

# Influences of Fluid Property Variation on Mhd Visco-Elastic Fluid Flow With Space And Temperature Dependent Heat Generation/Absorption Over A Non-Isothermal Stretching Surface

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## Abstract

The problem of MHD visco-elastic fluid flow over a non-isothermal stretching surface with variable thermal conductivity is analysed. Two different types of heating process are considered namely, (i) A surface with prescribed wall temperature (PST) ii) A surface with prescribed wall heat flux (PHF). The basic boundary layer equations for momentum and heat transfer, which are non-linear partial differential equations, are converted in to ordinary differential equations by means of similarity transformation. The resulting momentum equations is solved exactly and the solution of energy equation by considering thermal conductivity as function of temperature is solved numerically by shooting technique with fourth order Runge kutta method. The effects of different physical parameters like visco-elasticity, Magnetic parameter, space and temperature dependent heat generation/absorption, Prandtl number etc on temperature profile are thoroughly discussed.

**Keywords:** visco-elasticity, Magnetic parameter, thermal conductivity, space and temperature dependent heat generation/absorption

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## I. INTRODUCTION:

Boundary layer flow and heat transfer on a continuously moving stretching surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheet, cooling of an infinite metallic plate in cooling bath, in condensation process of liquid film and polymer sheet etc are the practical applications of moving stretching surface. In these processes it is very important to control the drag and the heat flux at the stretching surface in order to obtain good quality of products.

For liquid metals, it has been found that thermal conductivity varies with temperature in an approximately linear manner in the range from  $0^{\circ}\text{F}$  to  $400^{\circ}\text{F}$  (see Kays[5]).

The pioneering work in this area was done by Sakiadis [1,2], in his work, Sakiadis described the boundary layer assumptions and governing equations of the problem and the boundary layer flow on a continuously stretching surface with certain speed was investigated. His work was further verified by Tsou et.al [3] experimentally, at the same time, the thermal boundary layer for this flow configuration with constant wall temperature was discussed. Erickson et.al.[4] extended the work of sakiadis to account for mass transfer at the moving surface.

In view of these works the boundary conditions on the surface were extended by other researchers [5]-[10].

There are various applications in which significant temperature differences between the body surface and the ambient fluid exist. it is usually assumed that the sheet is inextensible, but in some different situations like in the polymer industries in which it is necessary to deal with stretching plastic sheet as mentioned by Crane[11].Chen and Char [7] have examined the heat transfer behaviors in this flow, considering the effect of suction and injection where the boundary surface is maintained with variable temperature. Considering the effect of temperature difference between the surface and the ambient fluid, some work have been carried out by Vajravelu and Rollins [12], Vajravelu and Nayfeh [13] on the flow and heat transfer introducing temperature dependent heat source/sink.

All the above investigations restricted their analyses to the flow of Newtonian fluids. However, in recent past, the study of non-Newtonian fluid flows has shown immense interest because of its ever-increasing application of plastic films and artificial fibers in industry.

In view of this the above study of boundary layer flow problem has been further channelised to the non-Newtonian fluid flow. Considering the survey of literature, it is noticed that Rajgopal et.al [14] considered the study of visco-elastic second order fluid flow over a stretching sheet by solving the boundary layer equation numerically. Battacharya et .al [15] and Nataraja et.al [16] have presented the problem of heat transfer in a visco-elastic fluid over a stretching sheet. Abel et.al [17] studied the effect of magnetic field on visco-elastic fluid flow and heat transfer over non- isothermal stretching sheet with internal heat generation. The study of heat generation or absorption effect is important in view of several physical problems such fluids undergoing

exothermic or endothermic chemical reactions (Vajravelu and Hadjinicalaou [18]). Although, exact modeling of internal heat generation or absorption is quite difficult. Some simple mathematical models can express its average behavior for most of physical situations, as heat generation or absorption has been assumed to be constant with space dependent or temperature dependent heat source/sink. Crepeau and Clark Sean [19] have used a space dependent exponentially decaying heat generation or absorption in their work on flow and heat transfer from a vertical plate. Postelnieu et al[20,21] have presented their work on convective flows with internal heat generation/absorption for both viscous fluid and fluid saturated porous media, Vajravelu and Hadjinicalaou [18] have consider hydro- magnetic convective heat transfer from a stretching surface with uniform free stream in the presence of temperature dependent heat generation/absorption, Abo-Eldahab and EL-Gendy[22] extended the work of Vajravelu and Hadjinicalaou[18] to include the radiation and variable viscosity effect. Abo-Eldahab[23] analyzed the problem of free convection heat transfer due to simultaneous action of buoyancy, radiation and transverse magnetic field near an isothermal stretching sheet in the presence of temperature dependent heat generation or absorption. Abo-Eldahab and Md.Aziz[24] analyzed the mixed convection boundary layer flow over an inclined continuously stretching surface with internal heat generation /absorption in presence of magnetic field with suction /blowing effect. In above all studies the physical properties of the fluid are assumed to be constant, but Chiam [25] has done his work by taking the thermal conductivity as a function of temperature. Prasad et.al.[26] analyzed the effect of momentum and heat transfer of visco elastic fluid flow over a non isothermal stretching sheet assuming the thermal conductivity varying linearly with temperature. Most of the researchers like A.Raptis [27], Cperdikis [28], A.J.Chamkha [29] have done their work in the field of variable thermal conductivity. In view of all above literature, in present work we have analysed the effect of space and temperature dependent internal heat generation/absorption in presence of variable thermal conductivity. Most of the existing studies for this problem are based on the physical properties of the ambient fluid, however it is observed that the properties may change with temperature. To accurately predict the flow and heat transfer rates, it is necessary to take in to account this variation of thermal conductivity.

## II. MATHEMATICAL FORMULATION:

A steady 2D-incompressible MHD boundary layer flow of viscoelastic Walter's liquid B fluid above a heated horizontal stretching sheet is considered. The flow is assumed to be in the x-direction and y-axis is normal to the stretching sheet. Two equal and opposite forces are introduced along x-axis so that the sheet is stretched keeping the origin fixed. The flow field is then exposed under the influence of transverse magnetic field where the induced magnetic field is assumed to be negligible. The internal heat generation / absorption which are space and temperature dependent are present in the equation of energy. The fluid properties are assumed to be isotropic except for thermal conductivity, which is function of temperature. Under all the above assumptions in addition to boundary layer approximations, the momentum and thermal boundary layer equation takes the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

the boundary conditions governing the flow are:

$$\begin{aligned} u &= bx & v &= 0 & \text{at } y &= 0 \\ u &\rightarrow 0 & \frac{\partial u}{\partial y} &\rightarrow 0 & \text{as } y &\rightarrow \infty \end{aligned} \tag{3}$$

With  $b > 0$  here  $u$  and  $v$  are velocity components along  $x$  and  $y$  direction,  $\sigma$  is the electrical conductivity,  $B_0$  is the applied magnetic field,  $k_0$  is the visco-elastic parameter of the Walter's liquid B. The other quantities have their usual meanings.

## III. FLOW ANALYSIS:

The flow is created due to the stretching of the sheet and the free stream velocity being zero. We introduce a similarity transformation, which converts a nonlinear partial differential equation in to an ordinary differential equation.

$$\text{i.e. } u = bx f'(\eta), \quad v = -\sqrt{b\gamma} f(\eta) \quad \text{Where } \eta = \sqrt{\frac{b}{\gamma}} \cdot y \quad (4)$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non linear ordinary differential equation.

$$f'^2 - f f'' = f''' - k_1 \{2f' f''' - f f'''' - f''^2\} - M_n f' \quad (5)$$

Where

$$k_1 = \frac{k_o b}{\gamma}, \quad M_n = \frac{\sigma B_o^2}{b \rho} \quad (6)$$

Where  $k_1$  and  $M_n$  are non-dimensional visco-elastic parameter and Magnetic parameter respectively. Similarly the boundary condition takes the form

$$\begin{aligned} f = 0 \quad f' = 1 & \quad \text{at } \eta = 0 \\ f' \rightarrow 0 \quad f'' \rightarrow 0 & \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (7)$$

Where prime denotes differentiation w.r.t  $\eta$ .

The exact solution of equation (5) corresponding to the boundary conditions (7) is obtained as:

$$f = \frac{1}{\alpha} (1 - e^{-\alpha \eta}), \quad \text{Where } \alpha = \sqrt{\frac{1 + M_n}{1 - k_1}} \quad (8)$$

The solution for velocity field are obtained as:

$$u = bx e^{-\alpha \eta}, \quad v = -\sqrt{b\gamma} \frac{1 - e^{-\alpha \eta}}{\alpha} \quad (9)$$

#### IV. HEAT TRANSFER ANALYSIS

The energy equation in presence of variable thermal conductivity with space and temperature dependent internal heat generation /absorption for the two dimensional flow is:

$$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + q''' \quad (10)$$

Where  $\rho$  is the density,  $c_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity, which is assumed to be variable with temperature is given by

$$k = k_\infty (1 + \varepsilon \theta), \quad \text{Where } \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (\text{PST CASE})$$

$$k = k_\infty (1 + \varepsilon g), \quad \text{where } g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (\text{PHFCASE})$$

$$\text{and } \varepsilon = \frac{k_w - k_\infty}{k_\infty} \quad (11)$$

where  $\varepsilon$  is very small parameter. The internal heat generation /absorption term  $q'''$  is modeled as

$$q''' = \frac{k u_w(x)}{x\nu} \left[ A^* (T_w - T_\infty) e^{-\alpha \eta} + B^* (T - T_\infty) \right] \quad (12)$$

Where  $A^*$  and  $B^*$  is coefficient of space and temperature dependent internal heat generation /absorption respectively. In equation (12), the first term represents the dependence of the internal heat generation/absorption on the space coordinates while the latter term represents its dependence on the temperature. Note that when  $A^* > 0$  &  $B^* > 0$ , this case corresponds to internal heat generation while for both  $A^* < 0$  and  $B^* < 0$ , the case corresponds to internal heat absorption. The thermal boundary conditions depends upon the type of the heating process, here we considered two different type of heating process namely:

- (1) Prescribed surface temperature                      (2) Prescribed power law heat flux

**CASE (1): PRESCRIBED SURFACE TEMPERATURE:**

In this case we consider the boundary conditions

$$T = T_w = T_\infty + A x^l \quad \text{at} \quad y = 0$$

$$T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \tag{13}$$

$T_w$  is wall temperature,  $A$  is a constant and  $l$  is wall temperature parameter.

Using (11) and (12), equation (10) and (13) becomes,

$$(1 + \varepsilon \theta) \theta'' + Pr f \theta' - Pr l f' \theta + \varepsilon (\theta')^2 + (1 + \varepsilon \theta) [A^* e^{-\infty \eta} + B^* \theta] = 0 \tag{14}$$

Where prime denotes differentiation w.r.t  $\eta$  and  $Pr = \frac{\mu c_p}{k_\infty}$  with boundary conditions

$$\theta(\eta) = 1 \quad \text{at} \quad \eta = 0$$

$$\theta(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \tag{15}$$

**CASE2: PRESCRIBED POWER LAW HEAT FLUX:**

For this heating process, the boundary conditions are

$$-k \frac{\partial T}{\partial y} = B x^s \quad \text{at} \quad y = 0$$

$$T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \tag{16}$$

Where  $s$  is the wall heat flux parameter,  $B$  is constant.

Defining

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{where} \quad T_w - T_\infty = \frac{B x^s}{k} \sqrt{\frac{\gamma}{b}} \tag{17}$$

and  $k = k_\infty (1 + \varepsilon g)$ ,

With this change of variable equation (10) and corresponding boundary conditions (16) becomes:

$$(1 + \varepsilon g) g'' + \varepsilon (g')^2 + pr f g' - pr s f' g + (1 + \varepsilon g) [A^* e^{-\infty \eta} + B^* g] = 0 \tag{18}$$

And the boundary conditions are

$$g'(\eta) = -1 \quad \text{at} \quad \eta = 0$$

$$g(\eta) \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty \tag{19}$$

Our interest lies in investigation of the flow behavior and heat transfer characteristics by analyzing the non-dimensional Nusselt number (Nu). These non-dimensional parameters are defined as:

$$Nu = \frac{-h}{T_w - T_\infty} T_y = -\theta'(0) \quad \text{PST case}$$

$$Nu = \frac{-h}{T_w - T_\infty} T_y = \frac{1}{g(0)} \quad \text{PHF case}$$

In the next section we solve equations (14) and (18) subject to the boundary conditions (15) and (19) respectively.

**Numerical solution:**

Since equation (14) and (18) are non-linear ordinary differential equations and we follow most efficient numerical shooting technique with fourth order Runge kutta algorithm to solve them.

## V. RESULTS AND DISCUSSION:

Numerical computation of the model has been carried out for different physical parameters like Visco-elastic parameter ( $k_I$ ), Hartmann number ( $M_n$ ), space dependent term ( $A^*$ ), temperature-dependent parameter  $B^*$  and Prandtl number (pr). For detailed discussion of the results, the numerical values are plotted graphically in the fig.1-10. Results for prescribed temperature (PST) are drawn in Fig.1-5 and for prescribed power law heat flux (PHF) are drawn in Fig.6-10.

Table-1: shows the values of wall temperature gradient  $-\theta'(0)$  in PST case and wall temperature  $g(0)$  in PHF case for different physical parameters, from this table values, we observe that as the Visco-elastic parameter ( $k_I$ ), Hartmann number ( $M_n$ ), space dependent parameter  $A^*$  and temperature dependent parameter  $B^*$  increases the wall temperature gradient  $-\theta'(0)$  decreases in PST case and also wall temperature  $g(0)$  is decreases in PHF case.

Fig1(a) and Fig1(b) give the graphical representation of the temperature  $\theta(\eta)$  vs.  $\eta$  in presence ( $\epsilon = 0.1$ ) and absence ( $\epsilon = 0$ ) of thermal conductivity respectively, for different values of Visco-elastic parameter ( $k_I$ ), it is noticed that the temperature distribution is unchanged at the wall (unity) with change of physical parameter. Where it tends to zero in the free stream, and further we observe that the temperature profile increases when  $k_I$  increases. This is due to fact that the thickening of thermal boundary layer occurs due to the increase of Visco-elastic normal stress.

Fig2(a) and Fig2(b) illustrates the effect of Hartmann number ( $M_n$ ) on temperature profile in presence ( $\epsilon = 0.1$ ) and absence ( $\epsilon = 0$ ) of thermal conductivity. The presence of magnetic field in an electrically conducting fluid gives rise to a resistive type of force called Lorentz force. This force has tendency to slow down the motion of the fluid in the boundary layer and to increase the temperature. Also the effect on the flow and thermal fields become more, so as the strength of the magnetic field increases temperature profile increases.

The influences of the presence of space dependent internal heat generation ( $A^* > 0$ ) or absorption ( $A^* < 0$ ) in the boundary layer on temperature field both in presence ( $\epsilon = 0.1$ ) and absence ( $\epsilon = 0$ ) of thermal conductivity is presented in Fig3(a) and Fig3(b). It is clear from these figures that there is increase in temperature distribution of the fluid, when  $A^*$  increases. This is expected since the presence of heat source ( $A^* > 0$ ) in the boundary layer generates energy, which causes the temperature of the fluid increases. However as the heat source effect get large ( $A^* = 2$ ) a distinctive peak in the temperature profile occurs in the fluid adjacent to the wall. This means the temperature of the fluid near the sheet is higher than the sheet temperature and consequently, heat is expected to transfer to the wall. The influence of the temperature dependent internal heat generation ( $B^* > 0$ ) or absorption ( $B^* < 0$ ) in the boundary layer on the flow and temperature field is the same as that of space dependent internal heat generation or absorption. Namely, for ( $B^* > 0$ ) (heat source), the temperature of the fluid increase while they decreases for  $B^* < 0$  (heat sink) both in presence ( $\epsilon = 0.1$ ) and absence ( $\epsilon = 0$ ) of thermal conductivity. These behaviors are depicted in figures 4(a) and 4(b).

In Fig5(a) and Fig5(b) represent the temperature profile  $\theta(\eta)$  vs.  $\eta$  for different values of Pr. We infer from these figures that the temperature profile decreases with increase in Prandtl number (pr). This is because of the fact that the thermal boundary thickness decreases with increase in Prandtl number (pr). Comparison of Fig [1-5](a) with [1-5](b) reveals the fact that the magnitudes of temperature profile  $\theta(\eta)$  is more compare to in absence of thermal conductivity.

The graphs for the situation when the boundary has been prescribed with heat flux (PHF) are shown in Figure 6-10. It is noticed from these figures that the wall temperature is not unity, it is changed at the wall with the change of physical parameters like Visco-elastic parameter ( $k_I$ ), Hartmann number ( $M_n$ ), space dependent term ( $A^*$ ), temperature –dependent parameter  $B^*$  and Prandtl number (pr), and these parameters have same qualitative effects as those we found in PST case but quantitatively the magnitude of wall temperature is more in PHF case. Comparison of fig [6-10](a) with [6-10](b) reveals the fact that the magnitude of temperature profile  $g(\eta)$  is more compare to in absence of thermal conductivity.

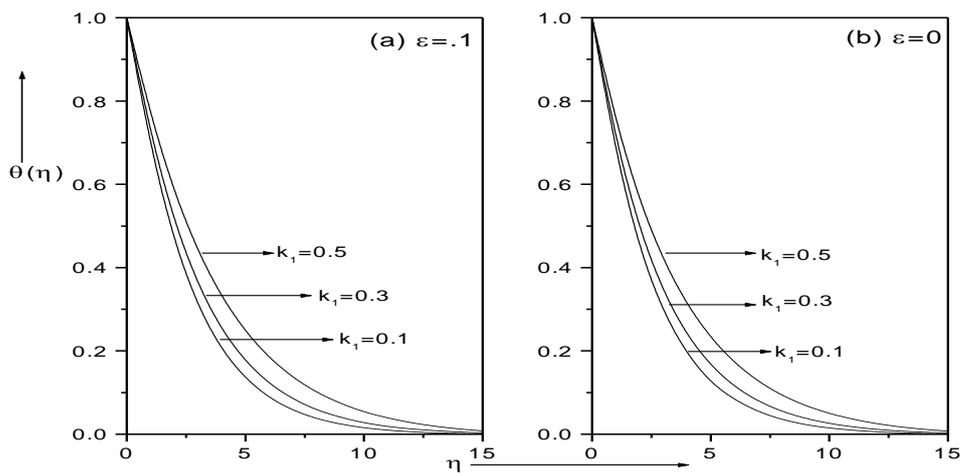


Fig1:variation of temperature profile  $\theta(\eta)$  Vs  $\eta$  for different values of visco-elastic parameter  $k_1$ ,when  $Mn=1.1, A^*=B^*=.01$  and  $Pr=1$ (PST Case)

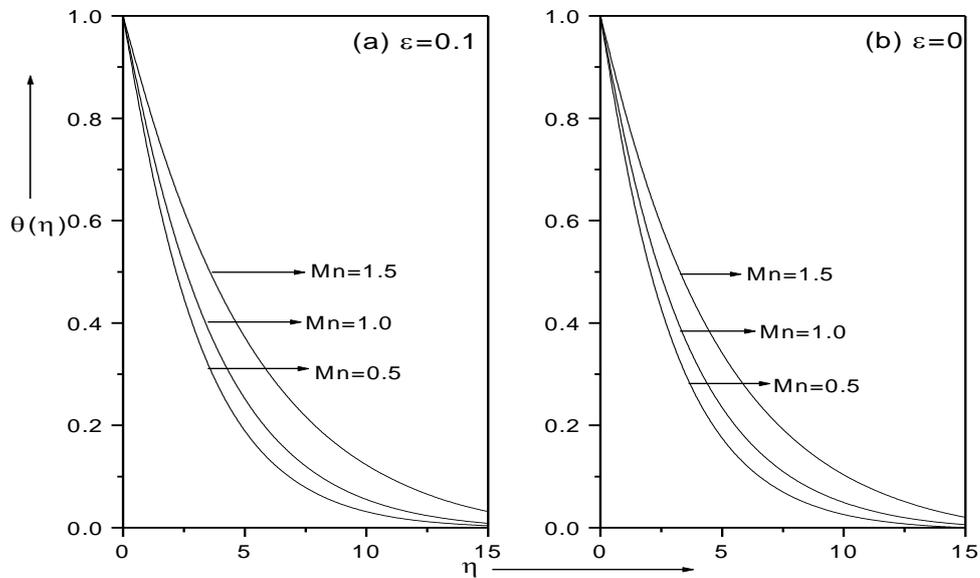


Fig2:variation of temperature profile  $\theta(\eta)$  Vs  $\eta$  for different values of  $Mn$  when  $k_1=1.1, A^*=B^*=.01$  and  $Pr=1$ (PSTcase)

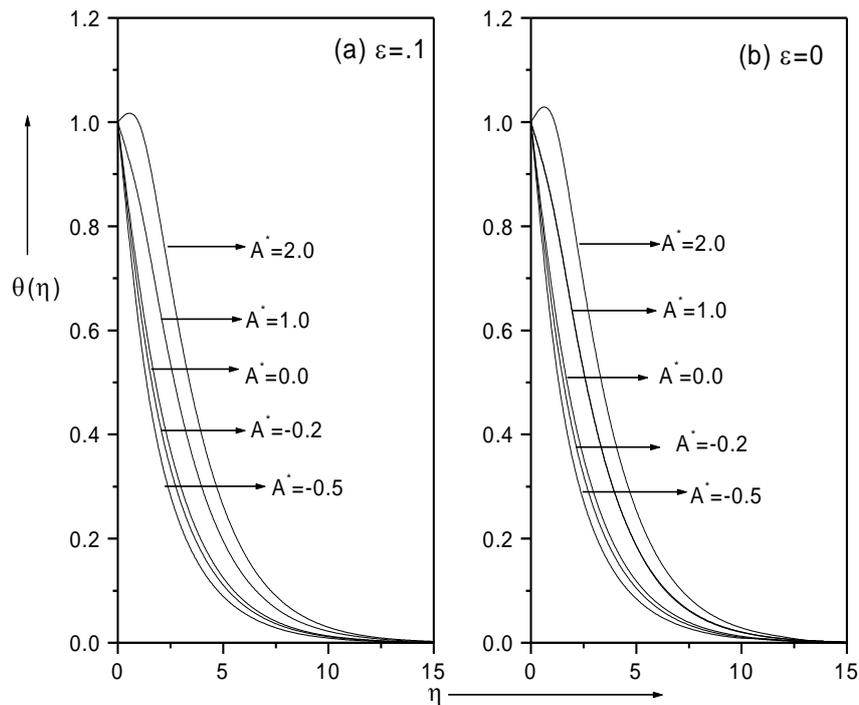


Fig3:variation of temperature profile  $\theta(\eta)$  Vs  $\eta$  for different values of  $A^*$  when  $k_1=1, Mn=1, B^*=0.01$  and  $Pr=1$  (PST case)

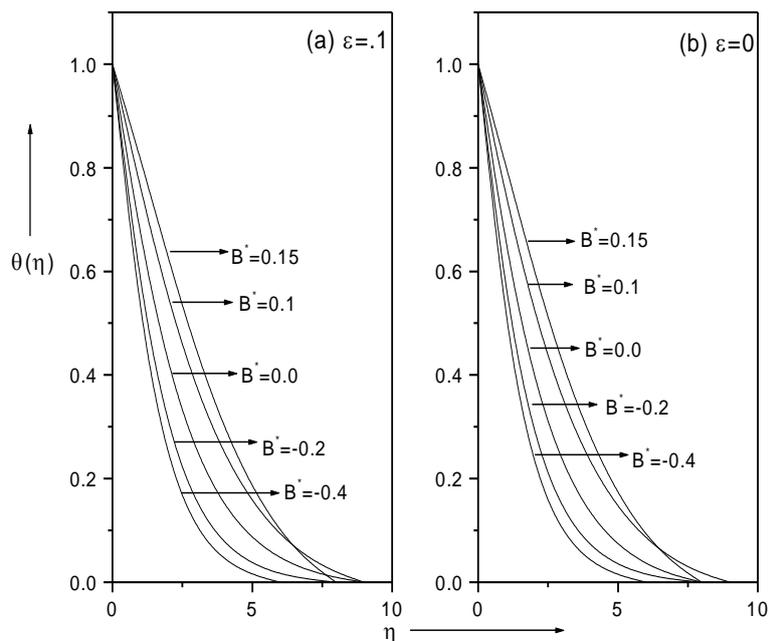


Fig4:variation of temperature profile  $\theta(\eta)$  Vs  $\eta$  for different values of  $B^*$  when  $k_1=Mn=1, A^*=0.01$  and  $Pr=1$  (PST case)

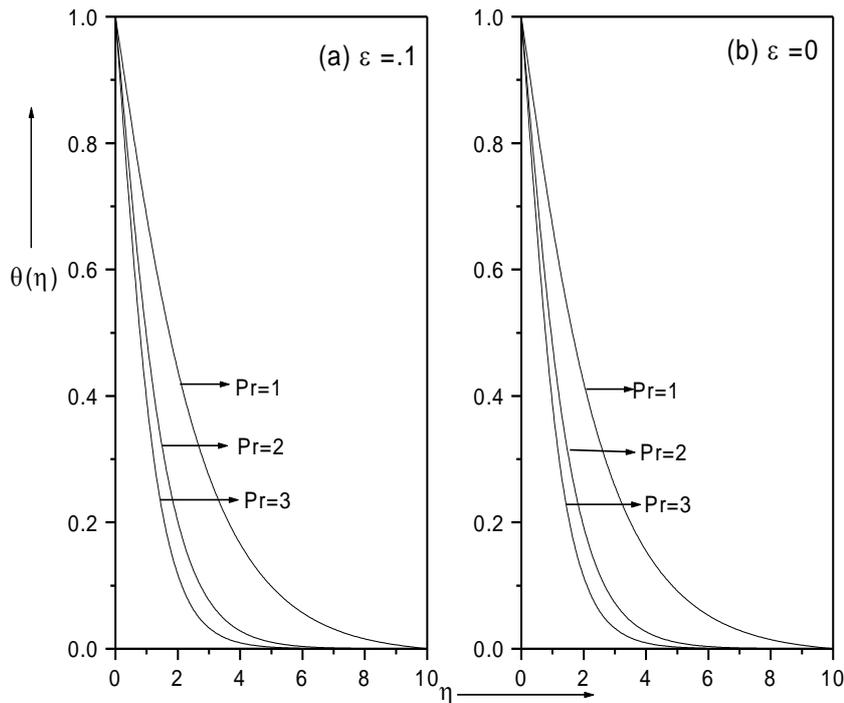


Fig5: variation of  $\theta(\eta)$  vs.  $\eta$  for different values of  $Pr$ , when  $k_1 = Mn = l = 1$  and  $A^* = B^* = .01$  (PST case)

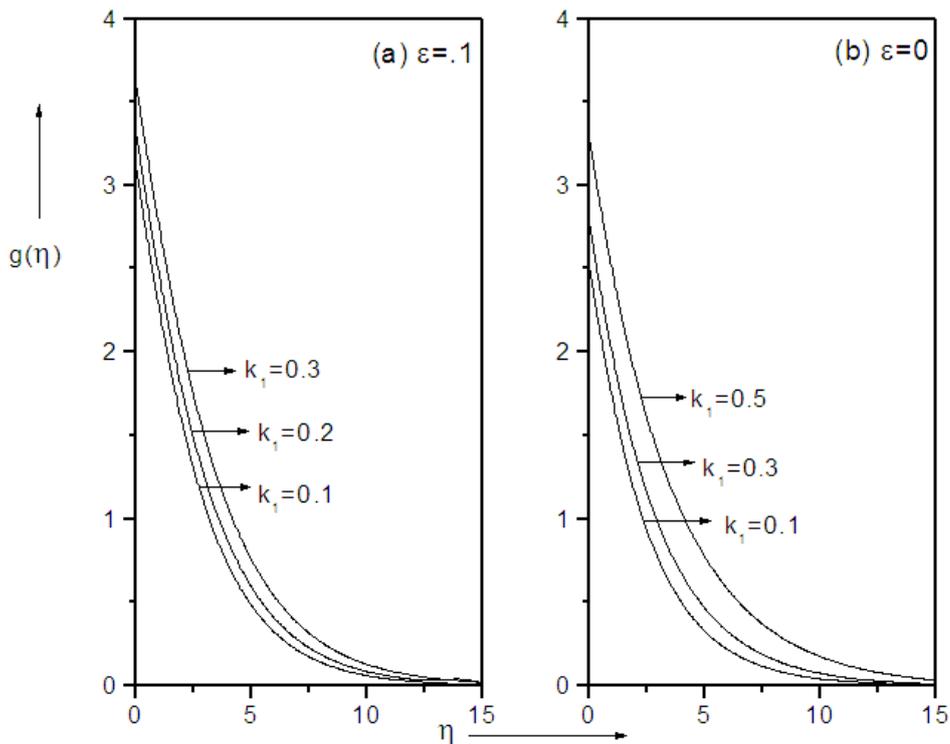


Fig6: variation of temperature profile  $\theta(\eta)$  Vs  $\eta$  for different values of visco-elastic parameter  $k_1$ , when  $Mn = s = 1, A^* = B^* = .01$  and  $Pr = 1$  (PHF Case)

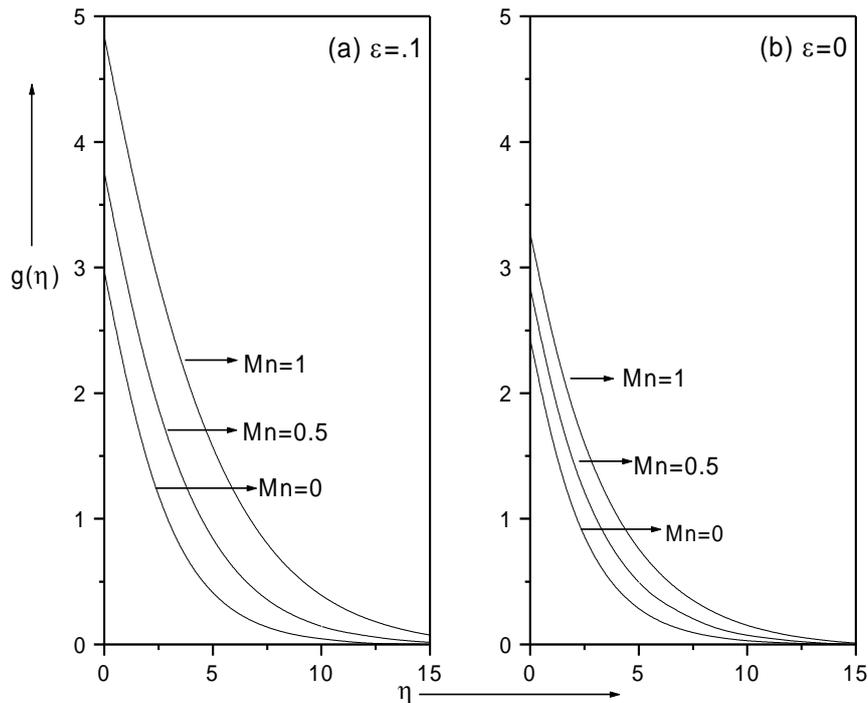


Fig7:variation of temperature profile  $g(\eta)$  Vs  $\eta$  for different values of  $Mn$  when  $k_1=s=0.1, A^*=B^*=0.01$  and  $Pr=1$ (PHFcase)

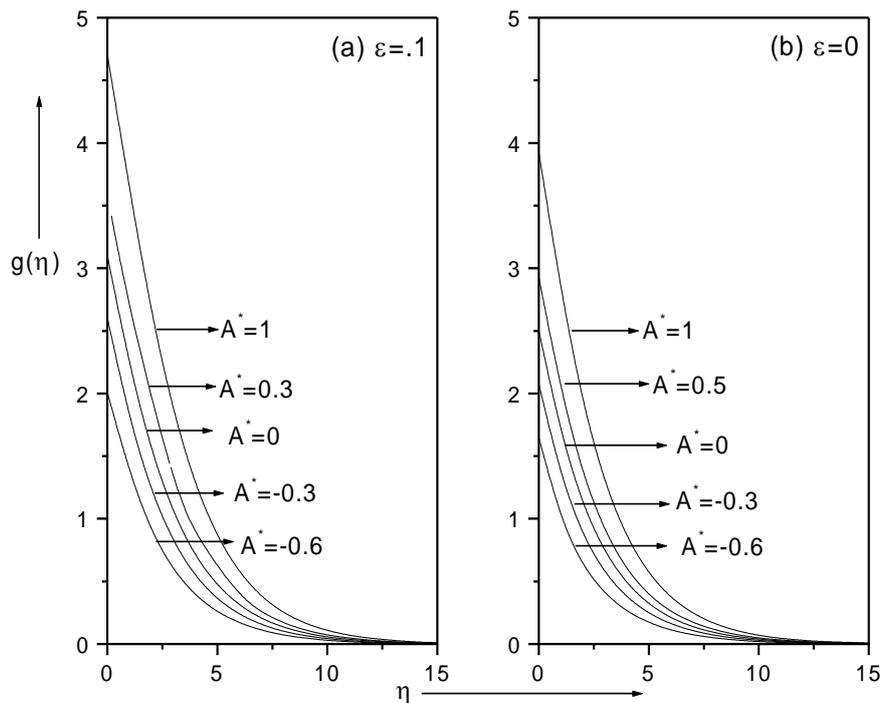


Fig8:variation of temperature profile  $g(\eta)$  Vs  $\eta$  for different values of  $A^*$  when  $k_1=Mn=s=1, B^*=0.01$  and  $Pr=1$ (PHFcase)

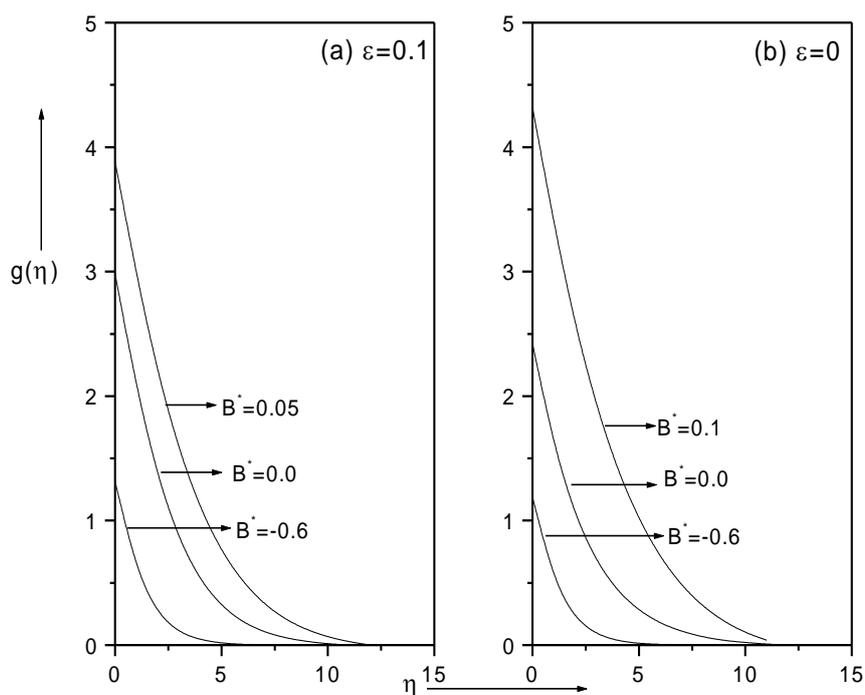


Fig9:variation of temperature profile  $g(\eta)$  Vs  $\eta$  for different values of  $B^*$  when  $k_1=Mn=s=1, A^*=0.01$  and  $Pr=1$  (PHFcase)

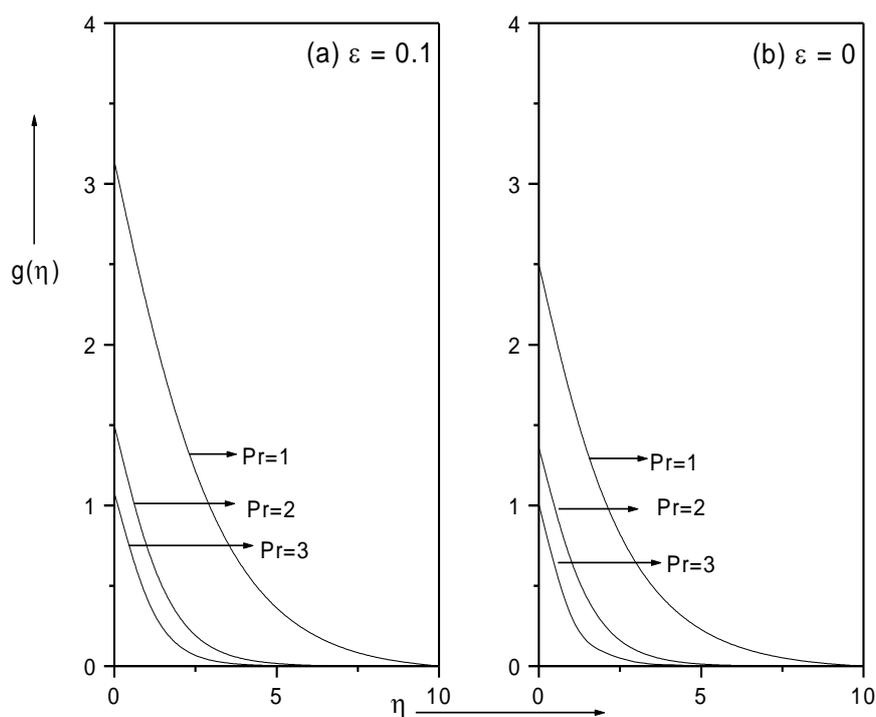


Fig10:variation of  $g(\eta)$  vs.  $\eta$  for different values of  $pr$ , when  $k_1=Mn=s=0.1$  and  $A^*=B^*=0.01$ (PHFcase)

Table-1: Values of wall temperature gradient  $-\theta'(0)$  in PST case and wall temperature  $g(0)$  in PHF case.

pr	$M_n$	$k_I$	$A^*$	$B^*$	$-\theta'(0)$			$g(0)$		
					$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.2$	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.2$
1	0.1	0	0.01	0.01	0.411946	0.381345	0.354656	2.406980	2.980563	4.024848
		0.2			0.382480	0.352856	0.327023	2.591538	3.317935	-- ----
		0.4			0.342727	0.314587	0.290074	2.8900632	3.933906	-----
1	0	0.1	0.01	0.01	0.410654	0.380094	0.353441	2.414523	2.993789	4.055947
	1				0.315707	0.288719	0.265250	3.136796	4.521318	-----
	2				0.260058	0.235933	0.215086	3.804576	-----	---- --
1	0.1	0.1	-2	0.01	0.513389	0.474074	0.439873	2.189284	2.745340	3.716489
			-1		0.456679	0.422940	0.393493	2.332219	2.909158	3.920328
			0		0.404002	0.371690	0.346945	2.475227	3.072034	4.123374
			.1		0.343259	0.320325	0.300134	2.618089	3.234041	4.325830
			.2		0.286549	0.268848	0.253158	2.761024	3.395342	4.527809
1	0.1	0.1	0.01	-2	0.606259	0.564619	0.528317	1.673528	1.917316	-----
			-1	0.519770	0.484375	0.453477	1.938361	2.269963	2.74022	
			0	0.399969	0.379615	0.354693	2.416684	2.971897	3.911681	
			.1	0.197211	0.175578	0.156017	3.848595	-----	-----	

Tabulated values of temperature profile for different values of visco-elastic parameter  $kl$  in presence and absence of thermal conductivity parameter  $\varepsilon$

$\eta$	$\theta(\eta)$ for PST case					
	$\varepsilon = 0.0$			$\varepsilon = 0.1$		
	$k_I=0.1$	$k_I=0.2$	$k_I=0.3$	$k_I=0.1$	$k_I=0.2$	$k_I=0.3$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.672527	0.686732	0.703419	0.688392	0.702694	0.719453
2	0.443600	0.464268	0.488905	0.460554	0.482073	0.507644
3	0.291703	0.313012	0.338969	0.305337	0.327965	0.355460
4	0.191526	0.210655	0.234517	0.201476	0.222002	0.247575
5	0.125491	0.141425	0.161783	0.132426	0.149611	0.171571
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.						
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$\eta$	$g(\eta)$ for PHF case					
	$\varepsilon = 0.0$			$\varepsilon = 0.1$		
	$k_I=0.1$	$k_I=0.2$	$k_I=0.3$	$k_I=0.1$	$k_I=0.2$	$k_I=0.3$

0	2.502122	2.618095	2.767395	3.087741	3.294491	3.577813
1	1.675533	1.792036	1.942137	2.210980	2.417975	2.701847
2	1.099749	1.207507	1.347333	1.519441	1.713161	1.981109
3	0.718298	0.810627	0.932086	1.020924	1.188937	1.425317
4	0.466949	0.542242	0.642957	0.675352	0.812904	1.010575
5	0.301389	0.36081	0.441692	0.440632	0.548753	0.707595
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Tabulated values of temperature profile for different values of Magnetic parameter  $M_n$  in presence and absence of thermal conductivity parameter  $\varepsilon$

$\eta$	$\theta(\eta)$ for PST case					
	$\varepsilon = 0.0$			$\varepsilon = 0.1$		
	$M_n = 0.5$	$M_n = 1$	$M_n = 1.5$	$M_n$	$M_n$	$M_n$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.710944	0.748846	0.778844	0.726994	0.764786	0.794431
2	0.500136	0.557773	0.604585	0.519270	0.578583	0.626258
3	0.350992	0.414402	0.467847	0.368166	0.434828	0.490465
4	0.245763	0.306893	0.360535	0.259609	0.324780	0.381513
5	0.171548	0.226296	0.276344	0.182100	0.241005	0.294548
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$\eta$	$g(\eta)$ for PHF case					
	$\varepsilon = 0.0$			$\varepsilon = 0.1$		
	$M_n = 0.1$	$M_n = 0.2$	$M_n = 0.3$	$M_n = 0.1$	$M_n = 0.2$	$M_n = 0.3$
0	2.432414	2.843914	3.263249	2.971537	3.733403	4.698753
1	1.605906	2.019188	2.441266	2.095101	2.857912	3.826220
2	1.037413	1.419832	1.817901	1.413788	2.129818	3.059235
3	0.668188	0.996248	1.350507	0.934158	1.558991	2.407080
4	0.429897	0.697500	1.000092	0.610139	1.125249	1.865721
5	0.276119	0.486834	0.737409	0.395476	0.802601	1.425453
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Tabulated values of temperature profile for different values of space dependent heat generation/absorption parameter  $A^*$  in presence and absence of thermal conductivity parameter  $\varepsilon$

$\eta$	$\theta(\eta)$ for PST case							
	$\varepsilon = 0.0$				$\varepsilon = 0.1$			
	$A^* = -0.2$	$A^* = 0.0$	$A^* = 1.0$	$A^* = 2.0$	$A^* = -0.2$	$A^* = 0.0$	$A^* = 1.0$	$A^* = 2.0$
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0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.632759	0.670634	0.860005	1.029152	0.649055	0.686532	0.865289	1.049376
2	0.406202	0.441819	0.619905	0.800068	0.422297	0.458741	0.634846	0.797994
3	0.263733	0.290372	0.423564	0.567607	0.276272	0.303957	0.439004	0.556756
4	0.172085	0.190601	0.283180	0.387056	0.181116	0.200508	0.295667	0.375760
5	0.112405	0.124871	0.187202	0.258676	0.118669	0.131771	0.196351	0.249534
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<i>g(η) for PHF case</i>								
$\eta$	$\varepsilon = 0.0$				$\varepsilon = 0.1$			
	$A^* = -0.3$	$A^* = 0.0$	$A^* = 0.3$	$A^* = -0.3$	$A^* = 0.0$	$A^* = 0.3$		
	0	2.074770	2.500103	2.925483	2.602861	3.095587	3.581274	
1	1.334576	1.676621	2.018713	1.796238	2.221327	2.644219		
2	0.863208	1.104538	1.345918	1.211770	1.533118	1.857733		
3	0.562446	0.725888	0.889381	0.809243	1.037163	1.270363		
4	0.367621	0.476441	0.585311	0.536898	0.693198	0.854549		
5	0.240318	0.312105	0.383942	0.354258	0.459341	0.568453		
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Tabulated values of temperature profile for different values of temperature dependent heat generation/absorption parameter  $B^*$  in presence and absence of thermal conductivity parameter  $\varepsilon$

$\eta$	<i>θ(η) for PST case</i>							
	$\varepsilon = 0.0$				$\varepsilon = 0.1$			
	$B^* = -0.2$	$B^* = 0.0$	$B^* = 1.0$	$B^* = 0.15$	$B^* = -0.2$	$B^* = 0.0$	$B^* = 1.0$	$B^* = 0.15$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.540265	0.644455	0.750953	0.862993	0.558040	0.662174	0.768457	0.880245
2	0.276985	0.395140	0.535664	0.697541	0.291528	0.414269	0.559705	0.726587
3	0.139526	0.237317	0.370317	0.535494	0.148286	0.252036	0.393088	0.568029
4	0.069770	0.140617	0.249058	0.392431	0.074514	0.150518	0.267329	0.422313
5	0.034681	0.082008	0.162231	0.273886	0.037129	0.088194	0.175445	0.297932
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$\eta$	<i>g(η) for PHF case</i>					
	$\varepsilon = 0.0$			$\varepsilon = 0.1$		
	$B^* = -0.6$	$B^* = 0.0$	$B^* = 0.05$	$B^* = -0.6$	$B^* = 0.2$	$B^* = 0.3$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.632759	0.670634	0.860005	1.029152	0.649055	0.686532
2	0.406202	0.441819	0.619905	0.800068	0.422297	0.458741
3	0.263733	0.290372	0.423564	0.567607	0.276272	0.303957
4	0.172085	0.190601	0.283180	0.387056	0.181116	0.200508
5	0.112405	0.124871	0.187202	0.258676	0.118669	0.131771
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		Pr	Emad et al [24]	Present study		
		0.7	-0.454449	-0.303640		
		1.0	-0.58201	-0.402053		
		10.0	-0.11077	0.00000		
0	1.188868	2.428794	2.986615	1.316316	2.973464	3.979791
1	0.515448	1.609590	2.125931	0.599783	2.104110	3.068545
2	0.212219	1.047295	1.482423	0.252214	1.430328	2.280480
3	0.085988	0.679643	1.027514	0.103031	0.953038	1.654144
4	0.034639	0.440340	0.708819	0.041630	0.627470	1.177774
5	0.013895	0.284562	0.486086	0.016711	0.409540	0.825424
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Comparison of the values of  $-\theta'(0)$  for various values of Pr with  $M=A^*=B^*=\epsilon=1=0$

### VI. SUMMARY AND CONCLUSIONS:

The governing equations for steady laminar flow of an incompressible and electrically conducting visco-elastic fluid over non-isothermal stretching surface in presence of space and temperature dependent internal heat generation/absorption with variable thermal conductivity was formulated. The resulting partial differential equations are transformed into ordinary differential equations by using similarity transformations. Numerical results for temperature profile is presented graphically for various parametric conditions. In addition, numerical data for  $-\theta'(0)$  in PST case and  $g(0)$  in PHF case are tabulated for various values of Visco-elastic parameter ( $k_f$ ), Magnetic field parameter ( $Mn$ ), space dependent term  $A^*$ , temperature-dependent term  $B^*$  both in absence and presence of variable thermal conductivity.

The important findings of our study are as follows:

1. The space and temperature dependent parameters strongly affect the temperature at the wall. It is found that temperature gradient  $-\theta'(0)$  decrease when both space and temperature dependent parameters  $A^*$  and  $B^*$  increases, It is also true in absence of thermal conductivity.
2. It is noted that the negative heat transfer rates are obtained for higher values of  $A^*$  ( $A^*=1.5$ ) and  $B^*$  ( $B^*=0.2$ ), Negative values indicates that heat is transferred from the fluid to the moving surface. In PHF case opposite effect is observed i.e.  $g(0)$  increase when both space and temperature dependent parameters  $A^*$  and  $B^*$  increases.

### REFERENCES:

- [1]. **B.C.Sakiadis:** Boundary layer flow behavior on continuous solid surfaces; I Boundary layer equations for two dimensional & axis-symmetric flow, Am inst.chem.engg.j.7, (1966) 26-28
- [2]. **B.C.Sakiadis:** Boundary layer flow behavior on continuous solid surfaces; I Boundary layer on a continuous flat surface, Am inst.chem.engg.j.7, (1961) 221-225
- [3]. **F.K.Tsou, F.K.Sparrow, R.J.Goldstein:** Flow & Heat transfer in the boundary layer on a continuous moving surface, Int.J. Heat Mass transfer 10 (1967) 219-23
- [4]. **L.E.Erickson, L.T.Fan, V.G.Fox:** Heat & Mass transfer on a moving continuous flat plate with suction or injection, Ind.engg.chem.fundamentals. 5,(1966)19-25
- [5]. **B.K.Dutta, P.Roy, A.S.Gupta:** Temperature field in flow over a stretching sheet with uniform heat flux, Int.comm.Heat Mass Transfer 12(1985)89-94
- [6]. **L.J.Grubka, K.M.Bobba:** Heat transfer characteristic of a continuous stretching surface with variable temperature, ASME J.Heat Transfer, 107(1985) 298-250
- [7]. **C.K.Chen, M.I.Char:** Heat Transfer of a continuous stretching surface with suction & blowing, J.Math.Anal.Appl, 135(1988) 568-580
- [8]. **M.E.Ali:** on thermal boundary layer on a power law stretching surface with suction or injection. Int.J.Heat Fluid flow, 16(1995) 280
- [9]. **E.M.A.Elbashbeshy:** Heat transfer over stretching surface with variable surface heat flux, J.Phys.D: Appl.phys, 31(1998) 1951-54
- [10]. **V.M.Soundalgekar, T.V.Ramanmurthy:** Heat transfer in the flow past a continuous moving plate with variable temperature. Wärme und stoffübertragung, 14(1980) 91-93
- [11]. **L.J.Crane:** Flow past a stretching plate,Z. Angew.Math.phys.21(1970)645-647
- [12]. **K.Vajravelu, D.Rollins:** Heat transfer in electrically conducting fluid over stretching surface, Int.J.Non-linear Mech.27(2)(1992)265-277
- [13]. **K.Vajravelu, J.Nayfeh:** Convective heat transfer at stretching sheet, Acta .Mech . 96 (1993) 47-54

- [14]. **K.R.Rajgopal, T.Y.Na, A.S.Gupta:** Flow of visco-elastic fluid over a stretching sheet, *Rheol. Acta* 23(1984)213-215
- [15]. **S.Bhattacharya, A. Pal, A.S.Gupta:** Heat transfer in the flow of a visco-elastic fluid over a stretching surface, *int.j. Heat & mass transfer.*34(1998)41-45
- [16]. **H.R.Natraja, M.S.Sharma, B.Nageswara rao:** Non-similar solution for flow & heat transfer in a visco-elastic fluid over a stretching ,*Int. j. Non- linear mechanics* 33(1998)357-361
- [17]. **M.S.Abel, K.V.Prasad,S.K.Khan:** Momentum & heat transfer in visco-elastic fluid flow in a porous medium over a non isothermal stretching sheet, *int.j.Numer.methods for Heat & fluid flow*,10(2001)784-780
- [18]. **Vajravelu, K Hadjinalaou:** A.*Int. J.engg.sci.* 35,1237(1997)
- [19]. **J.C.Crepeau, R.,Clarksean:** Similarity solution of natural convection with internal heat generation. *Heat transfer* 119(1997)183-185
- [20]. **A.Postelnieu, I. Pop:** Similarity solutions of free convection boundary layer over vertical & horizontal surfaces in porous media with internal heat generation. *Int.comm. Heat Mass transfer.* 26(8)(1999)1183-1191
- [21]. **A.Postelnieu, T.Grosan, I. Pop:** Free convection boundary layer over a vertical permeable flat plate in a porous medium with internal heat generation.*Int.comm..Heat Mass transfer.*27(5)(2000)729-738.
- [22]. **Abo-Eldahab, Md.Aziz:** Blowing /suction effect on hydro magnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation /absorption. *Int. J. Therm.sci*,43(2004)709-719.
- [23]. **Abo-Eldahab & El.Gendy,M.S:** Radiation effect on M.H.D free convective flow of a gas past a semi infinite vertical plate with variable thermo physical properties for high temperature differences. *Canada.J.phys.*80 (2002) 1609-1619.
- [24]. **Abo-Eldahab, E.M.Can:** Heat transfer due to the simultaneous action of buoyancy, radiation & magnetic field near an isothermal stretching sheet in the presence of temperature dependent heat generation/absorption.*J.Phys.*79,(2001)1
- [25]. **Chiam,T.C:** Heat transfer with variable conductivity in a stagnation point flow towards a stretching sheet.*Acta ,mecha .*129(1998)63-72
- [26]. **Raptis.A:** *int .Commun .Heat & mass transfer .*26 , 889-895 (1999)
- [27]. **Cperdikis. Raptis.A:** *Zamm* 78 , 277-279 (1998)
- [28]. **Chamka,A.J:** Hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium.*Int.J.Engg.sci.*38(2000)1699-1712