IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN:2319-765X. Volume 9, Issue 3 (Nov. – Dec. 2013), PP 18-23
www.iosrjournals.org

Implicit Finite Difference Solution of MHD Boundary Layer Heat Transfer over a Moving plate

Satish V Desale [1], V H Pradhan[2]
[1]*(R.C.Patel Institute of Technology, Shirpur, 425405)*
[2]*(S.V.National Institute of Technology, Surat, 395007)*

Abstract: *In this paper, magneto hydrodynamics boundary layer heat transfer over a moving flat plate is discussed, using similarity transformation momentum and energy equations that are reduced in to nonlinear ordinary differential equations. The nonlinear differential equations are solved using implicit finite difference Keller box method. Graphical results of fluid velocity and temperature profile are presented and discussed for various parameters.*
Keywords: *MHD boundary layer, moving surface, heat transfer, Keller box method.*

I. INTRODUCTION

Magnetohydrodynamics (MHD) is the study of the collaboration of conducting fluids with electromagnetic phenomena. The flow of an electrically conducting fluid in the occurrence of a magnetic field is important in various areas of technology and engineering such as MHD power generation, MHD flow meters, MHD pumps, etc.

Rossow [1], was discussed MHD boundary layer flow past a flat plate. He discussed two cases namely, magnetic field is fixed relative to play and magnetic fluid is fixed relative to fluid. Heat transfer of MHD boundary layer flow past a flat plate has been discussed by Rossow [1], Carrier and Green Span [2], and Afzal [3]. Rossow[1] has studied transverse magnetic field whereas Carrier et al. [2] have considered the effect of a longitudinal magnetic field effect on the velocity and the temperature profile distribution.

Sakiadis [4] studied the problem of forced convection along an isothermal moving plate. Erickson et al. [5] discussed the heat and mass transfer on a moving continuous flat plate with suction or injection. Tsou et al. [6] considered flow and heat transfer in the boundary layer on a continuously moving surface.

In this paper, MHD boundary layer heat transfer flow over a continuously moving plate is considered and solved with the help of implicit finite difference Keller box method and various results are discussed graphically.

II. GOVERNING EQUATIONS

Consider the steady flow of an electrically conducting, viscous, incompressible fluid past a continuously moving flat plate with even velocity and surface temperature in the presence of uniform transverse magnetic field. The magnetic Reynolds number of the flow is taken to be very small so that the induced magnetic field can be ignored. The fluid properties are assumed to be isotropic and constant. Therefore, under the usual boundary layer approximations, the governing equations of motion are [7]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 u}{\partial y^2}+\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0{}^2 u^2}{\rho C_p} \tag{3}$$

where u and v are velocity components in x and y directions respectively, ν is the kinematic viscosity , σ is the electrical conductivity, T is the temperature, α is the thermal diffusivity of the fluid and C_p is the specific heat at constant pressure.

The boundary conditions are

$$y=0;\ \ u=U_w\ \ ,\ \ \ v=0\ \ ,\ \ T=T_w \tag{4}$$

$$y \to \infty;\ u \to U_\infty\ ,\quad T \to T_\infty \tag{5}$$

where U_∞ and U_w are constants and denote the free stream and sheet velocities, respectively.

Define the stream function $\psi(x, \mathrm{y})$ such that

$$u = \frac{\partial \psi}{\partial y}; v = -\frac{\partial \psi}{\partial x}$$

Using the following similarity transformation

$$\psi(x, y) = \sqrt{2\nu x U}\, f(\eta)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

where $\eta = y\sqrt{\frac{U}{2\nu x}}$ and U=$U_w\ + U_\infty$ is reference velocity.

The momentum and energy equation reduces to

$$f^{'''} + ff^{''} - Mf^{'} = 0 \tag{6}$$

$$\theta^{''} + \Pr f\theta^{'} + \Pr Ec\, f^{''2} + M\Pr Ec\ f^{'2} = 0 \tag{7}$$

With the boundary conditions

$$\eta = 0 : f = 0\,,\ f^{'} = 1 - r\,,\ \theta = 1$$
$$\eta \to \infty :\quad f^{'} \to r\,,\ \theta \to 0 \tag{8}$$

Where $r = \frac{U_\infty}{U_w\ + U_\infty}$ is moving parameter $(0 \le r \le 1)$ and prime denotes differentiation with respect to η .

Here $M = \frac{2\sigma x B_0^{\ 2}}{\rho U}$ is the magnetic parameter.

$Pr = \frac{\nu}{\alpha}$ is Prandtl number.

$Ec = \frac{U^2}{C_p\left(T_w - T_\infty\right)}$ is the Eckert Number.

III. KELLER BOX METHOD

Equation (6)-(7) subject to the boundary conditions (8) is solved numerically using implicit finite difference method that is known as Keller-box in combination with the Newton's linearization techniques as described by Cebeci and Bradshaw [8]. This method is completely stable and has second-order accuracy.

In this method the transformed differential equation (6)-(7) are writes in terms of first order system, for that introduce new dependent variable u, v such that

$$f' = u \tag{9}$$
$$u' = v \tag{10}$$
$$\theta' = w \tag{11}$$

where prime denotes the differentiation w.r.to η .

Equation (6)-(7) become

$$v' + fv - Mu = 0 \tag{12}$$

$$w' + \Pr fw + \Pr Ecv^2 + M \Pr Ecu^2 = 0 \tag{13}$$

with new independent boundary conditions are

$$f(0) = 0, u(0) = 1 - r, \theta(0) = 1$$
$$u(\infty) = r, \theta(\infty) = 0 \tag{14}$$

Now write the finite difference approximations of the ordinary differential equations (9)-(11) for the midpoint $\left(x^n, \eta_{j-\frac{1}{2}}\right)$ of the segment using centered difference derivatives, this is called centering about $\left(x^n, \eta_{j-\frac{1}{2}}\right)$.

$$\frac{f_j^n - f_{j-1}^n}{h_j} = \frac{u_j^n + u_{j-1}^n}{2} \tag{15}$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = \frac{v_j^n + v_{j-1}^n}{2} \tag{16}$$

$$\frac{\theta_j^n - \theta_{j-1}^n}{h_j} = \frac{w_j^n + w_{j-1}^n}{2} \tag{17}$$

Ordinary differential equations (12)-(13) is approximated by the centering about the mid-point $\left(x^{n-\frac{1}{2}}, \eta_{j-\frac{1}{2}}\right)$ of the rectangle.

This can be done in two steps. In first step, approximate equation at $\left(x^{n-\frac{1}{2}}, \eta\right)$ without specifying η .

$$(v')^n + (fv)^n - M(u)^n = -\left[(v')^{n-1} + (fv)^{n-1} - M(u)^{n-1}\right]$$

$$(w')^n + \Pr(fw)^n + \Pr \mathrm{Ec}(v^2)^n + M \Pr \mathrm{Ec}(u^2)^n = -\left[(w')^{n-1} + \Pr(fw)^{n-1} + \Pr \mathrm{Ec}(v^2)^{n-1} + M \Pr \mathrm{Ec}(u)^{n-1}\right]$$

In second step approximate equations at $j - \dfrac{1}{2}$ (for simplicity, remove n)

$$\left(\frac{v_j - v_{j-1}}{h_j}\right) + \left[\frac{f_j + f_{j-1}}{2}\right]\left[\frac{v_j + v_{j-1}}{2}\right] - M\left[\frac{u_j + u_{j-1}}{2}\right]\left[\frac{u_j + u_{j-1}}{2}\right] = -\left[\left(\frac{v^{n-1}{}_j - v^{n-1}{}_{j-1}}{h_j}\right) + (fv)^{n-1} - M(u)^{n-1}\right]_{j-\frac{1}{2}} \tag{18}$$

$$\left(\frac{w_j - w_{j-1}}{h_j}\right) + \Pr\left[\frac{f_j + f_{j-1}}{2}\right]\left[\frac{w_j + w_{j-1}}{2}\right] + \Pr \mathrm{Ec}\left[\frac{v_j + v_{j-1}}{2}\right]\left[\frac{v_j + v_{j-1}}{2}\right] + M \Pr \mathrm{Ec}\left[\frac{u_j + u_{j-1}}{2}\right]\left[\frac{u_j + u_{j-1}}{2}\right]$$
$$= -\left[\left(\frac{w^{n-1}{}_j - w^{n-1}{}_{j-1}}{h_j}\right) + \Pr(fw)^{n-1} + \Pr \mathrm{Ec}(v^2)^{n-1} + M \Pr \mathrm{Ec}(u)^{n-1}\right]_{j-\frac{1}{2}} \tag{19}$$

Now linearize the nonlinear system of equations (15)-(19) using the Newton's quasi-linearization method [5]
For that use,

$$f_j^{n+1} = f_j^n + \delta f_j^n$$
$$u_j^{n+1} = u_j^n + \delta u_j^n$$
$$w_j^{n+1} = w_j^n + \delta w_j^n$$
$$\theta_j^{n+1} = \theta_j^n + \delta \theta_j^n$$

Equation (15)-(19) reduces to

$$\begin{aligned}
&\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta u_j + \delta u_{j-1}\right) = \left(r_1\right)_j \\
&\delta u_j - \delta u_{j-1} - \frac{h_j}{2}\left(\delta v_j + \delta v_{j-1}\right) = \left(r_2\right)_j \\
&\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2}\left(\delta w_j + \delta w_{j-1}\right) = \left(r_3\right)_j \\
&(a_1)_j \delta v_j + (a_2)_j \delta v_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta u_j + (a_6)_j \delta u_{j-1} = \left(r_4\right)_j \\
&(b_1)_j \delta w_j + (b_2)_j \delta w_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta u_j + (b_6)_j \delta u_{j-1} + (b_7)_j \delta v_j + (b_8)_j \delta v_{j-1} = \left(r_5\right)_j
\end{aligned} \tag{20}$$

The linearized difference equation of the system (20) has a block tridiagonal structure. In a vector matrix form, it can be written as

$$\begin{bmatrix}
[A_1] & [C_1] & & & & \\
[B_2] & [A_2] & [C_2] & & & \\
 & [B_3] & [A_3] & [C_3] & & \\
 & & : & & : & \\
 & & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\
 & & & & [B_J] & [A_J]
\end{bmatrix}
\begin{bmatrix} [\delta_1] \\ [\delta_2] \\ [\delta_3] \\ : \\ : \\ [\delta_J] \end{bmatrix}
=
\begin{bmatrix} [r_1] \\ [r_2] \\ [r_3] \\ : \\ : \\ [r_J] \end{bmatrix}$$

This block tridiagonal structure can be solved using LU method explained by Na [9]

IV. RESULTS AND DISCUSSION

Graphically, effects of magnetic parameter (M), moving parameter (r) and Eckert Number (Ec) on velocity profile as well as on temperature profile are shown in following figures.

Fig.1 and Fig 2 shows, the fluid velocity profile decreases as the magnetic parameter increases for constant value of moving parameter r =0 and r =0.1 respectively. Fig 3 presents velocity profile for different values of moving parameter. In Fig 4 and Fig 5, as the magnetic parameter M and Eckert number (Ec) increases the temperature profile increases. While in Fig 6, temperature profile decreases as Prandtl number increases.

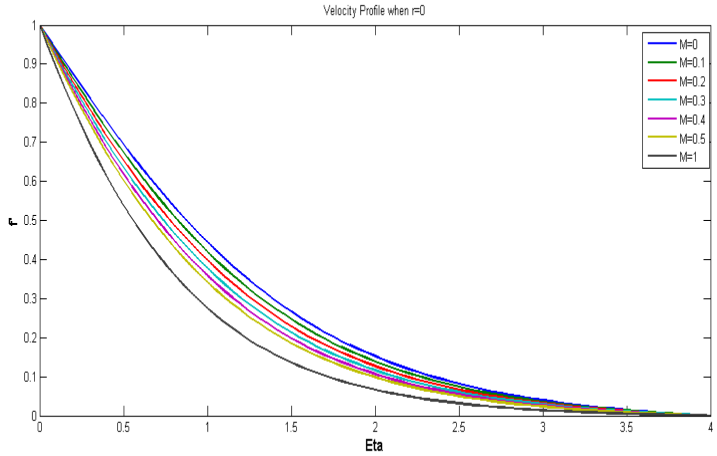

Figure 1 Effect of magnetic parameter on velocity profile when r=0, Pr=0.7,Ec=0.2.

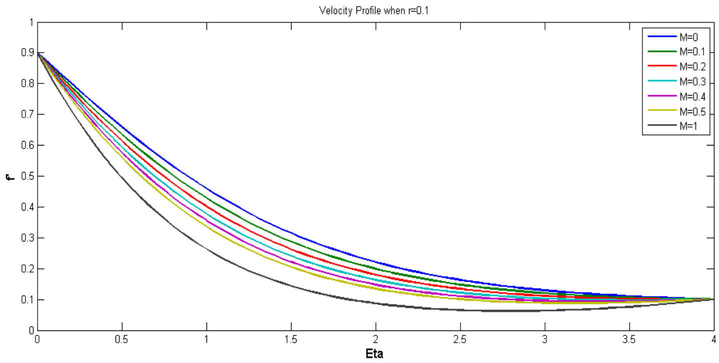

Figure 2 Effect of magnetic parameter on velocity profile when r=0.1, Pr=0.7,Ec=0.2.

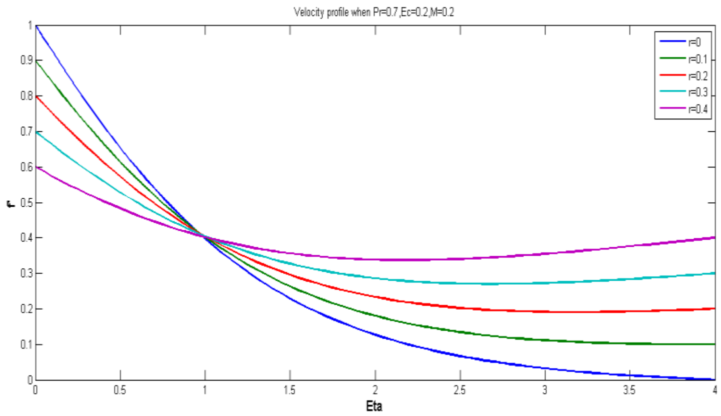

Figure 3 Effect of moving parameter (r) on velocity profile when M=0.2, Pr=0.7,Ec=0.2.

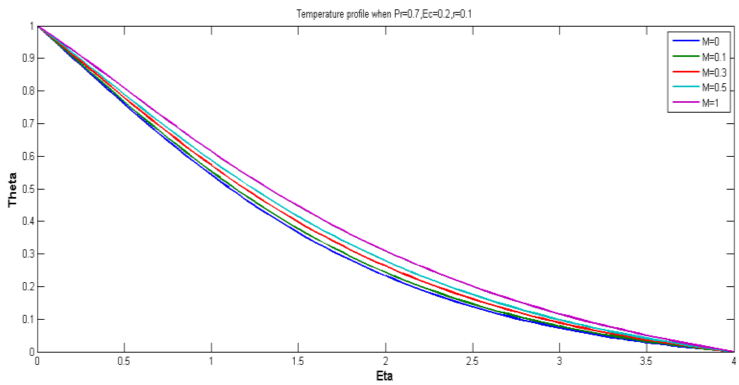

Figure 4 Effect of magnetic parameter (M) on temperature profile when r=0.1, Pr=0.7,Ec=0.2.

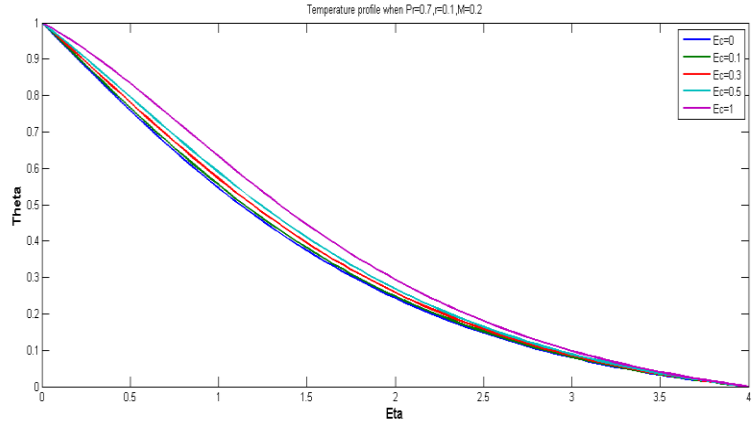

Figure 5 Effect of Eckert Number (Ec) on temperature profile when r=0.1, Pr=0.7, M=0.2.

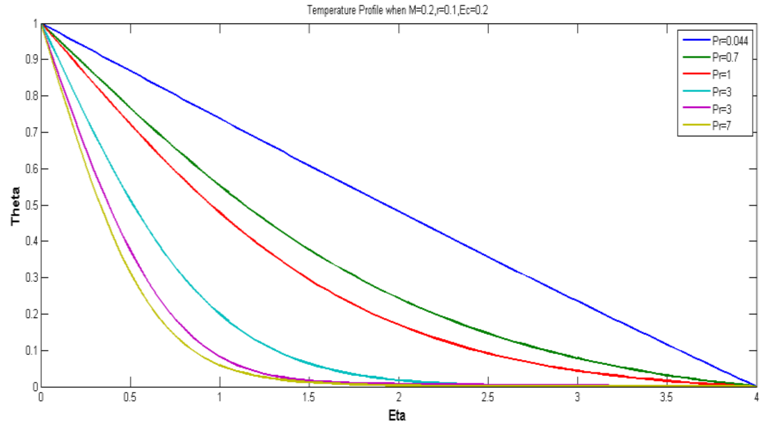

Figure 6 Effect of Prandtl Number (Pr) on temperature profile when r=0.2, EC=0.2, M=0.2.

REFERENCES

[1] Rossow VJ, On the flow of electrically conducting fluid over a flat plate in the presence of transverse magnetic field.-MACA, Rept.(1958),1358

[2] Carrier GF and Greenspan HP, J. Fluid Mech. 67, 1959, 77.

[3] Afzal N, Int. J. Heat Mass Trans. 15, 1972, 863.

[4] Sakiadis B C, Boundary layer behaviour on continuous moving solid surfaces: I. Boundary layer equations for two dimensional and axi-symmetric flow, II. Boundary layer on continuous flat surface, III. Boundary layer on a continuous cylindrical surface. AICHE Journal, 7, 1961, 26-28, 221-225, 467-472.

[5] Erickson L E, Fan L T and Fox V G, Heat and mass transfer on a moving continuous flat plate with suction or injection, Ind. Engg. Chem. Fundam.5, 1966, 19-25.

[6] Tsou F K, Sparrow E M and Goldstein R J, Flow and heat transfer in the boundary layer on a continuous moving surface, Int J Heat Mass Transfer, 10, 1967, 219- 235.

[7] R. N. Jat, Abhishek Neemawat,Dinesh Rajotia, MHD Boundary Layer Flow and Heat Transfer over a Continuously Moving Flat Plate, International Journal of Statistika and Mathematika,3(3),2012,102-108.

[8] Cebeci T., Bradshaw P. Physical and computational Aspects of Convective Heat Transfer, (New York: Springer, 1988), 391-411.

[9] Na T.Y., Computational Methods in Engineering Boundary Value Problem, (New York: Academic Press 1979), 111-118.