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Kaprekar's Constant as a Fixed Point of Iterative Function

Prof. Hiremath Suresh Shivarudrappa[1],
Dr. Kakde Ramesh kumar Vishwambharrao[2]
Brahmdevdada Mane Institute of Technology, Solapur (Maharashtra), India
Sant Gadage Maharaj Mahavidyalaya, Loha (Maharashtra), India

***Abstract**: In this paper we have tried to state that Kaprekar's constant is a fixed point of iterative function defined on set of all positive four digit numbers where none of its any three digits are equal. Hence consequently we have given the definition of Kaprekar's constant in terms of function.*
***Keywords:** Kaprekar's constant, Fixed point, Iterative function, Four digit numbers.*

I. Introduction:

D R Kaprekar invented different number properties throughout his life. He was not well known, however despite many of his papers being reviewed in Mathematical Reviews. International fame came in1975 when Martin Gardener wrote about Kaprekar and his numbers in his 'Mathematical Games' column in the March issue of Scientific American. The fascination for numbers which Kaprekar had as a child continued throughout his life. He realized that he was addicted to number theory and he would say of himself:

"A drunkard wants to go on drinking wine to remain in that pleasurable state.
The same is the case with me in so far as numbers are concered".

Kaprekar's name today is well known and many mathematicians have found themselves intrigued by the ideas about numbers which Kaprekar found so addictive. The best known of Kaprekar's results is the following which relates to the number 6174, today called Kaprekar's Constant. One starts with any four digit number whose any three digits are not equal. Rearrange the digits to form the largest and smallest numbers with these digits and subtract the smaller from the larger. Continue this process with the resulting number. Finally we obtain the four digit number 6174 where this process terminates.

Fixed Points:

Let X be a set and a function $f : X \to X$. The property that f brings the points of X closer together is enough to ensure that X has one and only one point which is not moved by f; that is a unique $x \in X$ with $x = f(x)$. These so called 'fixed points' of functions are invaluable in solving equations.
Here we explain the process of obtaining Kaprekar's constant by using iterative method.

Notations:

If x is a positive four digit number then $\vec{x}$ denotes the number obtained from x by rearranging its digits in descending order and $\overset{\leftarrow}{x}$ denotes the number obtained from x by rearranging its digits in ascending order.
The Kaprekar's procedure of obtaining constant 6174 is given by the following iterative function.

$f : X \to X$ such that $f(x) = \vec{x} - \overset{\leftarrow}{x}$ where X is the set of all positive four digit numbers where none of any three digits are equal.
According to Kaprekar's procedure the above iterative function converges to fixed number 6174.

That is $f(x) = \vec{x} - \overset{\leftarrow}{x} = x_1$

$$f(x_1) = f(f(x)) = f^2(x) = \vec{x}_1 - \overset{\leftarrow}{x}_1 = x_2$$

$$f(x_2) = f(f(f(x))) = f^3(x) = \vec{x}_2 - \overset{\leftarrow}{x}_2 = x_3$$

And so on

$$f(x_n) = x_n \quad \text{where } x_n = 6174$$

Thus we have the following definition for Kaprekar's Constant.

Definition:

Let X be the set of all positive four digit numbers. Define $f: X \to X$ such that $f(x) = \vec{x} - \overset{\leftarrow}{x}$, then there exists a unique element $x \in X$ such that $f(x) = x$. This unique four digit number is called Kaprekar's Constant and it is a fixed point of iterative function $f: X \to X$ and is given by 6174.

Illustration:

If we start with any element belongs to X, suppose x=5643

Therefore, f(x) = 6543-3456=3087

f(3087) =8730-0378=8352

f(8352) =8532-2358=6174

f(6174) =7641-1467=6174

Thus f(6174) =6174

That is the number 5643 reached to Kaprekar's Constant 6174 which is unique element of set X.

II. Summary:

Here we have tried to put the concept of Kaprekar's constant into structure of function with fixed point. Using which we can able to apply metric space structure to the set X. Hence the set X of all four digit numbers plays important role in this study.

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