 2(H''^2 + G''^2) + \tau_1R_z (4H'H''^2 + 4H'G'') 
+ 4HH''H'' + 4HG'G'') + 6\tau_2R_z (H'H''^2 + H'G'') \right] \]

\[\cdots(19)\]

The appropriate form for \(T^*\) as suggested from equation (19) is

\[T^* = T_{a*} + \phi(\zeta) + \xi^2\psi(\zeta) \quad \cdots(20)\]

where \(T_{a*}\) denotes the dimensionless temperature on the lower disc and \(\phi, \psi\) are the dimensionless functions of \(\zeta\). Substituting equation (20) into (19) and equating the coefficients of \(\xi^2\) and terms independent of \(\xi\) on both sides of the equation thus obtained, we get

\[
\frac{1}{P} \left( 4\psi + \phi'' \right) = 2R_m \left[ M'\psi - \left\{ \left( \frac{R_l}{R_m} \right) G'L' - H''M'' \right\} \right] + 2R_z \left[ H\phi' - 6H'^2 \right]
\]

\[-\frac{1}{2} \tau_1R_m \left\{ \left( \frac{R_l}{R_m} \right) (6H'L'G' + 6LG'H'' + 2HG' + 2HL'G'') \right\} 
+ (-6H'H'M'' - 2HH'M'' - 2M'H''^2 - 2HM'H'' + 4M'G'') \right\} 
+ \frac{1}{2} \tau_2R_m \left\{ (-3M'H''^2 - 6H'H''M'' + 3M'G'') + \left( \frac{R_l}{R_m} \right) (6H'L'G' \right)
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\[
\frac{1}{P} \psi'' = R_\zeta [-2H'\psi + 2H\psi' - H'\psi'' - G'G''] - R^2_\zeta [2\tau_1 (H'H''^2 + H'G'^2 + H'H''^2) + 3\tau_2 (H'H''^2 + H'G'^2)]...(22)
\]

The expressions for \(H, G, L\) and \(M'\) have been considered up to \((R_\zeta)^2\) and hence the appropriate forms for \(\phi\) and \(\psi\) should be as follows:

\[
\begin{align*}
\phi &= \phi_0 + R_\zeta \phi_1 + R^2_\zeta \phi_2 \\
\psi &= \psi_0 + R_\zeta \psi_1 + R^2_\zeta \psi_2
\end{align*}
\]

where \(\phi_0, \psi_0\) etc. are functions of \(\zeta\).

The boundary conditions on the temperature in terms of \(\phi_0, \psi_0\) etc. can be written as:

\[
\begin{align*}
\zeta = 0 : & \quad \phi_0 = 0, \quad \psi_0 = 0, \quad \phi_1 = 0, \quad \psi_1 = 0, \quad \phi_2 = 0, \quad \psi_2 = 0 \\
\zeta = 1 : & \quad \phi_0 = S, \quad \phi_1 = 0, \quad \phi_2 = 0; \quad \psi_0 = 0, \quad \psi_1 = 0, \quad \psi_2 = 0
\end{align*}
\]

where \(S = \frac{(T_b - T_a)C_v}{(v_1\Omega)}\)

Substituting (23) into (21) and (22) and equating the terms independent of \(R_\zeta\), coefficient of \(R_\zeta\) and \(R^2_\zeta\) on both sides of the resulting equations, we get a set of equations containing \(\phi_0, \psi_0\) etc. and their derivatives. The solution of this set of equations satisfying the boundary condition (24) is given by:

\[
\begin{align*}
\psi_0 &= 0, \quad \psi_1 = \frac{1}{2} P(N - 1)^2 \zeta (1 - \zeta), \quad \psi_2 = 0 \\
\phi_0 &= S \zeta \\
\phi_1 &= \frac{1}{6} P(N - 1)^2 (\zeta^4 - 2\zeta^3 + \zeta) + PR_m \left[ \frac{1}{10} P(N - 1)^2 (2\zeta^6 - 6\zeta^5 + 5\zeta^4 - \zeta) \\
&\quad - \frac{1}{15} \left( \zeta^6 - 9\zeta^5 + 21\zeta^4 - 27\zeta^3 + 18\zeta^2 - 4\zeta + N^2 (\zeta^6 + 3\zeta^5 - 9\zeta^4 + 13\zeta^3 \\
&\quad - 12\zeta^2 + 4\zeta) - 2N (\zeta^6 - 3\zeta^5 + 6\zeta^4 - 7\zeta^3 + 3\zeta^2) \right) \right] - \left\{ 2\tau_1 (\zeta^4 - 2\zeta^3 \\
&\quad + 2\zeta^2 - \zeta) + \frac{1}{2} \tau_2 (5\zeta^4 - 10\zeta^3 + 8\zeta^2 - 3\zeta) \right\} (N - 1)^2 \right] \\
\phi_2 &= \frac{PS}{(12600)} \left[ (10\zeta^7 - 70\zeta^6 + 147\zeta^5 - 105\zeta^4 + 18\zeta) + N^2 (10\zeta^7 - 63\zeta^5 \\
&\quad + 70\zeta^4 - 17\zeta) - N (20\zeta^7 - 70\zeta^6 + 84\zeta^5 - 35\zeta^4 + \zeta) \right] \quad ...(25)
\end{align*}
\]

Using equations (20), (23) and (25) we get the solution of equation (19) the dimensionless form of the temperature as:
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\[
\frac{T - T_a}{T_b - T_a} = \frac{T^* - T_a^*}{T_b^* - T_a^*} = \zeta + \left( \frac{PR^2}{12600} \right) \left\{ 10\zeta^7 - 70\zeta^6 + 147\zeta^5 - 105\zeta^4 + 18\zeta^3 + N^2(10\zeta^7 - 63\zeta^5 + 70\zeta^4 - 17\zeta) - N(20\zeta^7 - 70\zeta^6 + 84\zeta^5 - 35\zeta^4 + \zeta) \right\} + EP \left\{ \frac{1}{6} (N - 1)^2 (2\zeta^6 - 6\zeta^5 + 5\zeta^4 - \zeta) - 2 \left( \zeta^6 - 9\zeta^5 + 21\zeta^4 - 27\zeta^3 + 18\zeta^2 - 4\zeta \right) + N^2 (\zeta^6 + 3\zeta^5 - 9\zeta^4 + 13\zeta^3 - 12\zeta^2 + 4\zeta) + 2N (\zeta^6 + 3\zeta^5 - 6\zeta^4 + 7\zeta^3 - 3\zeta^2) \right\} - \left\{ 60\tau_1 (\zeta^4 - 2\zeta^3 + 2\zeta^2 - \zeta) + 15\tau_2 (5\zeta^4 - 10\zeta^3 + 8\zeta^2 - 3\zeta) \right\} \left( N - 1 \right)^2 + \frac{1}{2} \xi^2 (N - 1)^2 \zeta (1 - \zeta) \right\} \quad \ldots (26)
\]

where \( E \left( = \frac{\Omega z_0^2}{c_v(T_b - T_a)} \right) \) is the Eckert number.

The amount of heat transfer from the lower and upper discs are

\[
Q_a = \frac{1}{\pi(\xi^2 - \xi_0^2)} \int_{\xi_0}^{\xi} 2\pi \xi q_a d\xi \quad \ldots (27)
\]

and

\[
Q_b = \frac{1}{\pi(\xi^2 - \xi_0^2)} \int_{\xi_0}^{\xi} 2\pi \xi q_b d\xi \quad \ldots (28)
\]

respectively.

\[
q_a = - \left( \frac{k}{z_0} \right) \frac{\partial T}{\partial \xi} \bigg|_{\xi=0} \quad \text{and} \quad q_b = - \left( \frac{k}{z_0} \right) \frac{\partial T}{\partial \xi} \bigg|_{\xi=1}
\]

are heat fluxes on the lower and upper discs. Therefore the average Nusselt numbers on the lower and upper discs are

\[
Nu_a = \frac{Q_a z_0}{(T_b - T_a)k} = \left[ \frac{PR^2}{12600} \right] \left\{ 18 - 17N^2 - N \right\} + EP \left\{ \frac{1}{6} (N - 1)^2 \right\} + \frac{R_m}{30} \left\{ -3N (N - 1)^2 + 8 - 8N^2 + 60\tau_1 (N - 1)^2 + 45\tau_2 (N - 1)^2 \right\} + \frac{1}{4} (\xi^2 + 2\zeta_0^2)(N - 1)^2 \right\} \quad \ldots (29)
\]

\[
Nu_b = \frac{Q_b z_0}{(T_b - T_a)k} = \left[ \frac{PR^2}{12600} \right] \left\{ -17 + 18N^2 - N \right\} + EP \left\{ \frac{1}{6} (N - 1)^2 \right\}
\]
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\[
+ \frac{R_m}{30} \left\{ 3P(N - 1)^2 + 8 - 8N^2 - 60\tau(N - 1)^2 - 45\tau_2(N - 1)^2 \right\} - \frac{1}{4} (\xi^2 + \xi_0^2)(N - 1)^2 \right\} \] ...
(30)

IV. Results And Discussion

The dimensionless form of the radii at which there is no recirculation for the cases of net radial outflow \( (R_m > 0) \) and net radial inflow \( (R_m < 0) \) respectively, satisfying the following conditions

I \[ R_m > 0 \] \( \frac{\partial U}{\partial \zeta} \) \( \zeta = 0 \) \( \geq 0 \) \( \frac{\partial U}{\partial \zeta} \) \( \zeta = 1 \) \( \leq 0 \) ...
(31)

II \[ R_m = -R_n < 0 \] \( \frac{\partial U}{\partial \zeta} \) \( \zeta = 0 \) \( \leq 0 \) \( \frac{\partial U}{\partial \zeta} \) \( \zeta = 1 \) \( \geq 0 \) ...
(32)

Case I: When both the discs rotate in the same sense with equal angular velocity \( (N = 1) \): It is evident from equation (26) that there is no effect of the second-order parameter \( \tau_1, \tau_2 \) and \( \xi, E, R_m \) on the temperature profile. It is also observed that the temperature variation with \( \zeta \) is linear. The average Nusselt numbers in this case are constants for the constant value of \( R_m, P \).

Case II: When both the discs rotate with different angular velocities: The maximum value of the dimensionless radii \( \xi \) for no-recirculation viz. \( \xi_1 \) (for \( m > 0 \)) and \( \xi_2 \) (for \( m < 0 \)) are obtained from the condition (31) and (32) respectively as:

\[
\xi_1^2 = \frac{25200 - R_m^2 [6720(N - 1)^2 \tau^2 + 8\tau(239 - 181N^2 - 58N) + 1 - 89N^2 - 92N]}{70R_m^2 (4N^2 + 2N - 6)}
\]

\[
\xi_2^2 = \frac{25200 - R_m^2 [6720(N - 1)^2 \tau^2 - 8\tau(181 - 239N^2 + 58N) + N^2 - 92N - 89]}{70R_m^2 (-4 + 6N^2 - 2N)}
\]

where \( \tau = \tau_1 + \tau_2 \) and \( R_m = -R_n (R_n > 0) \) for \( m < 0 \).

The behaviour of the dimensionless temperature \( \frac{(T - T_n)}{(T_b - T_n)} \) with \( \zeta \) at \( P = 20 \),

\( E = 0.02, R = 0.5, N = 2 \) for different values of \( \tau_2 = 0, 1, 2 \) in cases of \( R_m (= -0.02) < 0; R_m (= 0.02) > 0 \) when \( \xi = 20 \) and \( \xi = 1 \) is represented through figure-1 and figure-2 respectively.

**Table-1** Variation of Temperature with \( \zeta \) for different values of \( \tau_2 \) at \( N = 2 \) in Recirculation region \( (\xi = 20) \) in case of \( R_m < 0 \) and \( R_m > 0 \)

<table>
<thead>
<tr>
<th>( R_m = -0.02 )</th>
<th>( R_m = 0.02 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>( \tau_2 = 0 )</td>
</tr>
<tr>
<td>0.0</td>
<td>7.30652</td>
</tr>
<tr>
<td>0.1</td>
<td>13.012049</td>
</tr>
<tr>
<td>0.2</td>
<td>17.116175</td>
</tr>
<tr>
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<td>20.616826</td>
</tr>
<tr>
<td>0.4</td>
<td>25.519262</td>
</tr>
</tbody>
</table>
Heat Transfer In The Steady Flow Of A Second-Order Fluid Between Two Enclosed Discs Rotating

<table>
<thead>
<tr>
<th>0.7</th>
<th>17.515198</th>
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<th>17.509577</th>
<th>17.511904</th>
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</thead>
<tbody>
<tr>
<td>0.9</td>
<td>8.105617</td>
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<td>8.103265</td>
<td>8.104391</td>
<td>8.105518</td>
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</tbody>
</table>

Table-2 Variation of Temperature with $\zeta$ for different values of $\tau_2$ at $N = 2$

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
<th>$\tau_2 = 2$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
<th>$\tau_2 = 2$</th>
</tr>
</thead>
<tbody>
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<td>0.123864</td>
<td>0.123209</td>
<td>0.12045</td>
<td>0.121106</td>
<td>0.121762</td>
</tr>
<tr>
<td>0.2</td>
<td>0.244049</td>
<td>0.243071</td>
<td>0.242093</td>
<td>0.237667</td>
<td>0.238645</td>
<td>0.238645</td>
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<tr>
<td>0.3</td>
<td>0.358175</td>
<td>0.357068</td>
<td>0.355961</td>
<td>0.349522</td>
<td>0.350629</td>
<td>0.350629</td>
</tr>
<tr>
<td>0.4</td>
<td>0.466627</td>
<td>0.465482</td>
<td>0.464338</td>
<td>0.457834</td>
<td>0.457834</td>
<td>0.457834</td>
</tr>
<tr>
<td>0.5</td>
<td>0.569264</td>
<td>0.568114</td>
<td>0.566964</td>
<td>0.55923</td>
<td>0.56038</td>
<td>0.56038</td>
</tr>
<tr>
<td>0.6</td>
<td>0.66079</td>
<td>0.664935</td>
<td>0.66379</td>
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<td>0.658294</td>
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<tr>
<td>0.7</td>
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<td>0.751465</td>
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<tr>
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<td>0.841924</td>
<td>0.840946</td>
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<td>0.838682</td>
<td>0.83966</td>
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<tr>
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<td>0.922311</td>
<td>0.921271</td>
<td>0.921927</td>
<td>0.922583</td>
</tr>
</tbody>
</table>

Table-2 Variation of Temperature with $\zeta$ for different values of $\tau_2$ at $N = 2$

It is clear from the numerical values of the temperature that the temperature decreases with an increase in $\tau_2$ in case of net radial inflow ($R_m < 0$) whenever increases in case of net radial outflow ($R_m > 0$) through out the gap length in both the figures-1 ($\zeta = 20$) and figure-2 ($\zeta = 1$). Due to small increments in temperature with respect to $\tau_2$ the branches of the graphs are overlapping in both the figures. In figure-1 the temperature is maximum at the middle of the gap length and minimum (equal to zero) at the lower disc. Temperature is increasing near the lower disc and after attaining its maximum value in the middle it starts decreasing and attains the value 1 on the upper disc whenever in figure-2 it is zero at the lower disc and attains its maximum value 1 on the upper disc.

Figure-2 Variation of Temperature with $\zeta$ for different values of $\tau_2$ at $N = 2$

In figure-3 ($\zeta = 20$) and figure-4 ($\zeta = 1$), the value of the temperature at $N = 1$ is equal to $\zeta$ for $R_m < 0$ and $R_m > 0$. At $P = 20$, $E = 0.02$, $\zeta = 0.5$, $\tau_2 = 2$ for different values

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of \( N = 1, 2, 3 \) the temperature is increasing with an increase in the value of the ratio of angular velocities in case of net radial inflow and outflow in both the figures 3 and 4. The graph of the temperature is linear at \( N = 1 \) with minimum at lower disc and maximum at the upper disc. At \( N = 2 \) and \( N = 3 \), the temperature is maximum in the middle of the gap length in figure-3 and maximum on the upper disc in figure-4.

**Table-3** Variation of Temperature with \( \zeta \) for different values of \( N \) at \( \tau_2 = 2 \) in Recirculation region (\( \xi = 20 \)) in case of \( R_m < 0 \) and \( R_m > 0 \)

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>( N = 1 )</th>
<th>( N = 2 )</th>
<th>( N = 3 )</th>
<th>( N = 1 )</th>
<th>( N = 2 )</th>
<th>( N = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>0.1</td>
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<td>28.923027</td>
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<td>28.918385</td>
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<tr>
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<td>13.008311</td>
<td>51.445034</td>
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<td>13.008427</td>
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</tr>
<tr>
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<td>0.3</td>
<td>17.111523</td>
<td>67.562813</td>
<td>0.3</td>
<td>17.111069</td>
<td>67.542693</td>
</tr>
<tr>
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<td>0.4</td>
<td>19.613482</td>
<td>77.274048</td>
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<td>0.5</td>
<td>20.513964</td>
<td>80.5774</td>
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<td>20.51338</td>
<td>80.551071</td>
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<td>0.6</td>
<td>19.812933</td>
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<td>0.7</td>
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</table>

**Figure-3** Variation of Temperature with \( \zeta \) for different values of \( N \) at \( \tau_2 = 2 \) in Recirculation region (\( \xi = 20 \)) in case of \( R_m < 0 \) and \( R_m > 0 \)

**Table-4** Variation of Temperature with \( \zeta \) for different values of \( N \) at \( \tau_2 = 2 \) in No-Recirculation region (\( \xi = 1 \)) in case of \( R_m > 0 \) and \( R_m < 0 \)

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>( N = 1 )</th>
<th>( N = 2 )</th>
<th>( N = 3 )</th>
<th>( N = 1 )</th>
<th>( N = 2 )</th>
<th>( N = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0.1</td>
<td>0.123209</td>
<td>0.195027</td>
<td>0.1</td>
<td>0.121762</td>
<td>0.190386</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.242093</td>
<td>0.373036</td>
<td>0.2</td>
<td>0.238645</td>
<td>0.360683</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3</td>
<td>0.355961</td>
<td>0.530814</td>
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<td>0.350629</td>
<td>0.510691</td>
</tr>
<tr>
<td>0.4</td>
<td>0.4</td>
<td>0.464338</td>
<td>0.66605</td>
<td>0.4</td>
<td>0.457834</td>
<td>0.640706</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.569694</td>
<td>0.777403</td>
<td>0.5</td>
<td>0.56038</td>
<td>0.751069</td>
</tr>
<tr>
<td>0.6</td>
<td>0.6</td>
<td>0.66379</td>
<td>0.864532</td>
<td>0.6</td>
<td>0.658294</td>
<td>0.841874</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7</td>
<td>0.754986</td>
<td>0.928122</td>
<td>0.7</td>
<td>0.751465</td>
<td>0.912831</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.840946</td>
<td>0.969887</td>
<td>0.8</td>
<td>0.83966</td>
<td>0.963302</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9</td>
<td>0.922311</td>
<td>0.992581</td>
<td>0.9</td>
<td>0.922583</td>
<td>0.992523</td>
</tr>
</tbody>
</table>

0.1 | 0.1 | 0.123209 | 0.195027 | 0.1 | 0.121762 | 0.190386 |
| 0.2 | 0.2 | 0.242093 | 0.373036 | 0.2 | 0.238645 | 0.360683 |
| 0.3 | 0.3 | 0.355961 | 0.530814 | 0.3 | 0.350629 | 0.510691 |
| 0.4 | 0.4 | 0.464338 | 0.66605 | 0.4 | 0.457834 | 0.640706 |
| 0.5 | 0.5 | 0.569694 | 0.777403 | 0.5 | 0.56038 | 0.751069 |
| 0.6 | 0.6 | 0.66379 | 0.864532 | 0.6 | 0.658294 | 0.841874 |
| 0.7 | 0.7 | 0.754986 | 0.928122 | 0.7 | 0.751465 | 0.912831 |
| 0.8 | 0.8 | 0.840946 | 0.969887 | 0.8 | 0.83966 | 0.963302 |
| 0.9 | 0.9 | 0.922311 | 0.992581 | 0.9 | 0.922583 | 0.992523 |
**Figure-4** Variation of Temperature with $\zeta$ for different values of $N$ at $\tau_2 = 2$

in No-Recirculation region ($\zeta = 1$) in case of $R_m < 0$ and $R_m > 0$.

Figure-5 represents the variation of $Nu_a$ (Nusselt number at the lower disc) with $\xi$ at $P = 20$, $E = 0.02$, $R_e = 0.5$, $\xi_0 = 5$ for different values of $\tau_2 = 0, 1, 2$. The average Nusselt number ($Nu_a$) on the lower disc is negative throughout the entire radial region and increases with an increase in $\tau_2$ in case of $R_m < 0$ whenever decreases in case of $R_m > 0$. It is also clear that $Nu_a$ is decreasing with an increase in $\xi$ in both the cases ($R_m > 0$ and $R_m < 0$). The behaviour of $Nu_b$ (average Nusselt number on the upper disc) in figure-6 is just reverse to that of $Nu_a$ (see figure-5). $Nu_b$ is positive throughout the entire radial region and attains its minimum value at the lower disc and maximum at the upper disc.

**Table-5** Variation of $Nu_a$ with $\xi$ for different values of $\tau_2$ at $N = 2$, $\xi_0 = 5$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
<th>$\tau_2 = 2$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
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</thead>
<tbody>
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</tr>
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<td>-6.03243</td>
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</tr>
</tbody>
</table>

**Figure-5** Variation of $Nu_a$ with $\xi$ for different values of $\tau_2$ at $N = 2$, $\xi_0 = 5$

in case of $R_m < 0$ and $R_m > 0$.
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Table-6 Variation of $\bar{Nu}_b$ with $\xi$ for different values of $\tau_2$ at $N = 2$, $\xi_0 = 5$

in case of $R_m > 0$ and $R_m < 0$

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
<th>$\tau_2 = 2$</th>
<th>$\tau_2 = 0$</th>
<th>$\tau_2 = 1$</th>
<th>$\tau_2 = 2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.537635</td>
<td>1.536035</td>
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<td>1.553635</td>
</tr>
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<td>1.653635</td>
</tr>
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<td>1.946435</td>
<td>1.937635</td>
<td>1.936035</td>
<td>1.944835</td>
<td>1.953635</td>
</tr>
<tr>
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<td>2.446435</td>
<td>2.437635</td>
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Figure-6 Variation of $\bar{Nu}_b$ with $\xi$ for different values of $\tau_2$ at $N = 2$, $\xi_0 = 5$

in case of $R_m < 0$ and $R_m > 0$

The variation $\bar{Nu}_a$ and $\bar{Nu}_b$ with $\xi$ at $P = 20$, $E = 0.02$, $R_z = 0.5$, $\xi_0 = 5$ for different values of $N = 1, 2, 3$ through figure-7 and 8 respectively. The value of $\bar{Nu}_a$ is negative throughout the radial region for all values of $N$. In case of $N = 1$ both $\bar{Nu}_a$ and $\bar{Nu}_b$ for net radial inflow and outflow have same values $-1$ for all values of $\xi$. In case of $N = 2, 3$ $\bar{Nu}_a$ have negative values whenever $\bar{Nu}_b$ have positive values. $\bar{Nu}_a$ is decreasing with an increase in velocity ratio $N$ in both cases $R_m < 0$ and $R_m > 0$ (see figure-7) whenever the behaviour of $\bar{Nu}_b$ (see figure-8) is just reverse to that of $\bar{Nu}_a$.

Table-7 Variation of $\bar{Nu}_a$ with $\xi$ at $\tau_2 = 2$, $\xi_0 = 5$ for different values of $N$

in case of $R_m < 0$ and $R_m > 0$

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<th>$R_m = 0.02$</th>
</tr>
</thead>
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</tr>
<tr>
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<td>-3.65083</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-3.95083</td>
</tr>
</tbody>
</table>
Heat Transfer In The Steady Flow Of A Second-Order Fluid Between Two Enclosed Discs Rotating

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>-4.45083</td>
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Figure-7 Variation of $N_{u_a}$ with $\xi$ at $\tau_2 = 2$, $\xi_0 = 5$ for different values of $N$ in case of $R_m < 0$ and $R_m > 0$

Table-8 Variation of $N_{u_b}$ with $\xi$ at $\tau_2 = 2$, $\xi_0 = 5$ for different values of $N$ in case of $R_m < 0$ and $R_m > 0$

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<th>$R_m = -0.02$</th>
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Figure-8 Variation of $N_{u_b}$ with $\xi$ at $\tau_2 = 2$, $\xi_0 = 5$ for different values of $N$ in case of $R_m < 0$ and $R_m > 0$
V. Conclusions

It is evident from the expressions of the functions $H$, $G$, $L$, $M'$ and the graphs of this problem that at $N = 1$ the results are identical to those obtained by Sharma and Singh\textsuperscript{[6]}.

Since $Nu_\alpha$ is negative throughout the entire radial region it follows that the heat is being transferred from the fluid to the lower disc and the amount of heat transfer decreases with an increase in $\tau_2$ in case of $R_m > 0$ whenever increases in case of $R_m < 0$. It is also concluded that due to $Nu_\beta > 0$ for all values of $\xi$ heat is being transferred from the fluid towards the upper disc and the amount of heat transfer is being increased with an increase in the angular velocity of the upper disc.

References