IOSR Journal of Mathematics (IOSRJM)
Volume 1, Issue 1 (May-June 2012), PP 01-10
www.iosrjournals.org

Triple integral relations involving certain special functions

V.B.L. Chaurasia* and R.C. Meghwal**
**Department of Mathematics, University of Rajasthan, Jaipur-302004, India*
***Department of Mathematics, Government Post Graduate College, Neemuch-458441, India*

Abstract. *The main aim of this paper is to obtain new triple integral relations that involve* $\overline{H}$ *-function and the multivariable H-function. The main results of our paper are unified in nature and capable of yielding several cases of interests (New and known).*

Key Words and Phrases: $\overline{H}$ *-function, H-function, Multivariable H-function*

I. Introduction

The $\overline{H}$ -function [2] occurring in this paper will be defined and represented in the following manner

(1.1) $$\overline{H}_{p,q}^{m,n}[z] = \overline{H}_{p,q}^{m,n}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q} \end{matrix} \right. \right]$$

$$= \frac{1}{(2\pi\omega)} \int_{-\omega\infty}^{+\omega\infty} \theta(s) z^s \, ds, \qquad \text{where}$$

(1.2) $$\theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma(1-a_j+\alpha_j s)^{A_j}}{\prod_{j=m+1}^{q} \Gamma(1-b_j+\beta_j s)^{B_j} \prod_{j=n+1}^{p} \Gamma(a_j - \alpha_j s)}$$

For details about this function we may refer to [2].

The multivariable H-function due to Srivastava and Panda [5] is defined and represented as follows

2000 Mathematics Subject Classification: 30C45, 33C60, 33C65

(1.3) $$H[z_1,\ldots,z_r] = H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix} [(a):\theta',\ldots,\theta^{(r)}]:[b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}]; \\ [(c):\psi',\ldots,\psi^{(r)}]:[d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}]; \end{matrix} \; z_1,\ldots,z_r \right]$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \ldots \int_{L_r} R_1(s_1)\ldots R_r(s_r).T(s_1,\ldots,s_r) z_1^{s_1}\ldots z_r^{s_r} \, ds_1\ldots ds_r, \; \omega=\sqrt{-1}.$$

where

(1.4) $$R_i s_i = \frac{\prod_{j=1}^{U^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)} s_i) \prod_{j=1}^{V^{(i)}} \Gamma(1-b_j^{(i)} + \phi_j^{(i)} s_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1-d_j^{(i)} + \delta_j^{(i)} s_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \phi_j^{(i)} s_i)}, \forall i = 1,\ldots, r$$

(1.5) $$T(s_1,\ldots,s_r)=\frac{\prod_{j=1}^{\lambda}\Gamma\left(1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}s_i\right)}{\prod_{j=\lambda+1}^{A}\Gamma\left(a_j-\sum_{i=1}^{r}\theta_j^{(i)}s_i\right)\prod_{j=1}^{C}\Gamma\left(1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}s_i\right)},$$

For more details about this function, we may refer to [7].
For the sake of brevity,

(1.6) $$\Delta=\left(\frac{\pi}{2^{2+\sigma}}\right)\overline{H}_{p,q}^{m,n}\left[(\alpha')(2^{-k})\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right].$$

(1.7) $$\nabla=\frac{\ell_{d-1}\Gamma(\mu+1)\Gamma\left(\frac{1}{2}+\frac{\Im}{2}\right)}{(-1)^{\mu}\,2^{2+\mu}\,(\pi)^{\frac{1}{2}}\,\Gamma(\gamma)\Gamma\left(\gamma-\frac{\Im}{2}\right)}\left(\frac{\pi}{2^{2+\sigma}}\right)$$

$$\times\overline{H}_{3,3}^{1,3}\left[(-\alpha')(2^{-k})\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(1-\gamma,1;1),\left(1-\gamma+\frac{\Im}{2},1;1\right),(1-\eta,1;1+\mu)\\(0,1),\left(-\frac{\Im}{2},1;1\right),(-\eta,1;1),(-\eta,1;1+\mu),(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$

(1.8) $$\xi=\left(\frac{\pi}{2^{2+\sigma}}\right)H_{p,q}^{m,n}\left[(\alpha')(2^{-k})\left|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right.\right].$$

II. The Main Triple Integral Relations

Our main results of the present paper are the triple integral relations contained in the following Theorems.

THEOREM 1

(2.1) $$\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{(tu)^{\sigma}}{(t^2+u^2+z^2)^{\sigma}}\cos\left(2\delta\tan^{-1}\frac{z}{t}\right)$$

$$\overline{H}_{p,q}^{m,n}\left[\frac{(\alpha')(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$

$$H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix}y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1}\\y_2(t^2+u^2+z^2)^{b_2}\\\vdots\\y_r(t^2+u^2+z^2)^{b_r}\end{matrix}\right]f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$=\Delta\int_0^\infty H^{0,\lambda\,:\,(U',V'+3)*}_{A,C:[B'+3,D'+2]*}\left[\begin{matrix} y_1\rho^{2b_1}2^{-\sigma_1} \\ y_2\rho^{2b_2} \\ \vdots \\ y_r\rho^{2b_r}\end{matrix}\left|\begin{matrix}[(a):\theta',...,\theta^{(r)}]: \\ \\ \\ [(c):\psi',...,\psi^{r}]:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\sigma-ks;s_1\right]* \\ \\ \left[-\frac{1}{2}-\sigma-ks\,;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]*\end{matrix}\right]\times\rho^2 f(\rho^2)\,d\rho,$$

where

$$\left[\mathrm{Re}(\sigma)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)-\sum_{i=1}^{r}b_i\sigma_i\max\mathrm{Re}\left(\frac{b_j^{(i)}-1}{\phi_j^{(i)}}\right)+\sigma_1 s_1\min\mathrm{Re}\left(\frac{d_j'}{\delta_j'}\right)-\frac{1}{2}\right]>0$$

$$\left[\mathrm{Re}(\sigma)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)+\sigma_1 s_1\min\left(\frac{d_j'}{\delta_j'}\right)+1\right]>0$$

$$\left[\mathrm{Re}(\sigma)+ks\min\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)-2\sum_{i=1}^{r}b_i\sigma_i\max\mathrm{Re}\left(\frac{b_j^{(i)}-1}{\phi_j^{(i)}}\right)+\sigma_1 s_1\min\mathrm{Re}\left(\frac{d_j'}{\delta_j'}\right)-1\right]>0,$$

where the asterisk * in (2.1) indicates that the parameters at these places are the same as the parameters of the H-function of several variables defined by (1.3)

THEOREM 2

(2.2) $$\int_0^\infty\int_0^\infty\int_0^\infty\frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\delta\tan^{-1}\frac{z}{t}\right)$$

$$\overline{H}^{m,n}_{p,q}\left[\frac{(\alpha')(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$

$$H^{0,\lambda:(U',V');...;(U^{(r)},V^{(r)})}_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}\left[\begin{matrix}y_1(tu)^{-\sigma_1}(t^2+u^2+z^2)^{b_1+\sigma_1} \\ y_2(t^2+u^2+z^2)^{b_2} \\ \vdots \\ y_r(t^2+u^2+z^2)^{b_r}\end{matrix}\right]f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$=\Delta\int_0^\infty H^{0,\lambda\,:\,(U'+3,V)*}_{A,C:[B'+2,D'+3]*}\left[\begin{matrix} y_1\rho^{2b_1}2^{\sigma_1} \\ y_2\rho^{2b_2} \\ \vdots \\ y_r\rho^{2b_r}\end{matrix}\middle|\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ \\ \\ [(c):\psi',\ldots,\psi^{r}]:\end{matrix}\right.$$

$$\left.\begin{matrix}\left[1+\frac{\sigma}{2}+\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right][1+\sigma+ks;s_1]* \\ \left[\frac{3}{2}+\sigma+ks\,;s_1\right]\left[1+\frac{\sigma}{2}+\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]*\end{matrix}\right]\times\rho^2\,f(\rho^2)\,d\rho,$$

valid under the conditions as obtainable from (2.1).

THEOREM 3

$$(2.3)\qquad \int_0^\infty\int_0^\infty\int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma}\cos\left(2\delta\tan^{-1}\frac{z}{t}\right)$$

$$\overline{H}^{m,n}_{p,q}\left[\frac{(\alpha')(tu)^k}{(t^2+u^2+z^2)^k}\middle|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right]$$

$$H^{0,\lambda:(U',V');\ldots;(U^{(r)},V^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left[\begin{matrix} y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1} \\ y_2(tu)^{\sigma_2}(t^2+u^2+z^2)^{b_2-\sigma_2} \\ \vdots \\ y_r(tu)^{\sigma_r}(t^2+u^2+z^2)^{b_r-\sigma_r}\end{matrix}\right] f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$=\Delta\int_0^\infty H^{0,\lambda+3\,*}_{A+3,C+2*}\left[\begin{matrix} y_1\rho^{2b_1}2^{-\sigma_1} \\ y_2\rho^{2b_2}2^{-\sigma_2} \\ \vdots \\ y_r\rho^{2b_r}2^{-\sigma_r}\end{matrix}\middle|\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: \\ \\ \\ [(c):\psi',\ldots,\psi^{r}]:\end{matrix}\right.$$

$$\left.\begin{matrix}\left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right][-\sigma-ks;s_1]* \\ \left[-\frac{1}{2}-\sigma-ks\,;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]*\end{matrix}\right]\times\rho^2\,f(\rho^2)\,d\rho.$$

where the condition of validity are same as surrounding Theorem 1.

Proof. To prove Theorem 1, first we change the left hand side of integral (2.1) from Cartesian to Polar form and then expressing the $\overline{H}$-function and the multivariable H-function in their contour form with the help of (1.1) and (1.3). Now changing the order of integration, we get the following form of integral say (χ)

(2.4) $$\chi = \frac{1}{(2\pi\omega)^r}\int_{L_1}\ldots\int_{L_r} T(s_1,\ldots,s_r)R_1(s_1)\ldots R_r(s_r)y_1^{s_1}\ldots y_r^{s_r}$$
$$\cdot\frac{1}{(2\pi\omega)}\int_L \theta(s)(\alpha')^s\left[\int_0^\infty\int_0^{\pi/2}\rho^{2+2b_1s_1+\ldots+2b_rs_r} f(\rho^2)(\sin\theta)^{\sigma+ks+\sigma_1s_1+1}\right.$$
$$\left.\cdot(\cos\theta)^{\sigma+ks+\sigma_1s_1}\left\{\int_0^{\pi/2}(\cos\phi)^{\sigma+ks+\sigma_1s_1}\cos(2\delta\phi)d\phi\right\}d\theta\, d\rho\right]ds.ds_1\ldots ds_r.$$

On evaluating the θ and ϕ integral occurring on the right hand side of (2.4) with the help of known result [3, Eq.5, p.16], see also [4, Eq.3.2.7, p.62] and using the well known Beta function, we easily arrive at the desired result (2.1) after a little simplification.

Theorems 2 and 3 can be proved on the similar way.

3. Special Cases

(I) Taking m = 1, n = 3 = p = q and replacing z by – z in the triple integral (2.1) through (2.3) and using

(3.1) $$g(\gamma,\eta,\Im,\mu;z) = \frac{\ell_{d-1}\Gamma(\mu+1)\Gamma\left(\frac{1}{2}+\frac{\Im}{2}\right)}{(-1)^{\mu}\,2^{2+\mu}\,(\pi)^{\frac{1}{2}}\,\Gamma(\gamma)\Gamma\left(\gamma-\frac{\Im}{2}\right)}$$
$$\times\overline{H}_{3,3}^{1,3}\left[-z\left|\begin{matrix}(1-\gamma,1;1),\left(1-\gamma+\frac{\Im}{2},1;1\right),(1-\eta,1;1+\mu)\\(0,1),\left(-\frac{\Im}{2},1;1\right),(-\eta,1;1+\mu)\end{matrix}\right.\right],$$

where $\ell_d = \dfrac{2^{1-d}(\pi)^{-\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}$ ([2], p.4121, Eqn. (5)).

The above function is connected with a certain class of Feynman integrals. We get

THEOREM 4

(3.2) $$\int_0^\infty\int_0^\infty\int_0^\infty \frac{(tu)^{\sigma}}{(t^2+u^2+z^2)^{\sigma}}\cos\left(2\delta\tan^{-1}\frac{z}{t}\right)$$
$$g\left(\gamma,\eta,\Im,\mu;\frac{(\alpha)'(tu)^k}{(t^2+u^2+z^2)^k}\right)$$
$$H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix}y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1}\\ y_2(t^2+u^2+z^2)^{b_2}\\ \vdots\\ y_r(t^2+u^2+z^2)^{b_r}\end{matrix}\right]f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$= \nabla \int_0^\infty H^{0,\lambda\,:\,(U',V'+3)*}_{A,C\,:\,[B'+3,D'+2]*}\left[\begin{matrix} y_1 \rho^{2b_1} 2^{-\sigma_1} \\ y_2 \rho^{2b_2} \\ \vdots \\ y_r \rho^{2b_r} \end{matrix}\left|\begin{matrix} [(a):\theta',...,\theta^{(r)}]: \\ \\ \\ [(c):\psi',...,\psi^{r}]: \end{matrix}\right.\right.$$

$$\left.\begin{matrix} \left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\sigma-ks;s_1\right]* \\ \\ \left[-\frac{1}{2}-\sigma-ks\,;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]* \end{matrix}\right] \times \rho^2 f(\rho^2)\, d\rho.$$

valid under conditions as obtainable from (2.1)

THEOREM 5

(3.3) $$\int_0^\infty \int_0^\infty \int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma} \cos\left(2\delta \tan^{-1}\frac{z}{t}\right)$$

$$g\left(\gamma,\eta,\Im,\mu;\frac{(\alpha)'(tu)^k}{(t^2+u^2+z^2)^k}\right)$$

$$H^{0,\lambda:(U',V)\,;...;(U^{(r)},V^{(r)})}_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}\left[\begin{matrix} y_1 (tu)^{-\sigma_1}(t^2+u^2+z^2)^{b_1+\sigma_1} \\ y_2 (t^2+u^2+z^2)^{b_2} \\ \vdots \\ y_r (t^2+u^2+z^2)^{b_r} \end{matrix}\right] f(t^2+u^2+z^2)\, dt\, du\, dz$$

$$= \nabla \int_0^\infty H^{0,\lambda\,:\,(U'+3,V')*}_{A,C\,:\,[B'+2,D'+3]*}\left[\begin{matrix} y_1 \rho^{2b_1} 2^{\sigma_1} \\ y_2 \rho^{2b_2} \\ \vdots \\ y_r \rho^{2b_r} \end{matrix}\left|\begin{matrix} [(a):\theta',...,\theta^{(r)}]: \\ \\ \\ [(c):\psi',...,\psi^{r}]: \end{matrix}\right.\right.$$

$$\left.\begin{matrix} \left[1+\frac{\sigma}{2}+\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{ks}{2};\frac{s_1}{2}\right]\left[1+\sigma+ks;s_1\right]* \\ \\ \left[\frac{3}{2}+\sigma+ks\,;s_1\right]\left[1+\frac{\sigma}{2}+\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]* \end{matrix}\right] \times \rho^2 f(\rho^2)\, d\rho.$$

valid under the conditions as needed for (2.2).

THEOREM 6

(3.4) $$\int_0^\infty \int_0^\infty \int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma} \cos\left(2\delta \tan^{-1}\frac{z}{t}\right)$$

$$g\left(\gamma, \eta, \Im, \mu; \frac{(\alpha)'(tu)^k}{(t^2+u^2+z^2)^k}\right)$$

$$H^{0,\lambda:(U',V');...;(U^{(r)},V^{(r)})}_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}\left[\begin{matrix} y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1} \\ y_2(tu)^{\sigma_2}(t^2+u^2+z^2)^{b_2-\sigma_2} \\ \vdots \\ y_r(tu)^{\sigma_r}(t^2+u^2+z^2)^{b_r-\sigma_r} \end{matrix}\right] f(t^2+u^2+z^2)\, dt\, du\, dz$$

$$=\nabla \int_0^\infty H^{0,\lambda+3:*}_{A+3,C+2:*}\left[\begin{matrix} y_1\rho^{2b_1}2^{-\sigma_1} \\ y_2\rho^{2b_2}2^{-\sigma_2} \\ \vdots \\ y_r\rho^{2b_r}2^{-\sigma_r} \end{matrix}\left|\begin{matrix} [(a):\theta',...,\theta^{(r)}]: \\ \\ [(c):\psi',...,\psi^r]: \end{matrix}\right.\right.$$

$$\left.\begin{matrix} \left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\sigma-ks;s_1\right]* \\ \left[-\frac{1}{2}-\sigma-ks;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]* \end{matrix}\right] \times \rho^2 f(\rho^2)\, d\rho.$$

(II) Putting $A_j = B_j = 1, \forall j$ the results from (2.1) to (2.3) reduce to a known result derived by Chaurasia and Saxena [1].

THEOREM 7

(3.5) $$\int_0^\infty \int_0^\infty \int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma} \cos\left(2\delta \tan^{-1}\frac{z}{t}\right)$$

$$H^{m,n}_{p,q}\left[\frac{(\alpha')(tu)^k}{(t^2+u^2+z^2)^k}\left|\begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix}\right.\right]$$

$$H^{0,\lambda:(U',V');...;(U^{(r)},V^{(r)})}_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}\left[\begin{matrix} y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1} \\ y_2(t^2+u^2+z^2)^{b_2} \\ \vdots \\ y_r(t^2+u^2+z^2)^{b_r} \end{matrix}\right] f(t^2+u^2+z^2)\, dt\, du\, dz$$

$$= \xi \int_0^\infty H^{0,\lambda\,:\,(U',V'+3)*}_{A,C:[B'+3,D'+2]*}\left[\begin{matrix} y_1 \rho^{2b_1} 2^{-\sigma_1} \\ y_2 \rho^{2b_2} \\ \vdots \\ y_r \rho^{2b_r} \end{matrix}\middle| \begin{matrix} [(a):\theta',...,\theta^{(r)}]: \\ \\ \\ [(c):\psi',...,\psi^{r}]: \end{matrix} \begin{matrix} \left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\sigma-ks;s_1\right]* \\ \left[-\frac{1}{2}-\sigma-ks\,;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]* \end{matrix}\right] \times \rho^2 f(\rho^2)\,d\rho.$$

THEOREM 8

(3.6) $$\int_0^\infty \int_0^\infty \int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma} \cos\left(2\delta \tan^{-1}\frac{z}{t}\right) H^{m,n}_{p,q}\left[\frac{(\alpha')\,(tu)^k}{(t^2+u^2+z^2)^k}\middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix}\right]$$

$$H^{0,\lambda:(U',V');...;(U^{(r)},V^{(r)})}_{A,C:[B',D'];...;[B^{(r)},D^{(r)}]}\left[\begin{matrix} y_1 (tu)^{-\sigma_1}(t^2+u^2+z^2)^{b_1+\sigma_1} \\ y_2\,(t^2+u^2+z^2)^{b_2} \\ \vdots \\ y_r\,((t^2+u^2+z^2)^{b_r} \end{matrix}\right] f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$= \xi \int_0^\infty H^{0,\lambda\,:\,(U'+3,V)*}_{A,C:[B'+2,D'+3]*}\left[\begin{matrix} y_1 \rho^{2b_1} 2^{\sigma_1} \\ y_2 \rho^{2b_2} \\ \vdots \\ y_r \rho^{2b_r} \end{matrix}\middle| \begin{matrix} [(a):\theta',...,\theta^{(r)}]: \\ \\ \\ [(c):\psi',...,\psi^{r}]: \end{matrix} \begin{matrix} \left[1+\frac{\sigma}{2}+\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}+\frac{\sigma}{2}+\frac{ks}{2};\frac{s_1}{2}\right]\left[1+\sigma+ks;s_1\right]* \\ \left[\frac{3}{2}+\sigma+ks\,;s_1\right]\left[1+\frac{\sigma}{2}+\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]* \end{matrix}\right] \times \rho^2 f(\rho^2)\,d\rho.$$

THEOREM 9

$$(3.7)\quad \int_0^\infty \int_0^\infty \int_0^\infty \frac{(tu)^\sigma}{(t^2+u^2+z^2)^\sigma} \cos\left(2\delta \tan^{-1}\frac{z}{t}\right) H_{p,q}^{m,n}\left[\frac{(\alpha')(tu)^k}{(t^2+u^2+z^2)^k}\middle|\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix}\right]$$

$$H_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}^{0,\lambda:(U',V');\ldots;(U^{(r)},V^{(r)})}\left[\begin{matrix} y_1(tu)^{\sigma_1}(t^2+u^2+z^2)^{b_1-\sigma_1}\\ y_2(tu)^{\sigma_2}(t^2+u^2+z^2)^{b_2-\sigma_2}\\ \vdots \\ y_r(tu)^{-\sigma_r}(t^2+u^2+z^2)^{b_r-\sigma_r}\end{matrix}\right] f(t^2+u^2+z^2)\,dt\,du\,dz$$

$$=\xi\int_0^\infty H_{A+3,C+2:*}^{0,\lambda+3:*}\left[\begin{matrix} y_1\rho^{2b_1}2^{-\sigma_1}\\ y_2\rho^{2b_2}2^{-\sigma_2}\\ \vdots\\ y_r\rho^{2b_r}2^{-\sigma_r}\end{matrix}\middle|\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:\\ \\ [(c):\psi',\ldots,\psi^r]:\end{matrix}\right.$$

$$\left.\begin{matrix}\left[-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[\frac{1}{2}-\frac{\sigma}{2}-\frac{ks}{2};\frac{s_1}{2}\right]\left[-\sigma-ks;s_1\right]*\\ \left[-\frac{1}{2}-\sigma-ks;s_1\right]\left[-\frac{\sigma}{2}-\frac{ks}{2}\pm\delta;\frac{s_1}{2}\right]*\end{matrix}\right] \times\rho^2 f(\rho^2)\,d\rho.$$

(III) By tacitly giving some values to $f(\rho^2)$ in (2.1) through (2.3); (3.5) through (3.7) and evaluate ρ integral by means of the known result [7, Eq. 2.4.1, p.15, we can get more triple integral relations.

(IV) A number of other special cases of our results can be obtained by specializing its parameters.

ACKNOWLEDGEMENT

The authors are grateful to Professor H.M. Srivastava (University of Victoria, Canada) for his kind help and valuable suggestion in the preparation of this paper.

REFERENCES

[1] V.B.L. Chaurasia and Vishal Saxena, Certain Triple Integral Relations Involving Multivariable H-function : Scientia, Series A: Mathematical Sciences **19** (2010), 69-75.

[2] A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals, II : A generalization of the H-function. J. Phys. A : Math. Gen. **20** (1987), 4109-4128.

[3] Y.L. Luke, The Special functions and their approximations, Academic Press, New York and London, I, (1969).

[4] A.M. Mathai and R.K. Saxena; The H-function with Applications, in Statistics and other Disciplines, Wiley Eastern Limited, New Delhi (1978).

[5] H.M. Srivastava and R. Panda, Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Angew. Math., 283/284, (1976), 265-274.

[6] H.M. Srivastava and M.C. Daoust, Certain generalized Neumann expansions associatged with the Kampé de Fériet's function, Nederl. Akad. Wetensch. Proc. Ser. A 72, Indag. Math., **31** (1969), 449-457.

[7] H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of One and Two Variables with Applications. South Asian Publishers, New Delhi (1982).