On Fuzzy Supra Semi $T_i=0, 1, 2$ Space In Fuzzy Topological Space On Fuzzy Set

1 Assist. Prof. Dr. Munir Abdul Khalik AL-Khafaji, 2 Roqaya Mohammed Hussien
1, 2(Department of Mathematics, College of Education, AL-Mustansiriya University, Iraq)

Abstract: This paper is devoted to introduce the notion of fuzzy supra semi $T_i=0, 1, 2$ space, fuzzy supra semi $D_i=0, 1, 2$ space, and use the notion of fuzzy quasi coincident in their definitions, study some properties and theorems related to these subjects.

Keywords: fuzzy supra semi open set, fuzzy supra semi $D$ set, fuzzy supra semi $T_i=0, 1, 2$ space, fuzzy supra semi $D_i=0, 1, 2$ space.

I. Introduction
The concept of fuzzy set and fuzzy set operation was first introduced by Zadeh[13]. Chakrabarty and Ahsanullah[2] introduced the notion of fuzzy topological space on fuzzy set. In 1986 Abd EL-Monsef and Ramadan[3] introduced fuzzy supra topological space. In this paper we introduced and study the concept of fuzzy supra semi $T_i=0, 1, 2$ space, fuzzy supra semi $D_i=0, 1, 2$ space in fuzzy topological space on fuzzy set.

1. Basic Definitions

Definition 1.1 [13]: Let $X$ be an empty set, a fuzzy set $A$ in $X$ is characterized by a membership function $\mu_A(x) : X \rightarrow I$, where $I$ is the closed unite interval $[0,1]$ which is written as $\bar{A} = \{ (x, \mu_A(x)) : x \in X, 0 \leq \mu_A(x) \leq 1 \}$, the collection of all fuzzy subsets in $X$ will be denoted by $I^*$, that is $I^* = \{ \bar{A} : \bar{A} \text{ is fuzzy subset of } X \}$ and $\mu_A(x)$ is called the membership function.

Proposition 1.2 [9, 12, 13]: Let $A$ and $B$ be two fuzzy sets in $X$ with membership function $\mu_A(x)$ and $\mu_B(x)$ respectively then for all $x \in X$.

- $\bar{A} \subseteq \bar{B}$ iff $\mu_A(x) \leq \mu_B(x)$
- $\bar{A} = \bar{B}$ iff $\mu_A(x) = \mu_B(x)$
- $\bar{A}$ is the complement of $\bar{A}$, with membership function $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$
- $\bar{C} = \bar{A} \cap \bar{B}$ iff $\mu_{\bar{C}}(x) = \min \{ \mu_A(x), \mu_B(x) \}$
- $\bar{B} = \bar{A} \cup \bar{B}$ iff $\mu_{\bar{B}}(x) = \max \{ \mu_A(x), \mu_B(x) \}$

Remark 1.3 [2, 8]: Let $A \in I^*$ then $p(A) = \{ B : B \in I^* and \mu_B(x) \leq \mu_A(x) \ \forall x \in X \}$.

Definition 1.4 [2]: A collection of fuzzy subset of $\bar{A}$, that is $\bar{A} \subseteq p(\bar{A})$ is said to be fuzzy topology on $\bar{A}$ if satisfied the following conditions:

- $\varnothing, \bar{A} \in \bar{T}$
- If $\bar{B}, \bar{C} \in \bar{T}$ then $\min \{ \mu_{\bar{B}}(x), \mu_{\bar{C}}(x) \} \in \bar{T}$.
- If $\bar{B}_i \in \bar{T}$, then $\max \{ \mu_{\bar{B}_i}(x) \ \forall i \} \in \bar{T}$. The pair $(\bar{A}, \bar{T})$ is said to be fuzzy topological space and every member of $\bar{T}$ is said to be fuzzy open set in $\bar{A}$, and a fuzzy set is called fuzzy closed set in $\bar{T}$ its complement is fuzzy open set in $\bar{A}$.

Remark 1.5 [2, 8]: If $(\bar{A}, \bar{T})$ is a fuzzy topological space and $\bar{B} \in p(\bar{A})$, the complement of $\bar{B}$ revered to $\bar{A}$, denoted by $\bar{B}^c$ is defined by $\mu_{\bar{B}^c}(x) = \mu_{\bar{B}}(x) - \mu_{\bar{B}}(x) \ \forall x \in X$.

Definition 1.6 [1]: A subfamily of $\bar{T}^*$ of $\bar{A}$ is said to be fuzzy supra topology on $\bar{A}$ if satisfied the following conditions:

- $\varnothing, \bar{A} \in \bar{T}^*$
- If $\bar{B}_j \in \bar{T}^*$, then $\max \{ \mu_{\bar{B}_j}(x) \ \forall J \} \in \bar{T}^*$. The pair $(\bar{A}, \bar{T}^*)$ is said to be fuzzy supra topological space, the element of $\bar{T}^*$ is said to be fuzzy supra open set in $\bar{A}$, and the complement of fuzzy supra open set is called fuzzy supra closed set.

Remark 1.7 [4]: Every fuzzy topological space is a fuzzy supra topological space.
On Fuzzy Supra Semi $\tilde{T}_{\sigma=0,1,2}$ Space In Fuzzy Topological Space On Fuzzy Set

**Definition 1.8** [4, 6, 7]: The support of a fuzzy set $\tilde{B}$ in $\tilde{A}$ will be denoted by $\text{Supp}(\tilde{B})$ and defined by $\text{Supp}(\tilde{B}) = \{x \in X : \mu_B(x) > 0\}$.

**Definition 1.9** [4, 6]: A fuzzy point $x_r$ in $X$ is a fuzzy set with membership function $\mu_{x_r}(x) = r$, if $x = v$ where $0 < r \leq 1$ and $\mu_{x_r}(x) = 0$ if $x \neq v$, such that $r$ is the value of $x_r$.

**Definition 1.10** [2, 5, 10]: Let $\tilde{B}, \tilde{C}$ be a fuzzy sets in $(\tilde{A}, \tilde{\tau})$, then:

- A fuzzy Point $x_r$ is said to be quasi coincident with a fuzzy set $\tilde{B}$ if there exists $x \in X$ such that $\mu_{x_r} + \mu_B(x) > \mu_A(x)$ and denoted by $x_r \tilde{B}$, if $\mu_{x_r}(x) + \mu_B(x) \leq \mu_A(x) \forall x \in X$. Then $x_r$ is not quasi coincident with a fuzzy set $\tilde{B}$ and denoted by $x_r \not\tilde{B}$.

- A fuzzy set $\tilde{B}$ is said to be quasi coincident with a fuzzy set $\tilde{C}$ if there exists $x \in X$ such that $\mu_B(x) + \mu_C(x) > \mu_A(x)$ and denoted by $\tilde{B} \tilde{C}$. If $\mu_B(x) + \mu_C(x) \leq \mu_A(x) \forall x \in X$, then $\tilde{B}$ is not quasi coincident with a fuzzy set $\tilde{C}$ and denoted by $\tilde{B} \not\tilde{C}$.

**Proposition 1.11** [6, 11]: Let $\tilde{B}, \tilde{C}, \tilde{D}$ be any fuzzy sets in $(\tilde{A}, \tilde{\tau})$, then:

- If $\min\{\mu_B(x), \mu_C(x)\} = \mu_{D}(x)$, then $\mu_B(x) + \mu_C(x) \leq \mu_D(x)$
- $\mu_B(x) + \mu_C(x) \leq \mu_{\tilde{A}}(x)$
- $\mu_B(x) + \mu_C(x) \leq \mu_{\tilde{A}}(x)$. If $\mu_B(x) + \mu_C(x) \leq \mu_{\tilde{A}}(x)$ then $\mu_B(x) + \mu_C(x) \leq \mu_{\tilde{A}}(x)$

2. Fuzzy supra semi open set

**Definition 2.1**: A fuzzy set $\tilde{B}$ of a fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is said to be fuzzy supra s-open (fuzzy supra s-closed) sets if $\mu_B(x) \leq \mu_{\text{suprsc}(\tilde{B})}(x) (\mu_B(x) \geq \mu_{\text{suprsc}(\tilde{B})}(x)) \forall x \in X$

The family of fuzzy supra s-open [fuzzy supra s-closed] sets is denoted by $\text{FSSO}(\tilde{A})$ [FSSC(\tilde{A})].

**Definition 2.2**: If $\tilde{B}$ is a fuzzy set in $(\tilde{A}, \tilde{\tau})$, then:

- supra s-closure of $\tilde{B}$ is denoted by $\text{suprsc}(\tilde{B})$ and defined by $\mu_{\text{suprsc}(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy supra s-closed set in $\tilde{A}, \mu_{\tilde{F}}(x) \leq \mu_{\tilde{B}}(x)\}$
- supra s-interior of $\tilde{B}$ is denoted by $\text{suprrint}(\tilde{B})$ and defined by $\mu_{\text{suprrint}(\tilde{B})}(x) = \max\{\mu_{\tilde{G}}(x) : \tilde{G}$ is a fuzzy supra s-open set in $\tilde{A}, \mu_{\tilde{G}}(x) \leq \mu_{\tilde{B}}(x)\}$

**Proposition 2.3**: Every fuzzy supra open set (resp. fuzzy supra closed set) in $(\tilde{A}, \tilde{\tau})$ is a fuzzy supra s-open set (resp. fuzzy supra s-closed set) in $(\tilde{A}, \tilde{\tau})$.

**Proof**: Obvious

**Remark 2.4**: The converse of **Proposition 2.3** is not true in general as shown in the following example.

**Example 2.5**: Let $X = \{a, b, c\}$, $\tilde{A} = \{(a, 0.6), (b, 0.4), (c, 0.4)\}$

$\tilde{B} = \{(a, 0.3), (b, 0.2), (c, 0.2)\}$, $\tilde{C} = \{(a, 0.5), (b, 0.4), (c, 0.3)\}$,

$\tilde{D} = \{(a, 0.1), (b, 0.0), (c, 0.1)\}$, be fuzzy sets in $\tilde{A}$, $\tilde{\tau} = \{\tilde{\tau}, \tilde{\tau}, \tilde{\tau}\}$, be a fuzzy topology on $\tilde{A}$.

Then $\tilde{C}$ is a fuzzy supra s-open set but not fuzzy supra open set and $\tilde{B}$ is a fuzzy supra s-closed set but not fuzzy supra closed set.

**Definition 2.6**: A fuzzy set $\tilde{B}$ of a fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is said to be fuzzy supra s-difference set (s-D set) if $\mu_B(x) = \mu_C(x) - \mu_H(x)$, where $\tilde{G}$, $\tilde{H}$ are fuzzy supra s-open sets and $\mu_C(x) \neq \mu_H(x)$.

**Proposition 2.7**: Every fuzzy supra s-open set is a fuzzy supra s-D-set

**Proof**: Obvious.

**Remark 2.8**: The converse of **Proposition 2.7** is not true in general as shown in the following example.

**Example 2.9**: Let $X = \{a, b, c\}$, $\tilde{A} = \{(a, 0.4), (b, 0.5), (c, 0.7)\}$

$\tilde{B} = \{(a, 0.3), (b, 0.3), (c, 0.4)\}$, $\tilde{C} = \{(a, 0.3), (b, 0.2), (c, 0.2)\}$,

$\tilde{D} = \{(a, 0.0), (b, 0.1), (c, 0.2)\}$, be fuzzy sets in $\tilde{A}$, $\tilde{\tau} = \{\tilde{\tau}, \tilde{\tau}, \tilde{\tau}\}$, be a fuzzy topology on $\tilde{A}$.

Then $\tilde{D}$ is a fuzzy supra s-D set but not fuzzy supra s-open set.
3. Fuzzy supra semi $\tilde{T}_{1,s}^{0,1,2}$ space

**Definitions 3.1:** A fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is said to be:

- **Fuzzy supra $s$-$\tilde{T}_0$ space** if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_A(x), \mu_{y_s}(x) < \mu_A(x)$, there exists fuzzy supra $s$-open set $G$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_G(x), y_s, qG$. or $\mu_{y_s}(x) < \mu_G(x), x_r, qG$.

- **Fuzzy supra $s$-$\tilde{T}_1$ space** if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_A(x), \mu_{y_s}(x) < \mu_A(x)$, there exists two fuzzy supra $s$-open sets $G, U$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_G(x), y_s, qG$ and $\mu_{y_s}(x) < \mu_U(x), x_r, qU$.

- **Fuzzy supra $s$-$\tilde{T}_2$ space** if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_A(x), \mu_{y_s}(x) < \mu_A(x)$, there exists two fuzzy supra $s$-open sets $G, U$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_G(x), \mu_{y_s}(x) < \mu_U(x)$ and $G \cap U$.

**Propositions 3.2:**
1. Every fuzzy supra $s$-$\tilde{T}_1$ space is a fuzzy supra $s$-$\tilde{T}_0$ space.
2. Every fuzzy supra $s$-$\tilde{T}_2$ space is a fuzzy supra $s$-$\tilde{T}_1$ space.

**Proof:** Obvious

**Remark 3.3:** The converse of Propositions 3.2 is not true in general as shown in the following examples.

**Examples 3.4:**
1. Let $X=[a, b], \tilde{A}=[(a, 0.8), (b, 0.7)]$, $\tilde{\tau}=[(a, 0.0), (b, 0.7)]$, be a fuzzy sets in $\tilde{A}$, then $(\tilde{A}, \tilde{\tau})$ is fuzzy supra $s$-$\tilde{T}_0$ space but not fuzzy supra $s$-$\tilde{T}_1$ space.
2. Let $X=[a, b], \tilde{A}=[(a, 0.5), (b, 0.4)]$, $\tilde{\tau}=[(a, 0.4), (b, 0.1)]$, be a fuzzy sets in $\tilde{A}$, then $(\tilde{A}, \tilde{\tau})$ is fuzzy supra $s$-$\tilde{T}_1$ space but not fuzzy supra $s$-$\tilde{T}_2$ space.

**Theorem 3.5:** A fuzzy topological space $(\tilde{A}, \tilde{\tau})$ is fuzzy supra $s$-$\tilde{T}_0$ space if and only if for each pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_A(x), \mu_{y_s}(x) < \mu_A(x)$, then $x_r, q$ suprascl($y_s$) or $y_s, q$ suprascl($x_r$)

**Proof:** Let $\mu_{x_r}(x) < \mu_A(x), \mu_{y_s}(x) < \mu_A(x)$, then there exist a fuzzy supra $s$-open set $G$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_G(x), y_s, qG$. or $\mu_{y_s}(x) < \mu_G(x), y_s, qG$. $x_r$. If $\mu_{x_r}(x) < \mu_G(x), y_s, qG$, then $x_r, q$ suprascl($y_s$) or $x_r, q$ suprascl($y_s$).

**Conversely**, let $x_r, q$ suprascl($y_s$) or $y_s, q$ suprascl($x_r$). Then $\mu_{x_r}(x) < \mu_{x_r}(x)$ or $\mu_{y_s}(x) < \mu_{y_s}(x)$. $x_r$. If $\mu_{x_r}(x) < \mu_{x_r}(x)$, then $x_r, q$ suprascl($y_s$) or $x_r, q$ suprascl($y_s$).

**Theorem 3.6:** If $(\tilde{A}, \tilde{\tau})$ is a fuzzy topological space then the following statements are equivalent:
1. $(\tilde{A}, \tilde{\tau})$ is fuzzy supra $s$-$\tilde{T}_1$ space
2. For each two distinct fuzzy points, $x_r, y_s$ then $x_r, q$ suprascl($y_s$) and $y_s, q$ suprascl($x_r$).

**Theorem 3.7:** If $(\tilde{A}, \tilde{\tau})$ is a fuzzy supra $s$-$\tilde{T}_2$ space then for each two fuzzy points $x_r, y_s$ in $\tilde{A}$ there exists two fuzzy supra $s$-closed sets $\tilde{F}_1, \tilde{F}_2$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{F}_1}(x), y_s, q\tilde{F}_1$, $\mu_{y_s}(x) < \mu_{\tilde{F}_2}(x), x_r, q\tilde{F}_2$ and max($\mu_{x_r}(x), \mu_{y_s}(x)$) = $\mu_{\tilde{F}_1}(x)$

**Proof:** Obvious
Theorem 3.8: If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space then the following statements are equivalent:

1. $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_2$ space
2. For each two distinct fuzzy points $x_r, y_s$ in $\tilde{A}$ there exist fuzzy supra s-open set $G$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_G(x) < \mu_{suprascl}(G)(x) < \mu_{y_s}(x)$

Proof: Let $(\tilde{A}, \tilde{T})$ be a fuzzy supra s-$T_2$ space, $x_r, y_s$ be two distinct fuzzy points such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{\tilde{H}}(x)$, and $G, U$ are fuzzy s-open set in $\tilde{A}$.

Such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{\tilde{U}}(x)$, and $\tilde{G} \subseteq \tilde{U}$

Since $\mu_{suprascl}(G)(x)$, $\min\{\mu_{\tilde{G}}(x) : \tilde{U}^c$ is fuzzy supra s-closed set, $\mu_{\tilde{G}}(x) \leq \mu_{\tilde{U}}(x)\}$

Therefore $\mu_{x_r}(x) < \mu_{\tilde{G}}(x) < \mu_{suprascl}(\tilde{G})(x) < \mu_{\tilde{U}}(x)\}$

(2) $\Rightarrow$ (1) Let $x_r, y_s$ be a distinct fuzzy points in $\tilde{A}$ and $\tilde{G}$ be a fuzzy supra s-open set in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{suprascl}(\tilde{G})(x) < \mu_{\tilde{G}}(x)$

since suprasint($\tilde{G}^c$) is a fuzzy supra s-open set and $\mu_{suprasint}(\tilde{G}^c)(x) \leq \mu_{\tilde{G}}(x)$.

Then there exists two fuzzy supra s-open sets $\tilde{G}$, $suprasint(\tilde{G}^c)$

such that $\mu_{\tilde{G}}(x) < \mu_{\tilde{U}}(x)$. $\mu_{\tilde{G}}(x) < \mu_{suprasint}(\tilde{G}^c)(x)$ and $\tilde{G} \subseteq \tilde{U}$

Hence the space $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_2$ space.

Theorem 3.9: A fuzzy topological space $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_1$ space if for every fuzzy point is a fuzzy supra s-closed set.

Proof: Let $x_r, y_s$ be two distinct fuzzy points in $\tilde{A}$ which are fuzzy supra s-closed set, then $x_r, y_s$ are fuzzy supra s-open sets

since $\mu_{x_r}(x) \leq \mu_{suprascl}(x_r)$, and $\mu_{y_s}(x) \leq \mu_{suprascl}(y_s)$.

Then $x_r \in [suprascl(x_r)]^c$ and $y_s \in [suprascl(y_s)]^c$

Let $\mu_{\tilde{G}}(x) = \mu_{suprascl}(x_r)$ and $\mu_{\tilde{G}}(x) = \mu_{suprascl}(y_s)$

Hence $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_1$ space.

Remark 3.10: The converse of Theorem 3.9 is not true in general as shown in the following example.

Example 3.11: The space $(\tilde{A}, \tilde{T})$ in the examples 3.4(2) is a fuzzy supra s-$T_1$ space but $a_{012}$ is not fuzzy supra s-closed set.

Theorem 3.12: If $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_2$ space then for each two fuzzy points $x_r, y_s$ in $\tilde{A}$ there exists two fuzzy supra s-closed sets $\tilde{F}_1$ and $\tilde{F}_2$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{F}_1}(x)$, $y_s \in \tilde{F}_1$, $\mu_{x_r}(x) < \mu_{\tilde{F}_2}(x)$, $x_r \in \tilde{F}_2$ and

max($\mu_{\tilde{F}_1}(x)$, $\mu_{\tilde{F}_2}(x)$) = $\mu_{\tilde{G}}(x)$

Proof: Obvious.

4. Fuzzy Supra Semi $D_i=0, 1, 2$ Space

Definition 4.1: A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be fuzzy supra s-$D_i$ space if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_{\tilde{G}}(x)$, $\mu_{x_r}(x) < \mu_{\tilde{H}}(x)$, there exists fuzzy supra s-D set $\tilde{B}$ in $\tilde{A}$ such that either $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s \in \tilde{B}$ or $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$, $x_r \in \tilde{B}$.

Example 4.2: The space $(\tilde{A}, \tilde{T})$ in the examples 3.4(1) is a fuzzy supra s-$D_0$ space.

Theorem 4.3: If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space then the following statements are equivalent:

1. $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$D_0$ space
2. $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_0$ space

Proof: (1) $\Rightarrow$ (2) Let $(\tilde{A}, \tilde{T})$ be a fuzzy supra s-$D_0$ space, then for each distinct fuzzy points $x_r, y_s \in \tilde{A}$, there exist fuzzy supra s-D set $\tilde{B}$ in $\tilde{A}$ such that $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s \in \tilde{B}$ or $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$.

Since $\tilde{B}$ is a fuzzy supra s-D set, then $\mu_{\tilde{B}}(x) = \mu_{\tilde{B}}(x) - \mu_{\tilde{B}}(x)$, where $\tilde{G}, \tilde{H}$ are fuzzy supra s-open set.

If $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $y_s \in \tilde{B}$, then $\mu_{x_r}(x) < \mu_{\tilde{B}}(x)$, $x_r \in \tilde{B}$, $\ldots$ (*)

Since $y_s \in \tilde{B}$ and $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$ and $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$.

If $y_s \in \tilde{B}$ and by (*) we get $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_0$ space and if $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$ and by (*) we get $(\tilde{A}, \tilde{T})$ is a fuzzy supra s-$T_0$ space.

Similarly if $\mu_{y_s}(x) < \mu_{\tilde{B}}(x)$, $x_r \in \tilde{B}$.

(2) $\Rightarrow$ (1) Obvious.

www.iosrjournals.org 4 | Page
Definition 4.4: A fuzzy topological space $(\tilde{A}, \tilde{r})$ is said to be fuzzy supra $sD_1$ space if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_{x_r}(x), \mu_{y_s}(x) < \mu_{y_s}(x)$, there exists two fuzzy supra $sD_2$ sets $\tilde{B}, \tilde{C}$ in $A$ such that $\mu_{x_r}(x) < \mu_{y_s}(x), y_s \notin \tilde{B}$ and $\mu_{y_s}(x) < \mu_{y_s}(x), x_r \notin \tilde{C}$.

Proposition 4.5: Every fuzzy supra $s-T_1$ space is a fuzzy supra $s-D_1$ space.

Proof: Obvious

Remark 4.6: The converse of proposition 4.5 is not true in general as shown in the following example.

Example 4.7: Let $X=\{a, b, c\}, \tilde{A}=\{(a,0.06), (b,0.5), (c,0.0)\}, \tilde{B}=\{(a,0.6), (b,0.1), (c,0.0)\}, \tilde{C}=\{(a,0.1), (b,0.5), (c,0.0)\}, \tilde{D}=\{(a,0.6), (b,0.5), (c,0.0)\}, \tilde{E}=\{(a,0.1), (b,0.1), (c,0.0)\}, \tilde{F}=\{(a,0.6), (b,0.4), (c,0.4)\}, \tilde{G}=\{(a,0.1), (b,0.4), (c,0.4)\}, \tilde{H}=\{(a,0.6), (b,0.4), (c,0.0)\}, \tilde{I}=\{(a,0.0), (b,0.0), (c,0.4)\}$, be a fuzzy sets in $A$, $\tilde{r}=\{\tilde{G}, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}, \tilde{G}, \tilde{H}\}$, be a fuzzy topology on $A$. Then $(\tilde{A}, \tilde{r})$ is a fuzzy supra $s-D_1$ space but not fuzzy supra $s-T_1$ space.

Proposition 4.8: Every fuzzy supra $s-D_1$ space is a fuzzy supra $s-D_2$ space.

Proof: Obvious

Remark 4.9: The converse of proposition 4.8 is not true in general as shown in the following example.

Example 4.10: The space $(\tilde{A}, \tilde{r})$ in the examples 3.4(1) is a fuzzy supra $s-D_0$ space but not fuzzy supra $s-D_1$ space.

Definition 4.11: A fuzzy topological space $(\tilde{A}, \tilde{r})$ is said to be fuzzy supra $s-D_2$ space if for every pair of distinct fuzzy points $x_r, y_s$ such that $\mu_{x_r}(x) < \mu_{x_r}(x), \mu_{y_s}(x) < \mu_{y_s}(x)$, there exists two fuzzy supra $s-D_2$ sets $\tilde{B}, \tilde{C}$ in $A$ such that $\mu_{x_r}(x) < \mu_{y_s}(x), x_r \notin \tilde{B}$ and $\mu_{y_s}(x) < \mu_{y_s}(x), y_s \notin \tilde{C}$.

Proposition 4.12: Every fuzzy supra $s-T_2$ space is a fuzzy supra $s-D_2$ space.

Proof: Obvious

Remark 4.13: The converse of proposition 4.12 is not true in general as shown in the following example.

Example 4.14: Let $X=\{a, b, c, d\}, \tilde{A}=\{(a,0.4), (b,0.4), (c,0.4), (d,0.4)\}, \tilde{B}=\{(a,0.4), (b,0.0), (c,0.0), (d,0.0)\}, \tilde{C}=\{(a,0.4), (b,0.4), (c,0.0), (d,0.0)\}, \tilde{D}=\{(a,0.4), (b,0.4), (c,0.0), (d,0.0)\}, \tilde{E}=\{(a,0.4), (b,0.4), (c,0.0), (d,0.0)\}, \tilde{F}=\{(a,0.4), (b,0.4), (c,0.0), (d,0.0)\}, \tilde{G}=\{(a,0.4), (b,0.0), (c,0.0), (d,0.0)\}, \tilde{H}=\{(a,0.4), (b,0.0), (c,0.0), (d,0.0)\}, \tilde{I}=\{(a,0.0), (b,0.0), (c,0.0), (d,0.4)\}$, be a fuzzy sets in $A$, $\tilde{r}=\{\tilde{F}, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}, \tilde{E}, \tilde{F}\}$, be a fuzzy topology on $A$. Then $(\tilde{A}, \tilde{r})$ is a fuzzy supra $s-D_2$ space but not fuzzy supra $s-T_2$ space.

Proposition 4.15: Every fuzzy supra $s-D_2$ space is a fuzzy supra $s-D_0$ space.

Proof: Obvious

Remark 4.16: The converse of proposition 4.15 is not true in general as shown in the following example.

Example 4.17: The space $(\tilde{A}, \tilde{r})$ in the example 4.7 is a fuzzy supra $s-D_1$ space but not fuzzy supra $s-D_2$ space.

Theorem 4.18: A fuzzy topological space $(\tilde{A}, \tilde{r})$ is a fuzzy supra $s-D_2$ space if for each two distinct fuzzy points $x_r, y_s$ in $\tilde{A}$ there exist fuzzy supra $s$-open set $\tilde{G}$ in $A$ such that $\mu_{x_r}(x) < \mu_{y_s}(x) < \mu_{supsucel}(\tilde{G})(x) < \mu_{y_s}(x)$.

Proof: Obvious

Propositions 4.19:

1. Every fuzzy supra $s-D_2$ space is a fuzzy supra $s-T_0$ space.
2. Every fuzzy supra $s-D_1$ space is a fuzzy supra $s-T_0$ space.

Proof: Obvious

Remark 4.20: The converse of propositions 4.19 is not true in general as shown in the following example.

Example 4.21: The space $(\tilde{A}, \tilde{r})$ in the examples 3.4(1) is a fuzzy supra $s-T_0$ space but not fuzzy supra $s-D_2$ space, fuzzy supra $s-D_1$ space.
On Fuzzy Supra Semi $T_{1,0,1,2}$ Space In Fuzzy Topological Space On Fuzzy Set

References