

New Solution of vector fuzzy transportation problem in interval integer form

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Abstract: A vector square fuzzy transportation problem is a special type of transportation problem of the network optimization problems has the special data. Transportation problem with fuzzy supply values of the suppliers and with fuzzy demand values of the receivers. In this paper we are changing in interval form then solving a vector fuzzy transportation problem. A solution concept is attractive from the standpoint of feasibility and efficiency is specified. An investigation of the stability set of parameters corresponding to one α - efficient solution of the ordinary problem – α - VTP to a vector fuzzy transportation problem is presented. We used ranking technique for solve trapezoidal fuzzy number. An illustrative example is given.

Keywords: Vector fuzzy transportation problem, square Trapezoidal fuzzy numbers, α - efficiency, Optimal solutions, interval number.

I. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers become stronger. How and when to send the product to the customers in the quantities. They want in a cost effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

The basic transportation problem was originally stated and latter discussed in [8]. A linear programming problem using L-R fuzzy number was given in [15]. An operator theory of parametric programming for general transportation problem was presented by [2]. A fuzzy transportation problem (FTP) is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and fuzzy demand limits to deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [7] introduced the notion of fuzziness. In the literature many researchers [4,7] have developed various algorithms to FTP with equality constraints.

Chanas et al gives a fuzzy approach to solving the transportation problem [5,6]. He gives the concept of optimal solution of the fuzzy transportation problem with fuzzy cost coefficients. ÖhEIGEARTAIGH, M. introduced an algorithm for solving transportation problem with fuzzy constraints in [12]. He investigates the relationship between the algebraic structures of the optimal solution of the deterministic problem. Sakawa et al described the profit and cost allocation for a production and transportation using fuzzy programming [14]. Jimenez gives the evolutionary algorithm based on a parametric approach to solve the fuzzy solid transportation problem [10]. Kikuchi shows a method that can be used to adjust the values of the variables in transportation problems [11]; the value of the variable is either a row data or derived from a model. The method uses the concept of fuzzy LP in such a way so as to find a set of values that respects the original observed values as much as possible. Ammar studied the fuzzy vector transportation problem with fuzzy numbers [1] and presents the concept that how can we solve the vector fuzzy transportation problem of trapezoidal number. According to Ammar E.E. and A. Kozae there are however several open points for future research in the area of vector fuzzy transportation problems, to be studied. Some of these points of interest are stated in the following:

- a) An algorithm is required for solving single-objective transportation problem with fuzzy parameters in the constraint function.
- b) Stability of the optimal solution should be investigated for single objective fuzzy transportation.
- c) An algorithm is required for solving multi-objective transportation with fuzzy parameters in (i) in the objective functions and (ii) in the constraint function.
- d) Stability of the efficient solution should be investigated for fuzzy multi objective fuzzy transportation problems.
- e) Computer codes are needed to be constructed to solve the problems recommended above.

On this base we are solving a vector fuzzy transportation problem of trapezoidal fuzzy numbers by changing in interval integer form. For this we study vector fuzzy transportation problem with fuzzy supply values of the suppliers and with fuzzy demand values of the receivers. We give characterizing of the active solution set of this problem. By use of concept of α - efficiency, we deduce the corresponding usual problem α -FVTP of triangular, trapezoidal fuzzy numbers with methods of linear programming and Vogel approximation method and also compare the results of problem.

MATHEMATICAL FORMULATION OF VECTOR FUZZY TRANSPORTATION PROBLEM

$$\text{Min}Z_k(x) = \sum_{i=1}^m \sum_{j=1}^n C_{ij}^k X_{ij}$$

$$\text{Subject to } = \sum_{j=1}^n X_{ij} = a_i \in (\tilde{a}_i)\alpha \quad i = 1,2,3,\dots,m$$

$$= \sum_{i=1}^m X_{ij} = b_j \in (\tilde{b}_j)\alpha \quad j = 1,2,3,\dots,n$$

$$X_{ij} \geq 0$$

Where $i = 1,2,3,\dots,m$ and $j = 1,2,3,\dots,n$

$$\sum_{j=1}^n a_i = \sum_{j=1}^m b_j \quad \forall a_i \in (\tilde{a}_i)\alpha, b_j \in (\tilde{b}_j)\alpha$$

The suppliers $\tilde{a}_i, i = 1,2,3,\dots,m$ (supply) and receivers $\tilde{b}_j, j = 1,2,3,\dots,n$ are fuzzy numbers characterized by membership functions $\mu_{\tilde{a}_i}(a_i)$ and $\mu_{\tilde{b}_j}(b_j)$ respectively.

Deterministic of a vector fuzzy transportation problem

For a certain $\alpha \in [0,1]$ the fuzzy vector transportation problem may be consider a deterministic transportation of α -VTP

$$\text{Min}Z_k(x) = \sum_{i=1}^m \sum_{j=1}^n C_{ij}^k X_{ij} \quad k = 1,2,3,\dots,k$$

$$\text{Subject to } = \sum_{j=1}^n X_{ij} = a_i \text{ for } \mu_{\tilde{a}_i}(a_i) \geq \alpha \quad i = 1,2,3,\dots,m$$

$$= \sum_{i=1}^m X_{ij} = b_j \text{ for } \mu_{\tilde{b}_j}(b_j) \geq \alpha \quad j = 1,2,3,\dots,n$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \quad \forall a_i \in (\tilde{a}_i)\alpha, b_j \in (\tilde{b}_j)\alpha$$

$$X_{ij} \geq 0 \quad i = 1,2,3,\dots,m, j = 1,2,3,\dots,n$$

Robust Ranking Technique:

Roubast ranking technique which satisfy compensation, linearity, and additive properties and provides results which are consist human intuition. If \tilde{a} is a fuzzy number then the Roubast Ranking is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L a_\alpha^U) d\alpha, \text{ where } (a_\alpha^L a_\alpha^U) \text{ is the } \alpha \text{ level cut of the fuzzy numbers } \tilde{a}$$

Where $(a_\alpha^L a_\alpha^U) = \{ [(b-a)\alpha + a], [d-(d-c)\alpha] \}$

Numerical Example:(Trapezoidal Squared fuzzy numbers)

A company has four sources S_1, S_2, S_3, S_4 and destinations D_1, D_2, D_3, D_4 . The fuzzy transportation cost for unit quantity of product from i^{th} sources j^{th} destinations is C_{ij}

$$\text{where } C_{ij} = \begin{pmatrix} (1, 4, 9, 16) & (4, 9, 16, 25) & (9, 16, 25, 36) & (16, 25, 36, 49) \\ (4, 9, 16, 25) & (9, 16, 25, 36) & (16, 25, 36, 49) & (25, 36, 49, 64) \\ (9, 16, 25, 36) & (16, 25, 36, 49) & (25, 36, 49, 64) & (36, 49, 64, 81) \\ (16, 25, 36, 49) & (25, 36, 49, 64) & (36, 49, 64, 81) & (25, 36, 49, 81) \end{pmatrix}$$

Fuzzy availability products at **source** are (36 49 64 81) (16 25 36 49) (4 16 25 36) (25 36 49 64) and there Fuzzy **demand** products and destination are (25 36 49 64) (4 16 25 36) (16 25 36 49) (36 49 64 81)

Solution:

The fuzzy Transportation problems are given in Table-1

Table-1

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4, 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(36 49 64 81)
S ₂	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(16 25 36 49)
S ₃	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(4 16 25 36)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(25 36 49 64)
Demand	(25 36 49 64)	(4,16,25,36)	(16 25 36 49)	(36 49 64 81)	(81,126,174,230)

$$\text{here } \sum_{i=1}^4 a_i = (81,126,174,230)$$

$$\text{here } \sum_{j=1}^4 b_j = (81,126,174,230)$$

$$\sum_{i=1}^4 a_i = \sum_{j=1}^4 b_j \text{ for each } a_i \in (\tilde{a}_i)\alpha \text{ and } b_j \in (\tilde{b}_j)\alpha$$

$$X_{ij} \geq 0 \quad i = 1,2,3,\dots,m, \quad j = 1,2,3,\dots,n$$

The vector fuzzy transportation problem in interval integer form

Table-2

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4, 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(36+13 α ,81-17 α)
S ₂	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(16+9 α ,49-13 α)
S ₃	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(4+12 α , 36-11 α)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(25+11 α ,64-25 α)
Demand	(25+11 α ,64-25 α)	(4+12 α ,36-11 α)	(16+9 α ,49-13 α)	(36+13 α ,81-17 α)	(81+45 α ,230-56 α)

The above fuzzy interval integer transportation problem is a balanced one. Using separation method [12], now we consider lower bound vector fuzzy transportation problem in integer form is given below:

Table-3

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4, 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(36+13 α)
S ₂	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(16+9 α)
S ₃	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(4+12 α)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(25+11 α)
Demand	(25+11 α)	(4+12 α)	(16+9 α)	(36+13 α)	(81+45 α)

Table-4

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4 , 9, 16) (21-α)	(4, 9,16,25)	(9,16,25,36) (16+α)	(16,25,36,49) (-1+5α)	(36+13α)
S ₂	(4, 9,16, 25)	(9,16,25,36) (4+2α)	(16,25,36,49)	(25,36,49,64) (12-3α)	(16+9α)
S ₃	(9,16,25,36) (4+12α)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(4+12α)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81) (25+11α)	(25+11α)
Demand	(25+11α)	(4+12α)	(16+9α)	(36+13α)	(81+45α)

Now the upper bound vector fuzzy transportation problem in integer form is given below:

Table-5

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4 , 9, 16)	(4, 9,16,25)	(9,16,25,36)	(16,25,36,49)	(81-17α)
S ₂	(4, 9,16, 25)	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(49-13α)
S ₃	(9,16,25,36)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(36-11α)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81)	(64-25α)
Demand	(64-25α)	(36-11α)	(49-13α)	(81-17α)	(230-56α)

Table-6

Sources	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(1, 4 , 9, 16) (28-14α)	(4, 9,16,25)	(9,16,25,36) (49-13α)	(16,25,36,49) (4+10α)	(81-17α)
S ₂	(4, 9,16, 25)	(9,16,25,36) (36-11α)	(16,25,36,49)	(25,36,49,64) (13-2α)	(49-13α)
S ₃	(9,16,25,36) (36-11α)	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(36-11α)
S ₄	(16,25,36,49)	(25,36,49,64)	(36,49,64,81)	(25,36,49,81) (64-25α)	(64-25α)
Demand	(64-25α)	(36-11α)	(49-13α)	(81-17α)	(230-56α)

Now using vogel approximation method, the optimal solution of the upper bound and lower bound fuzzy transportation problem is

$$X_{11}=(21-\alpha) \quad X_{13}=(16+\alpha) \quad X_{14}=(-1+5\alpha) \quad X_{22}=(4+2\alpha) \quad X_{24}=(12-3\alpha) \quad X_{31}=(4+12\alpha) \quad X_{44}=(25+11\alpha)$$

$$Y_{11}=(28-14\alpha) \quad Y_{13}=(49-13\alpha) \quad Y_{14}=(4+10\alpha) \quad Y_{22}=(36-11\alpha) \quad Y_{24}=(13-2\alpha) \quad Y_{31}=(36-11\alpha)$$

$$Y_{44}=(64-25\alpha)$$

Thus an fuzzy optimal solution for the vector fuzzy transportation problem is

$$[2237.75+1069.25\alpha, \quad 6413-1797\alpha]$$

Special cases Case 1:

when α=0

The interval cost is [2237.75, 6413] And the optimum value of the objective function of the problem is min (2237.75+ 6413) = 8650.75

Case 2:

when α=0.5

The interval cost is [2772.38 , 5514.5] And the optimum value of the objective function of the problem is min (2772.38 +5514.5) = 8286.88

Case 3:

when α=0.75

The interval cost is [3039.69 , 5065.25] And the optimum value of the objective function of the problem is min (3039.69 +5065.25) = 8104.94

Case 4:

When α=1

The interval cost is [3307 , 4611] And the optimum value of the objective function of the problem is min (3307 +4611) = 7923

II. Conclusion

In this paper, we obtained an optimal solution for vector fuzzy transportation with trapezoidal membership functions. The new arithmetic operations of trapezoidal square fuzzy numbers are employed to get the fuzzy optimal solutions. At first we change fuzzy transportation in integer form and solve by vogel approximation method. And also verified in the trapezoidal membership function, which would be a new attempt in solving the transportation problem in fuzzy environment. The same approach for solving the fuzzy problems may also be utilized in future studies in operational research. Also we solve the problem in the sense

of minimising problem using α - cuts. By trial or error method we get solutions on $\alpha= 0, 0.5, 0.75 , \& 1.0$ we get the optimum cost at is $\alpha= 7923$

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