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Fuzzy L –Filters

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Abstract: *In this paper, fuzzy L-filter and level fuzzy L-filter are defined. Also some elementary properties and theorems are derived. Some examples are provided.*
Key Words-*Fuzzy L-ideals, level fuzzy L-ideals, fuzzy L-filters and level fuzzy L-filters.*

I. Introduction

The concept of fuzzy sets was introduced in 1965 by L.A.Zadeh [1]. In that, the fuzzy group was introduced by Rosenfield [2]. Yuan and Wu [3] applied the concepts of fuzzy sets in lattice theory. The idea of fuzzy sublattice was introduced by Ajmal [4]. In paper [5], the definition of fuzzy L-ideal, level fuzzy L-ideal, union and intersection of fuzzy L-ideals, theorems, propositions and examples are given. In this present paper, fuzzy L-filters and level fuzzy L-filters are introduced. Some characterization theorems and propositions are derived. Some more results related to this topic are also established.

II. Preliminaries

Fuzzy L-ideal, level fuzzy L-ideal are defined and examples are given.

Definition: 2.1

A fuzzy subset $\mu: L \to [0,1]$ of L is called a fuzzy L-ideal of L if $\forall$ x, y $\in$ L,

(i) $\mu(x \vee y) \geq \min \{ \mu(x), \mu(y)\}$

(ii) $\mu(x \wedge y) \geq \max \{ \mu(x), \mu(y)\}$.

Example: 2.2

Let L = { 0,a,b,1}. Let $\mu: L \to [0,1]$ is a fuzzy set in L defined by $\mu(0) = 0.9$, $\mu(a) = 0.5$, $\mu(b) = 0.5$, $\mu(c) = 0.5$, $\mu(1) =0.5$.

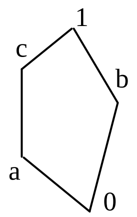

figure.1

Then μ is a fuzzy L-ideal of L.

Definition: 2.3

Let μ be any fuzzy L-ideal of a lattice L and let t$\in$ [0,1]. Then μ_t = { x$\in$ L / $\mu(x) \geq t$} is called level fuzzy L-ideal of μ.

Example : 2.4

Let L = { 0,a,b,1}. Let $\mu: L \to [0,1]$ is a fuzzy set in L defined by $\mu(0) = 0.7$, $\mu(a) = 0.5$, $\mu(b) = 0.5$,$\mu(c) = 0.5$, $\mu(1) =0.5$.

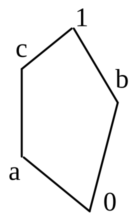

figure.2

Then μ is a fuzzy L-ideal of L.
In this example, let t = 0.5. Then $\mu_t = \mu_{0.5}$ = { a, b, c, 1 }.

III. Some Theorems On Fuzzy L-Filters

In this section, some properties of fuzzy L-filters are discussed and some theorems on fuzzy L-filters are derived.

Definition: 3.1
A fuzzy subset $\mu:L\rightarrow[0,1]$ of L is called a fuzzy L-filter of L if $\forall$ x, y $\in$ L,
(i) $\mu(x \vee y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(x \wedge y) \leq \min\{\mu(x), \mu(y)\}$.

Example: 3.2
Let L = { 0,a,b,1}. Let $\mu:L\rightarrow[0,1]$ is a fuzzy set in L defined by $\mu(0) = 0.3$, $\mu(a) = 0.3$, $\mu(b) = 0.3$, $\mu(c) = 0.3$, $\mu(1) = 0.7$.

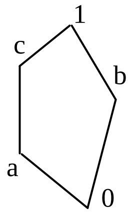

figure.3

Then μ is a fuzzy L-filter of L.

Definition: 3.3
Let μ be any fuzzy L-filter of a lattice L and let t$\in$ [0,1]. Then $\mu_t = \{ x\in L / \mu(x) \leq t \}$is called level fuzzy L-filter of μ.

Example : 3.4
Let L = { 0,a,b,1}. Let $\mu:L\rightarrow[0,1]$ is a fuzzy set in L defined by $\mu(0) = 0.3$, $\mu(a) = 0.3$, $\mu(b) = 0.3$, $\mu(c) = 0.3$, $\mu(1) = 0.5$.

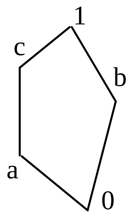

figure.4

Then μ is a fuzzy L-filter of L.
In this example, let t = 0.3. Then $\mu_t = \mu_{0.3} = \{ 0, a, b, c \}$.

Definition: 3.5
Let μ_1 and μ_2 be any two fuzzy L-filters of a lattice L. μ_1 is said to be contained in μ_2 if $\mu_1(x) \leq \mu_2(x)$, $\forall$ x$\in$ L and is denoted by $\mu_1 \subseteq \mu_2$.

Definition: 3.6
Let μ be any fuzzy L-filter of a lattice L, t$\in$[0,1] and $t \geq \mu(0)$. The fuzzy L-filter μ_t is called a level fuzzy L-filter of μ.

Proposition: 3.7
If μ is any fuzzy L-filter of a lattice L, then the following statement is true:
$\mu(1) \geq \mu(x) \geq \mu(0), \forall\, x\in L$.
Proof:
Let μ be any fuzzy L-filter of a lattice L.
(i) Let 0,1$\in$L. Then by definition
$\mu(1)=\mu(1\vee 0) \leq \max\{\mu(0), \mu(1)\}$
$\mu(0) =\mu(1\wedge 0) \leq \min\{\mu(0), \mu(1)\}$
$\Rightarrow \mu(1) \geq \mu(0)$ ----------------------(1)
$\mu(x) = \mu(0\vee x) \leq \max\{\mu(0), \mu(x)\}$

$\mu(0) = \mu(0\wedge x) \leq \min\{\mu(0), \mu(x)\}$
$\Rightarrow \mu(x) \geq \mu(0)$ ----------------------(2)
$\mu(1) = \mu(1\vee x) \leq \max\{\mu(1), \mu(x)\}$
$\mu(x) = \mu(1\wedge x) \leq \min\{\mu(1), \mu(x)\}$
$\Rightarrow \mu(1) \geq \mu(x)$ ----------------------(3)
Therefore (1), (2) & (3), we have
$\mu(1) \geq \mu(x) \geq \mu(0)$,for all $x \in L$.

Remark: 3.8
Every fuzzy L-filter is a fuzzy sublattice. But the converse need not be true. The following example prove this. Let L = { 0, a,b,c,1 }. Let μ:L→ [0,1] is a fuzzy set in L defined by $\mu(0) = 0.6$, $\mu(a) = 0.5$, $\mu(b) = 0.4$, $\mu(c) = 0.7$, $\mu(1) = 0.8$.

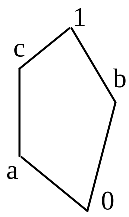

figure.5

Then μ is a fuzzy sublattice but not a fuzzy L-filter of L.

Lemma: 3.9
Let μ be a fuzzy L-filter of a lattice L and t, s $\in$ Imμ .Then $\mu_t = \mu_s$ iff t =s.
Proof:
If t = s, then clearly $\mu_t = \mu_s$.
Conversely, let $\mu_t = \mu_s$.
Since $t \in \text{Im}\mu$, $\exists\, x\in L$ such that $\mu(x) = t$.
$\Rightarrow x\in\mu_s$.
Hence $t = \mu(x) \leq s$ ---------------------------------(1)
Similarly we can prove $s \leq t$ ----------------------(2)
Therefore from (1) and (2), we get t = s.

Theorem: 3.10
Two level fuzzy L-filters μ_s and μ_t (with s<t) of a fuzzy L-filter μ of a lattice L are equal ifand only if there is no x in L such that $s \geq \mu(x) > t$.
Proof:
Let μ_s and μ_t be two level L-filters of a fuzzy L-filter of a lattice L, where s>t.
Assume that μ_s and μ_t are equal.
To prove:
There is no x in L such that $s \geq \mu(x) > t$.
On the contrary,
assume that $s \geq \mu(x) > t$, for some x in L.
$\Rightarrow \mu(x) \leq s$ and $\mu(x) > t$.
$\Rightarrow x\in\mu_s$ and $x \notin \mu_t$.
$\Rightarrow \mu_s \neq \mu_t$.
This is a contradiction to our assumption.
Hence there is no x in L such that $s \geq \mu(x) > t$.
Conversely, assume that there is no x in L such
that $s \geq \mu(x) > t$. ------------------------------------(1)
$\mu_s = \{ x\in L / \mu(x) \leq s \}$ and
$\mu_t = \{ x\in L / \mu(x) \leq t \}$ and s> t .
Then $\mu_s \subseteq \mu_t$ --(2)
It is enough to show that $\mu_t \subseteq \mu_s$.
Let $x\in\mu_s$ .Then $\mu(x) \leq s$.
$\Rightarrow \mu(x) \leq t$, by (1)
$\Rightarrow x\in \mu_s$.
$\Rightarrow \mu_t \subseteq \mu_s$ --(3)

From (2) and (3) , $\mu_s = \mu_t$.
Hence the two level L-filters are equal.

Theorem: 3.11
Let L be a lattice. If μ: L→ [0,1] is a fuzzy L-filter, then the level subset μ_t ,$t \in \text{Im}\mu$ is a level fuzzy L-filter of the lattice L.
PROOF:
Let x, y $\in \mu_t$.
Then $\mu(x) \leq t$; $\mu(y) \leq t$.
$\mu(x \vee y) \leq \max \{\mu(x), \mu(y)\} \leq t$
Therefore $x \vee y \in \mu_t$.
Let $x \in \mu_t$ & $y \in L$, $t \in \text{Im}\mu$.
Then $\mu(x) \leq t$.
$\mu(x \wedge y) \leq \min\{ \mu(x), \mu(y)\}$,since μ is a fuzzy filter.
$\leq t$
Hence $x \wedge y \in \mu_t$.
Therefore μ_t is a level fuzzy L-filter of L.

Theorem: 3.12
A fuzzy subset μ of a lattice L is a fuzzy L-filter of L iff the level subset μ_t, $t \in$ Im μ is a level fuzzy L-filter of L.
Proof:
Let μ be a fuzzy subset of a lattice L.
Assume that μ is a fuzzy L-filter of L.
Then μ_t, $t \in$ Im μ is a level fuzzy L-filter of L by theorem 3.11.
Conversely, assume that the level subsets μ_t,$t \in \text{Im}\mu$ is level fuzzy L-filter of L.
To prove: μ is a fuzzy L-filter of L.
It is enough to prove that

(i) $\mu(x \vee y) \leq \max \{ \mu(x), \mu(y) \}$

(ii) $\mu(x \wedge y) \leq \min \{ \mu(x), \mu(y) \}$

Now x, y $\in \mu_t$ => $\mu(x) \leq t$ and $\mu(y) \leq t$
Also $\min \{ \mu(x), \mu(y) \} \leq t$
$x \vee y \in \mu_t$ => $\mu(x \vee y) \leq t$
=>$\mu(x \vee y) \leq \max\{ \mu(x), \mu(y) \}$
Similarly,
$x \wedge y \in \mu_t$ => $\mu(x \wedge y) \leq t$
=> $\mu(x \wedge y) \leq \min\{ \mu(x), \mu(y) \}$
Hence μ is a fuzzy L-filter.

IV. Conclusion

In this paper, the definition with examples, properties and some theorems in fuzzy L-filters are given. Using these, various results can be developed under the topic fuzzy L-filter. These results may be used in various applications related to this field.

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