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Contra R*- Continuous And Almost Contra R*- Continuous Functions

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Abstract: *In this paper we present and study a new class of functions as a new generalization of contra continuity. Furthermore we obtain some of their basic properties and relationship with R*-regular graphs.*
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I. Introduction

In 1996,Dontchev [5] introduced the notion of contra continuity. Several new generalizations to this class were added by Dontchev and Noiri [6] as contra-continuous functions and S-closed spaces, contra semi continuous,contra δ-precontinuous functions etc. C.W.Baker [2] introduced and investigated the notion of contra β continuity, Jafari and Noiri [11] studied the contra precontinuous and contra α continuous functions.

Almost contra pre continuous function was introduced by Ekici [7].In this direction we will introduce the concept of almost contra R*-continuous functions. We include the properties of contra R*-continuous functions and the R*-regular graphs.

Throughout this paper, the spaces X and Y always mean the topological spaces (X,τ) *and* (Y,σ) respectively. For A $\subset$ X, the closure and the interior of A in X are denoted by cl(A) and int(A) respectively. Also the collection of all R*-open subsets of X containing a fixed point x is denoted by R*-O(X,x).

II. Preliminaries

Definition: 2.1. A subset A of a topological space (X, τ) is called
(1) a regular open [17] if A = int(cl(A)) and regular closed [17] if A = cl(int(A)).
The intersection of all regular closed subset of (X, τ) containing A is called the regular closure of A and is denoted by rcl(A) .

Definition :2.2. [4] A subset A of a space (X, τ) is called regular semi open set if there is a regular open set U such that U $\subset$ A $\subset$ cl(U). The family of all regular semi open sets of X is denoted by RSO(X).

Lemma:2.3. [5] In a space (X, τ) , the regular closed sets, regular open sets and clopen sets are regular semiopen.

Definition:2.4. A subset of a topological space (X, τ) is called
1. a regular generalized (briefly rg-closed) [17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
2. a generalized pre regular closed (briefly gpr-closed) [10] if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
3. a regular weakly generalized closed (briefly rwg-closed) [15] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in X.
4. a generalized regular closed (briefly gr-closed) [14] if rcl (A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
5. a regular generalized weak closed set (briefly rgw-closed) [19] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular semi open in X.

The complements of the above mentioned closed sets are their respectively open sets.

Definition:2.5 [12] A subset A of a space (X, τ) is called R*-closed if rcl(A) $\subset$ U whenever A $\subset$ U and U is regular semiopen in (X, τ). We denote the set of all R*- closed sets in (X, τ) by R*C(X).

Definition:2.6 [12]A function $f: X \to Y$ is called R*-continuous if $f^{-1}(V)$ is R*-closed in X for every closed set V of Y.

Definition:2.7[5] A function $f: X \to Y$ is called contra continuous if $f^{-1}(V)$ is closed in X for every open set V of Y.

Definition:2.8 A function $f: X \to Y$ is called

1. contra rg- continuous, if $f^{-1}(V)$ is rg- closed in X for each open set V of Y.
2. contra gpr- continuous , if $f^{-1}(V)$ is gpr- closed in X for each open set V of Y .
3. contra rwg-continuous, if $f^{-1}(V)$ is rwg- closed in X for each open set V of Y.
4. contra gr- continuous, if $f^{-1}(V)$ is gr- closed in X for each open set V of Y.
5. contra rgw-continuous, if $f^{-1}(V)$ is grw- closed in X for each open set V of Y.
6. an R-map [8] if $f^{-1}(V)$ is regular closed in X for each regular closed set V of Y.
7. perfectly continuous if [1,7] $f^{-1}(V)$ is clopen in X for each open set V in Y.
8. almost continuous if [20] $f^{-1}(V)$ is open in X for each regular open set V in Y
9. regular set connected if [9] $f^{-1}(V)$ is clopen in X for each regular open set V in Y.
10. RC-continuous [8] if $f^{-1}(V)$ is regular closed in X for each open set V in Y.

Definition:2.9 [21] A space is said to be weakly Hausdroff if each element of X is an intersection of regular closed sets .

Definition:2.10 [22] A space is said to be Ultra Hausdroff if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V containing x and y respectively.

Definition:2.11[22] A topological space X is called a Ultra normal space, if each pair of disjoint closed sets can be separated by disjoint clopen sets.

Definition:2.12[23] A topological space X is said to be hyperconnected if every open set is dense.

III. Contra R*-continuous functions

Definition: 3.1 A space X is called locally R*-indiscrete if every R*-open subset of X is closed.

Definition:3.2 A function $f: X \to Y$ is called contra R*-continuous if $f^{-1}(V)$ is R*-closed in X for every open set V of Y.

Definition :3.3 A function $f: X \to Y$ is strongly R*-open if the image of every R*-open set of X is R*-open in Y.

Definition :3.4 A function $f: X \to Y$ is almost R*-continuous if $f^{-1}(V)$ is R*-open in X for each regular open set V of Y.

Theorem:3.5 Every contra R*-continuous function is contra rg-continuous, contra gpr-continuous, contra rwg-continuous, contra gr-continuous, contra rgw-continuous but not conversely.

Proof :Obvious from definitions.

Example 3.6: Let
$X=\{a,b,c,d\}=Y, \tau=\{X,\phi,\{b\},\{d\},\{b,d\},\{a,b,d\},\{b,c,d\}\} \sigma=\{Y,\phi,\{a\},\{c\},\{a,c\},\{c,d\},\{a,c,d\}\}$

Define a mapping $f: X \to Y$ as the identity mapping. Here the function f is contra rg-continuous, contra gpr-continuous and contra rwg- continuous but not contra R*-continuous since $f^{-1}\{a\}=a and f^{-1}\{c\}=c$ are not R*-closed.

Example 3.7:Let
$X=\{a,b,c,d\}=Y, \tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}, \sigma=\{Y,\phi,\{b\},\{a,b\},\{b,c\},\{a,b,c\}\}$.

Define a mapping $f: X \to Y$ as $f(a)=c, f(b)=a, f(c)=d, f(d)=b,$ the function f is contra gr-continuous but not contra R*-continuous.

Example3.8:
$X=\{a,b,c,d\}=Y, \tau=\{X,\phi,\{a\},\{d\},\{a,d\},\{a,b\},\{a,b,d\}\}, \sigma=\{Y,\phi,\{c,d\},\{a,c,d\}\}$.

Define a mapping $f: X \to Y$ as $f(a)=b, f(b)=a, f(c)=d, f(d)=c,$ the function f is contra rgw-continuous but not contra R*-continuous.

Remark:3.9
Contra continuous and contra R*-continuous are independent concepts.

Example:3.10
Let X=Y={a,b,c} $\tau = \{X,\phi,\{a\},\{b,c\}\}$ $\sigma = \{Y,\phi,\{b\},\{c\},\{b,c\}\}$ R*-C(X) ={set of all subsets of X}. Define f: X→Y as the identity mapping .Here f is contra R*-continuous but not contra continuous since $f^{-1}\{b\} = \{b\}$ is not closed in X.

Example 3.11: Let X=Y={a,b,c,d} $\tau = \{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ $\sigma = \{Y,\phi,\{d\}\}$
$R^{*} - C(X) = \{X,\phi,\{a,b\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Define f: X→Y as the identity mapping. $f^{-1}\{d\} = \{d\}$ is not R*-closed and hence is not contra R*-continuous but contra continuous.

Theorem3.12: Every RC continuous function is contra R*-continuous but not conversely.
Proof : Straight forward.

Example 3.13: Let $X = Y = \{a,b,c\}$ $\tau = \{\{X\},\{\phi\},\{a\},\{b\},\{a,b\}\}, \sigma = \{Y,\phi,\{a\},\{a,b\},\{a,c\}\}$
Define $f : X \to Y$ as $f(a) = b, f(b) = c, f(c) = a$
Here f is contra R*-continuous but not RC-continuous.

Remark 3.14:
The composition of two contra R*-continuous functions need not be contra R*-continuous as seen in the following example.

Example3.15: let X=Y=Z ={a,b,c} τ = {X, ϕ ,{a},{c},{a,c}} σ = {Y, ϕ ,{a,b}}
η= {Z, ϕ ,{b},{a,c}} R*- C(X)={X, ϕ ,{b},{a,b},{b,c},{c,a}}
R*- C(Y)={X, ϕ ,{a},{b},{c},{a,b},{b,c},{c,a}}
Define f:X→Y by f(a) = b, f(b) = c, f(c) = a ,g:Y→Z by g(a) = b, g(b)= c, and g(c) = a,
Then f and g are contra R*-continuous but g ∘ f: X→Z is not contra R*-continuous since $(g \circ f)^{-1}\{b\} = f^{-1}(g^{-1}\{b\}) = f^{-1}(a)=\{c\}$ is not R*-closed .

Theorem3.16: If f:(X,τ) → (Y,σ) is a contra R*-continuous function and g: (Y,σ)→(Z,η) is a continuous function, then the function g ∘ f: X→Z is contra R*-continuous.
Proof: Let V be open in Z. Since g is continuous, $g^{-1}(V)$ is open in Y.f is contra R*-continuous,so $f^{-1}(g^{-1}(V)$ is R*-closed in X . Hence $(g \circ f)^{-1}(V)$ is R*-closed in X. i.e g ∘ f is contra R*-continuous.

Theorem 3.17: If f: (X,τ) → (Y,σ) is a contra R*-continuous map and g: (Y,σ) → (Z,η) is regular set connected function, then g ∘ f: X→Z is R*-continuous and almost R*-continuous.
Proof: Let V be regular open in Z.Since g is regular set connected, $g^{-1}(V)$ is clopen in Y. Since f is a contra R*-continuous $f^{-1}(g^{-1}(V)$ is R*-closed in X Hence g ∘ f is almost R*-continuous.

Theorem 3.18: If f : (X,τ) → (Y,σ) is R*- irresolute and g : (Y,σ) → (Z,η) is a contra R*– continuous function ,then g ∘ f:X→Z is contra R*-continuous.
Proof: Let V be open in Z. Since g is contra R*-continuous, $g^{-1}(V)$ is R*-closed in Y. Since f is a contra R*-irresolute, $f^{-1}(g^{-1}(V)$ is R*-closed in X. Hence g ∘ f is contra R*-continuous.

Theorem3.19: If f: (X,τ) → (Y,σ) is R*- continuous and the space X is R*-locally indiscrete, then f is contra continuous.
Proof : Let V be an open set in Y. Since f is R*-continuous $f^{-1}(V)$ is R*- open in X. And since X is locally R*-indiscrete, $f^{-1}(V)$ is closed in X. Hence f is contra continuous.

Theorem 3.20: If f: (X,τ) → (Y,σ) is contra R*-continuous, X is R*-$T_{1/2}$ space , then f is RC-continuous.
Proof : Let V be open in Y. Since f is contra R*- continuous, $f^{-1}(V)$ is R*-closed in X. And X is R*-$T_{1/2}$ space, hence $f^{-1}(V)$ is regular closed in X. Thus for every open set V of Y, $f^{-1}(V)$ is regular closed in X. Hence f is RC-continuous.

Theorem3.21: Suppose R*-O(X) is closed under arbitrary unions ,then the following are equivalent for a function f: (X,τ) → (Y,σ) ,
(i) f is contra R*-continuous
(ii) for every closed subset V of Y, $f^{-1}(V) \in$ R*-O(X) .
(iii) for each $x \in X$ and each $V \in C(Y, f(x))$,there exists a set $U \in R^*O(X,x)$ such that $f(U) \subset V$.

Proof : (i) $\Rightarrow$(ii): Let f be contra R*- continuous. Then $f^{-1}(V)$ is R*-closed in X for every open set V of Y. i.e $f^{-1}(V)$ is R*-open in X for every closed set V of Y. Hence $f^{-1}(V)\in R^*-O(X)$.

(ii)$\Rightarrow$(i) :Obvious.

(ii)$\Rightarrow$(iii) : For every closed subset V of Y, $f^{-1}(V)\in$ R*-O(X).Then for each x $\in$ X and each $V\in C(Y, f(x))$, there exists a set U$\in R^*-O(X)$ such that $f(U)\subset V$.

(iii)$\Rightarrow$(ii) : For each $x\in X$ and each $V\in C(Y, f(x))$,there exists a set $U_x\in R^*-O(X,x)$such that $f(U_x)\subset V$ .i.e $x\in f^{-1}(V)$ and $f(x)\subset V$ .So there exists $U\in R^*-O(X,x)$such that $f^{-1}(V)$= $\cup\{U_x : x\in f^{-1}(V)\}$and Hence $f^{-1}(V)$ is R*-open.

Definition 3.22:[7] For a function f: X→Y, the subset $\{(x, f(x)) : x\in X\}\subset X\times Y$ is called the graph of f and is denoted by G(f).

Lemma 3.23: [3] Let G(f) be the graph of f ,for any subset $A\subset X$ and $B\subset Y$,we have $f(A)\cap B=\varphi$ if and only if $(A\times B)\cap G(f)=\varphi$.

Definition 3.24: The graph G(f) of a function f: X→Y is said to be contra R*-closed if for each (x,y) $\in(X,Y)-G((f))$ there exists$U\in R^*O(X,x) and V\in C(Y,y)$such that$(U\times V)\cap G(f)=\varphi$.

Lemma 3.25: The graph G(f) of a function f: X→Y is said to be contra R*-closed if for each (x,y) $\in(X,Y)-G((f))$ there exists $U\in R^*O(X,x) and V\in C(Y,y)$such that $f(U)\cap V=\varphi$.

Proof: The proof is a direct consequence of definition 3.24 and lemma.3.23

IV. Almost contra R*-continuous function

Definition 4.1: A function *f* : X→Y is said to be almost contra R*-continuous is $f^{-1}$is R*-closed in X for each regular open set V in Y.

Theorem 4.2: If a function f: X→Y is almost contra R*-continuous and X is locally R *-indiscreet space , then f is almost continuous.

Proof: Let U be regular open set in Y. Since f is almost contra R*-continuous $f^{-1}(U)$ is R*-closed set in X and X is locally R*-indiscreet space, which implies $f^{-1}(U)$ is an open set in X. Therefore f is almost continuous.

Theorem 4.3: If a function f: X→Y is contra R*-continuous , then it is almost contra R*-continuous.

Proof: Obvious because every regular open set is open set.

Remark 4.4: The converse of the theorem need not be true in general as seen from the following example.

X={a,b,c}=Y, τ ={X,ϕ,{a},{b},{a,b}} σ ={ X,ϕ,{a},{b},{ab},{b,c}} R*-C(X) = { X,ϕ,{c},{ab},{b,c},{c,a}}.Define f(c) = a, f(a) = b, f(b)= c, $f^{-1}(\{b\})$ = {a} which is not R*-closed in X.

Theorem 4.5: The following are equivalent for a function f: X→Y

1. f is almost contra R*- continuous
2. for every regular closed set F of Y,$f^{-1}(F)$ is R*-open set of X.

Proof: (i) Let F be a regular closed set in Y, then Y-F is a regular open set in Y. By (i) $f^{-1}(Y-F)$ = X-$f^{-1}(F)$ is R*-closed in X. Therefore (ii) holds.

(ii)$\Rightarrow$(i). Let G be a regular open set in Y. Then Y –G is regular closed in Y. By (ii) $f^{-1}(Y-G)$ is an R*-open set in X. This implies X - $f^{-1}(G)$ is R*-open which implies, $f^{-1}(G)$ is R*-closed set in X. Therefore (i) holds.

Theorem 4.6: The following are equivalent for a function f:X→Y

1. f is almost contra R*-continuous.
2. $f^{-1}\left(int(cl(G))\right)$ is a R*-closed set in X for every open set G of Y.
3. $f^{-1}(cl(int(F)))$ is a R*-open set in X for every open subset F of Y.

Proof: (i)$\Rightarrow$(ii).Let G be an open set in Y. Then int (cl(G)) is regular open set in Y. By (i) $f^{-1}(int(cl(G)))\in$ R*-C(X).

(ii))$\Rightarrow$(i). Proof is obvious.

(i) $\Rightarrow$ (iii).Let F be a closed set in Y. Then cl(int(F)) is a regular closed set in Y. By (i) $f^{-1}\left(cl\left(\text{int}\left(F\right)\right)\right) \in$ R*-O(X).

(iii) $\Rightarrow$ (i).Proof is obvious.

Definition 4.7: A space X is said to be

1. R*-$T_{1/2}$ space [13] if every R*-closed set is regular closed.
2. R*-T_0 if for each pair of distinct points in X, there is an R*-open set of X containing one point but not the other.
3. R*-T_1 if for every pair of distinct points x and y,there exists R*-open sets G and H such that $x \in G, y \notin G and x \notin H, y \in H$.
4. R*-T_2 if for every pair of distinct points x and y ,there exists disjoint R*-open sets G and H such that $x \in G and y \in H$.

Theorem 4.8: If f: X→Y is an almost contra R*-continuous injection and Y is weakly Hausdroff then X is R*-T_1.

Proof: Suppose Y is weakly Hausdroff, for any distinct points x and y in X, there exists V and W regular closed sets in Y such that $f\left(x\right) \in V, f\left(y\right) \notin V and f\left(y\right) \in W and f\left(x\right) \notin W$ .Since f is almost contra R*-continuous $f^{-1}(V) and\, f^{-1}(W)$ are R*-open subsets of X such that $x \in f^{-1}(V), y \notin f^{-1}(V), y \epsilon\, f^{-1}(W) and\, x \notin f^{-1}(W)$.This shows that X is R*-T_1.

Corollary 4.9: If f: (X,τ)→(Y,σ) is a contra R*-continuous injection and Y is weakly Hausdroff, then X is R*-T_1 .

Proof: Every contra R*-continuous is almost contra R*-continuous and by the above theorem [4.8] the result follows.

Theorem:4.10 If f:X→Yis an almost contra R*-continuous injective function from a space X into the Ultra Haudroff space Y, then Y is an R*-T_2.

Proof: Let x and y be any two distinct points in X. Since f is an injective function such that f(x)≠ f(y) and Y is Ultra Hausdorff space, there exists disjoint clopen sets U and V containing f(x) and f(y) respectively .Then x $\in f^{-1}$(U) and $y \in f^{-1}$(V), were f^{-1}(U) and f^{-1}(V) are disjoint R*- open sets in X. Therefore Y is R*-T_2.

Definition:4.11 A topological space X is called a R*- normal space, if each pair of disjoint closed sets can be separated by disjoint R*-open sets.

Theorem:4.12 If f:X→Yis an almost contra R*-continuous, closed, injective function and Y is Ultra Normal, then X is R*-normal.

Proof: Let E and F be disjoint closed subsets of X. Since f is closed and injective f(E) and f(F) are disjoint closed sets in Y. Since Y is ultra normal there exists disjoint clopen sets in U and V in Y such that f(E) $\subset$ U and f(F) $\subset$ V .This implies E $\subset f^{-1}$(U) and F $\subset f^{-1}$(V).Since f is an almost contra R*-continuous injection f^{-1} (U) and f^{-1}(V) are disjoint R*-open sets in X.This show that X is R*-normal.

Theorem:4.13 For two functions f:X→Y and k :Y→Z, let k ∘ f :X → Z is a composition function. Then the following holds:

(1) If f is almost contra R*-continuous and k is an R-map, then $k \circ f$ is almost contra R*-continuous.

(2) If f is almost contra R*-continuous and k is perfectly continuous, then $k \circ f$ is R*-continuous and contra R*-continuous.

(3) If f is almost contra R*-continuous and k is almost continuous, then $k \circ f$ is almost contra R*-continuous.

Proof: (1) Let V be any regular open set in Z .Since k is an R-map, k^{-1} (V) is regular open in Y. Since f is almost contra R*- continuous $f^{-1}\left(k^{-1}(V)\right) = (k \circ f)^{-1}$(V) is R*-closed in X. Therefore $k \circ f$ is almost contra R*-continuous.

(2) Let V be an open set in Z. Since k is perfectly continuous,$k^{-1}(V)$ is clopen in Y.Since f is an almost contra R*- continuous $f^{-1}\left(k^{-1}(V)\right) = (k \circ f)^{-1}$(V) is R*-open and R*-closed set in X. Therefore $k \circ f$ is R*-continuous and contra R*- continuous.

(3) Let V be a regular open set in Z. Since k is almost continuous, $k^{-1}(V)$ is open in Y. Since f is contra R*-continuous $f^{-1}\left(k^{-1}(V)\right) = (k \circ f)^{-1}$(V) is R*-closed in X. Therefore $k \circ f$ is almost contra R*-continuous.

Theorem:**4.14** Let f: X→Y is a contra R*-continuous function and g :Y→Z is R*-continuous.If Y is R*-$T_{1/2}$,then $g \circ f : X \rightarrow Z$ is an almost contra R*-continuous function .

Proof: Let V be regular open and hence open set in Z. Since g is R*-continuous $g^{-1}(V)$ is R*- open in Y and Y is $T_{1/2}$-space implies $g^{-1}(V)$ is regular open in Y. Since f is almost contra R*- continuous $f^{-1}\left(g^{-1}(V)\right)=(g\circ f)^{-1}(V)$ is R*-closed set in X. Therefore $g\circ f$ is almost contra R*-continuous.

Theorem:4.15 If f: X→Y is surjective, strongly R*-open (or strongly R*-closed) and g :Y→Z is a function such that $g\circ f: X\to Z$ is almost contra R*- continuous ,then g is almost contra continuous.

Proof: Let V be any regular closed set (resp regular open) set in Z. Since $g\circ f$ is almost contra R*-continuous $(g\circ f)^{-1}(V)= f^{-1}(g^{-1}(V))$ is R*-open (resp R*-closed) in X .Since f is surjective and strongly R*-open (or strongly R*-closed). $f\left(f^{-1}\left(g^{-1}(V)\right)\right)=g^{-1}(V)$ is R*-open (resp R*-closed).Therefore g is almost contra R*-continuous.

Definition:4.16 A topological space X is said to be R*-ultra connected if every two non empty R*-closed subsets of X intersect.

Theorem:4.17 If X is R*-ultra connected and f: X→Y is an almost contra R*-continuous surjection ,then Y is hyperconnected.

Proof: Let X be R*-ultraconnected and f: X→Y is an almost contra R*-continuous surjection. Suppose Y is not hyperconnected. Then there is an open set V such that V is not dense in Y. Therefore there exists non empty regular open subsets B_1 = intcl(V) and B_2 =Y- cl(V) in Y. Since f is an almost contra R*-continuous surjection, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint R*-closed sets in X.This is contrary to the fact that X is R*-ultra connected .Therefore Y is hyperconnected.

V. R*-Regular graphs

Definition 5.1: A graph G(f) of a function f: X→Y is said to be R*-regular if for each (x,y)$\in (X,Y)-G((f))$ there exists $U\in R^*C(X,x) and V\in RO(Y,y)$ such that $(U\times V)\cap G(f)=\varphi$.

Lemma 5.2: The graph G(f) of a function f: X→Y is R*-regular (resp strong contra R* closed) in $X\times Y$ if and only if for each (x,y)$\in (X,Y)-G((f))$, there is an R* -closed (resp R*-open) set U in X containing x and $V\in RO(Y,y)(resp V\in RC(Y,y))$ such that $f(U)\cap V=\phi$.

Proof: Obvious.

Theorem 5.3: If a function f: X→Y is almost R*-continuous and Y is T_2 then G(f) is R*-regular in $X\times Y$.

Proof: Let $(x,y)\in (X,Y)-G(f)$. Then $y\neq f(x)$. Since Y is T_2, there exists open set V and W in Y such that $f(x)\in V, y\in W and V\cap W=\phi$.Then $\operatorname{int}(cl(V))\cap\operatorname{int}(cl(W))=\phi$. Since f is almost R*-continuous is $f^{-1}\left(\operatorname{int}(cl(V))\right)$ is R* -closed set in X containing x. Set $U=f^{-1}\left(\operatorname{int}(cl(V))\right)$, then $f(U)\subset\operatorname{int}(cl(V))$. Therefore $f(U)\cap\operatorname{int}(cl(W))=\phi$ .Hence G(f) is R*-regular in $X\times Y$.

Theorem:5.4 Let f: X→Y be a function and let g: $X\to X\times Y$ be the graph function of f, defined by g(x) = (x,f(x)) for every $x\in X$. If g is almost contra R*-continuous function, then f is an almost contra R*-continuous.

Proof: Let $V\in RC(Y)$, then

$$X\times V=X\times Cl\left(\operatorname{int}(V)\right)=Cl\left(\operatorname{int}(X)\right)\times Cl\left(\operatorname{int}(V)\right)=Cl\left(\operatorname{int}(X\times V)\right).$$

Therefore, $X\times V\in RC(X\times Y)$.Since g is almost contra R*-continuous , $f^{-1}(V)=g^{-1}(X\times V)\in R^*O(X)$. Thus f is almost contra R*-continuous.

Theorem:5.5 Let f : X→Y have a R*-regular G(f). If f is injective, then X is R*-T_0.

Proof: Let x and y be two distinct points of X. Then $(x, f(y))\in (X,Y)-G(f)$. Since $G(f) is R^*$-regular, there exists R*-closed set U in X containing x and $V\in RO(Y, f(y))$ such that $f(U)\cap V=\phi$ by lemma 5.2

and hence $U \cap f^{-1}(V) = \phi$.Therefore $y \notin U$. Thus $y \in X - U and x \notin of X - U$ and X-U is R*-open set in X. This implies that X is R*-T_0.

Theorem:5.6 Let f : X→Y have a R*-regular G(f). If f is surjective then Y is weakly T_2.

Proof:Let $y_1$and y_2 be two distinct points of Y. Since f is surjective f(x) = $y_1$for some $x \in X and (x, y_2) \in X \times Y - G(f)$. By lemma 5.2, there exists a R*-closed set U of X and $F \in RO(Y)$ such that $(x, y_2) \in U \times F and f(U) \cap F = \phi.$ Hence $Y_1 \notin of F$ Then $Y_2 \notin of Y - F \in RC(Y)$ and $Y_1 \in Y - F.$ This implies that Y is weakly T_2.

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