Interval valued fuzzy generalized semi-preclosed mappings

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Abstract: In this paper, we introduce interval valued fuzzy generalized semi-preclosed mappings and interval valued fuzzy generalized semi-preopen mappings. Also we investigate some of their properties.

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I. Introduction
The concept of fuzzy subset was introduced and studied by L. A. Zadeh [15] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [14]. Tapas Kumar Mondal and S. K. Samantha [9] introduced the topology of interval valued fuzzy sets and Jeyabalan, R. Arjunan. K. [7] introduced interval valued fuzzy generalized semi-preclosed sets. In this paper, we introduce interval valued fuzzy generalized semi-preclosed mappings and interval valued fuzzy generalized semi-preopen mappings and some properties are investigated.

II. Preliminaries

Definition 2.1 [9] Let X be a non empty set. A mapping \( \overline{A} : X \to D[0,1] \) is called an interval valued fuzzy set (briefly IVFS) on X, where D[0,1] denotes the family of all closed subintervals of [0,1] and 
\[ \overline{A}(x) = [A^- (x), A^+ (x)], \]
for all \( x \in X \), where \( A^- (x) \) and \( A^+ (x) \) are fuzzy sets of X such that 
\[ A^- (x) \leq A^+ (x), \]
for all \( x \in X \).

Thus \( \overline{A}(x) \) is an interval (a closed subset of [0,1]) and not a number form the interval [0,1] as in the case of fuzzy set.

Obviously any fuzzy set \( A \) on X is an IVFS.

Notation 2.2 \( D^X \) denotes the set of all interval valued fuzzy subsets of a non empty set X.

Definition 2.3 [9] Let \( \overline{A} \) and \( \overline{B} \) be any two IVFS of X, that is \( \overline{A} = \{ x, [A^- (x), A^+ (x)] : x \in X \} \), and \( \overline{B} = \{ x, [B^- (x), B^+ (x)] : x \in X \} \). We define the following relations and operations:

(i) \( \overline{A} \subseteq \overline{B} \) if and only if \( A^- (x) \leq B^- (x) \) and \( A^+ (x) \leq B^+ (x) \), for all \( x \in X \).

(ii) \( \overline{A} = \overline{B} \) if and only if \( A^- (x) = B^- (x) \) and \( A^+ (x) = B^+ (x) \), for all \( x \in X \).

(iii) \( (\overline{A})^c \equiv \overline{1 - A} = \{ x, [1 - A^+ (x), 1 - A^- (x)] : x \in X \} \).

(iv) \( \overline{A} \cap \overline{B} = \{ x, [\min \{ A^- (x), B^- (x) \}, \min \{ A^+ (x), B^+ (x) \}] : x \in X \} \).

(v) \( \overline{A} \cup \overline{B} = \{ x, [\max \{ A^- (x), B^- (x) \}, \max \{ A^+ (x), B^+ (x) \}] : x \in X \} \).
We denote by \( \overline{0}_x \) and \( \overline{1}_x \) for the interval valued fuzzy subsets \( \{< x, [0,0] > : x \in X \} \) and \( \{< x, [1,1] > : x \in X \} \) on a nonempty set \( X \) respectively.

**Definition 2.4** [9] Let \( X \) be a set and \( \mathcal{I} \) be a family of interval valued fuzzy sets \( (IVFSs) \) of \( X \). The family \( \mathcal{I} \) is called an interval valued fuzzy topology \( (IVFT) \) on \( X \) if and only if \( \mathcal{I} \) satisfies the following axioms:

(i) \( \overline{0}_x, \overline{1}_x \in \mathcal{I} \).

(ii) If \( \{\overline{A}_i : i \in I\} \subseteq \mathcal{I} \), then \( \bigcup_{i \in I} \overline{A}_i \in \mathcal{I} \).

(iii) If \( \overline{A}_1, \overline{A}_2, \overline{A}_3, \ldots, \overline{A}_n \in \mathcal{I} \), then \( \bigcap_{i=1}^n \overline{A}_i \in \mathcal{I} \).

The pair \( (X, \mathcal{I}) \) is called an interval valued fuzzy topological space \( (IVFTS) \). The members of \( \mathcal{I} \) are called interval valued fuzzy open sets \( (IVFOS) \) in \( X \).

An interval valued fuzzy set \( \overline{A} \) in \( X \) is said to be interval valued fuzzy closed set \( (IVFCS) \) in \( X \) if \( (\overline{A})^c \) is an \( IVFOS \) in \( X \).

**Definition 2.5** [9] Let \( (X, \mathcal{I}) \) be an \( IVFTS \) and \( \overline{A} = \{< x, [A^-(x), A^+(x)] > : x \in X \} \) be an \( IVFS \) in \( X \). Then the interval valued fuzzy interior and interval valued fuzzy closure of \( \overline{A} \) denoted by \( ivfint(\overline{A}) \) and \( ivfcl(\overline{A}) \) are defined by

\[
ivfint(\overline{A}) = \bigcup \{\overline{G} : \overline{G} \text{ is an IVFOS in } X \text{ and } \overline{G} \subseteq \overline{A}\},
\]

\[
ivfcl(\overline{A}) = \bigcap \{\overline{K} : \overline{K} \text{ is an IVFCS in } X \text{ and } \overline{K} \subseteq \overline{A}\}.
\]

For any \( IVFS \ \overline{A} \) in \( (X, \mathcal{I}) \), we have \( ivfcl(\overline{A}) = (ivfint(\overline{A}))^c \) and \( ivfint(\overline{A}) = (ivfcl(\overline{A}))^c \).

**Definition 2.6** An \( IVFS \ \overline{A} = \{< x, [A^-(x), A^+(x)] > : x \in X \} \) in an \( IVFTS \ (X, \mathcal{I}) \) is said to be an

(i) interval valued fuzzy semi-closed set \( (IVFSCS) \) if \( ivfint(\overline{A}) \subseteq \overline{A} \);

(ii) interval valued fuzzy preclosed set \( (IVFPSC) \) if \( ivfcl(\overline{A}) \subseteq \overline{A} \);

(iii) interval valued fuzzy \( \alpha \) closed set \( (IVF\alpha CS) \) if \( ivfcl(ivfint(\overline{A})) \subseteq \overline{A} \);

(iv) interval valued fuzzy \( \beta \) closed set \( (IVF\beta CS) \) if \( ivfint(ivfcl(\overline{A})) \subseteq \overline{A} \).

**Definition 2.7** An \( IVFS \ \overline{A} = \{< x, [A^-(x), A^+(x)] > : x \in X \} \) in an \( IVFTS \ (X, \mathcal{I}) \) is said to be an interval valued fuzzy generalized closed set \( (IVFGCS) \) if \( ivfcl(\overline{A}) = \overline{U} \), whenever \( \overline{A} \subseteq \overline{U} \) and \( \overline{U} \) in an \( IVFOS \).

**Definition 2.8** An \( IVFS \ \overline{A} = \{< x, [A^-(x), A^+(x)] > : x \in X \} \) in an \( IVFTS \ (X, \mathcal{I}) \) is said to be an
(i) interval valued fuzzy semi-preclosed set (IVFSPCS) if there exist \( \text{IVFPCS} \ B \), such that \( \text{ivfspcl} B \subseteq \bar{A} \subseteq B \);

(ii) interval valued fuzzy semi-preopen set (IVFPOS) if there exist \( \text{IVFPOS} \ B \), such that \( B \subseteq \bar{A} \subseteq \text{ivfcl}(B) \).

Definition 2.9 Let \( \bar{A} \) be an IVFS in an IVFTS \((X, \mathcal{F})\). Then the interval valued fuzzy semi-preinterior of \( \bar{A} \) (ivfspint\(\bar{A}\)) and the interval valued fuzzy semi-preclosure of \( \bar{A} \) (ivfspcl\(\bar{A}\)) are defined by

\[
\text{ivfspint}(\bar{A}) = \bigcup \{ \text{G} : \text{G} \text{ is an IVFPOS in } X \text{ and } \text{G} \subseteq \bar{A} \},
\]

\[
\text{ivfspcl}(\bar{A}) = \bigcap \{ \text{K} : \text{K} \text{ is an IVFSPCS in } X \text{ and } \bar{A} \subseteq \text{K} \}.
\]

For any IVFS \( \bar{A} \) in \((X, \mathcal{F})\), we have \( \text{ivfspcl}(\bar{A}^c) = (\text{ivfspint}(\bar{A}))^c \) and \( \text{ivfspint}(\bar{A}^c) = (\text{ivfspcl}(\bar{A}))^c \).

Definition 2.10 [7] An IVFS \( \bar{A} \) in IVFTS \((X, \mathcal{F})\) is said to be an interval valued fuzzy generalized semi-preclosed set (IVFGSPCS) if \( \text{ivfspcl}(\bar{A}) \subseteq \bar{U} \), whenever \( \bar{A} \subseteq \bar{U} \) and \( \bar{U} \) is an IVFOS in \((X, \mathcal{F})\).

Definition 2.11 [7] The complement \( \bar{A}^c \) of an IVFGSPCS \( \bar{A} \) in an IVFTS \((X, \mathcal{F})\) is called an interval valued fuzzy generalized semi-preopen set (IVFGSPOS) in \( X \).

Definition 2.12 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called interval valued fuzzy continuous (IVF continuous) mapping if \( g^{-1}(B) \) is IVFOS in \( X \) for all IVFOS \( \bar{B} \) in \( Y \).

Definition 2.13 An IVFTS \((X, \mathcal{F})\) is called an interval valued fuzzy semi-pre \( \text{T}_{1/2} \) space (IVFSPT\(_{1/2}\)). If every IVFGSPCS is an IVFSPCS in \( X \).

Definition 2.14 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called an interval valued fuzzy closed mapping (IVFC) mapping if \( g(\bar{A}) \) is an IVFCS in \( Y \) for each IVFCS \( \bar{A} \) in \( X \).

Definition 2.15 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called an interval valued fuzzy semi-closed mapping (IVFSC) mapping if \( g(\bar{A}) \) is an IVFSCS in \( Y \) for each IVFCS \( \bar{A} \) in \( X \).

Definition 2.16 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called an interval valued fuzzy preclosed mapping (IVFPC) mapping if \( g(\bar{A}) \) is an IVFPSC in \( Y \) for each IVFCS \( \bar{A} \) in \( X \).

Definition 2.17 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called an interval valued fuzzy semi-open mapping (IVFoS) mapping if \( g(\bar{A}) \) is an IVFSOS in \( Y \) for each IVFOS \( \bar{A} \) in \( X \).

Definition 2.18 [9] Let \((X, \mathcal{F})\) and \((Y, \sigma)\) be IVFTSs. Then a map \( g : X \rightarrow Y \) is called an interval valued fuzzy generalized semi-preopen mapping (IVFGSPO) mapping if \( g(\bar{A}) \) is an IVFGSPOS in \( Y \).
for each IVFOS $\overline{A}$ in $X$.

**Definition 2.19** Let $(X, \mathcal{S})$ and $(Y, \sigma)$ be IVFTSs. Then a map $g : X \rightarrow Y$ is called an interval valued fuzzy generalized semi-preclosed mapping (IVFGSPC) mapping if $g(\overline{A})$ is an IVFGSPCS in $Y$ for each IVFC $\overline{A}$ in $X$.

**Example 2.20** Let $X = \{a, b\}$, $Y = \{u, v\}$ and 
\[
\overline{K}_1 = \{a, [0.3, 0.4], b, [0.4, 0.6]\},
\overline{L}_1 = \{u, [0.1, 0.2], v, [0.3, 0.4]\}.
\]
Then $\mathcal{S} = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are IVFT on $X$ and $Y$ respectively. Define a mapping $g : (X, \mathcal{S}) \rightarrow (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then $g$ is an IVFGSPC mapping.

### III. Main Results

**Theorem 3.1** Every IVFC mapping is an IVFGSPC mapping.

**Proof.** Let $X$ and $Y$ be two IVFTSs. Assume that $g : X \rightarrow Y$ is an IVFC mapping. Let $\overline{A}$ be an IVFCS in $X$. Then $g(\overline{A})$ is an IVFCS in $Y$. Since every IVFCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPCS in $Y$ and hence $g$ is an IVFGSPC mapping.

**Remark 3.2** The converse of the above theorem 3.1 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and 
\[
\overline{K}_1 = \{a, [0.3, 0.4], b, [0.4, 0.6]\},
\overline{L}_1 = \{u, [0.1, 0.2], v, [0.3, 0.4]\}.
\]
Then $\mathcal{S} = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are IVFT on $X$ and $Y$ respectively.

Define a mapping $g : (X, \mathcal{S}) \rightarrow (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then $g$ is an IVFGSPC mapping but not an IVFC mapping, since $\overline{K}_1^c$ is an IVFCS in $X$ but $g(\overline{K}_1^c) = \{u, [0.6, 0.7], v, [0.4, 0.6]\}$ is not an IVFCS in $Y$, because $\text{ivfcl}(g(\overline{K}_1^c)) = \overline{L}_1^c \neq g(\overline{K}_1^c)$.

**Theorem 3.3** Every IVFαCS mapping is an IVFGSPC mapping.

**Proof.** Let $X$ and $Y$ be two IVFTSs. Assume that $g : X \rightarrow Y$ is an IVFαCS mapping. Let $\overline{A}$ be an IVFCS in $X$. Then $f(\overline{A})$ is an IVFαCS in $Y$. Since every IVFαCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPCS in $Y$ and hence $g$ is an IVFGSPC mapping.

**Remark 3.4** The converse of the above theorem 3.3 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and 
\[
\overline{K}_1 = \{a, [0.3, 0.4], b, [0.4, 0.6]\},
\overline{L}_1 = \{u, [0.1, 0.2], v, [0.3, 0.4]\}.
\]
Then $\mathcal{S} = \{\overline{0}_X, \overline{K}_1, \overline{1}_X\}$ and $\sigma = \{\overline{0}_Y, \overline{L}_1, \overline{1}_Y\}$ are IVFT on $X$ and $Y$ respectively.

Define a mapping $g : (X, \mathcal{S}) \rightarrow (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then $g$ is an IVFGSPC mapping but not an IVFαCS mapping, since $\overline{K}_1^c$ is an IVFCS in $X$ but $g(\overline{K}_1^c) = \{u, [0.6, 0.7], v, [0.4, 0.6]\}$ is not an IVFαCS in $Y$, because $\text{ivfcl}(g(\overline{K}_1^c)) = \text{ivfcl}(\text{ivfint}(\text{ivfcl}(g(\overline{K}_1^c)))) = \text{ivfcl}(\text{ivfint}(\overline{L}_1^c)) = \text{ivfcl}(\overline{K}_1^c) = \overline{L}_1^c \neq g(\overline{K}_1^c)$. 

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Theorem 3.5 Every IVFSC mapping is an IVFGSPC mapping.
Proof. Assume that $g : X \rightarrow Y$ is an IVFSC mapping, where $X$ and $Y$ be two IVFTSSs. Let $\overline{A}$ be an IVFCS in $X$. Then $g(\overline{A})$ is an IVFSCS in $Y$. Since every IVFSCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPCS in $Y$ and hence $g$ is an IVFGSPC mapping.

Remark 3.6 The converse of the above theorem 3.5 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $\overline{K} = \{< a, [0.7, 0.8] \}, < b, [0.8, 0.9] \}$. Then $\overline{K} \subseteq \overline{I}$. By hypothesis, let $\overline{A}$ be an IVFCS in $X$. Then $g(\overline{A})$ is an IVFSCS in $Y$. Since every IVFSCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPC mapping. Let $\overline{K}$ be an IVFCS in $X$ but $g(\overline{K}^c) = \{< u, [0.2, 0.3] \}, < v, [0.3, 0.4] \}$ is not an IVFSCS in $Y$, because $\text{ivfint}(\text{ivfcl}(g(\overline{K}^c))) = \text{ivfint}(\overline{I}^c) = \overline{I} \supseteq g(\overline{K}^c)$.

Theorem 3.7 Every IVFPC mapping is an IVFGSPC mapping.
Proof. Assume that $g : X \rightarrow Y$ is an IVFPC mapping, where $X$ and $Y$ be two IVFTSSs. Let $\overline{A}$ be an IVFCS in $X$. Then $g(\overline{A})$ is an IVFPCS in $Y$. Since every IVFPCS is an IVFGSPCS, $g(\overline{A})$ is an IVFGSPCS in $Y$ and hence $g$ is an IVFGSPC mapping.

Remark 3.8 The converse of the above theorem 3.7 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $\overline{K} = \{< a, [0.1, 0.3] \}, < b, [0.2, 0.4] \}$. Then $\overline{K} \subseteq \overline{I}$. By hypothesis, let $\overline{A}$ be an IVFCS in $X$ but $g(\overline{K}^c) = \{< u, [0.2, 0.3] \}, < v, [0.3, 0.4] \}$ is not an IVFPCS in $Y$, because $\text{ivfcl}(\text{ivfint}(g(\overline{K}^c))) = \text{ivfcl}(\overline{I}^c) = \overline{I} \supset g(\overline{K}^c)$.

Theorem 3.9 Let $g : X \rightarrow Y$ be an IVFSPC mapping between two IVFTSSs $X$ and $Y$. Then for every IVFS $\overline{A}$ of $X$, $g(\text{ivfcl}(\overline{A}))$ is an IVFSPCS in $Y$.
Proof. Let $\overline{A}$ be any IVFS in $X$. Then $\text{ivfcl}(\overline{A})$ is an IVFCS in $X$. By hypothesis, $g(\text{ivfcl}(\overline{A}))$ is an IVFGSPC in $Y$.

Theorem 3.10 Let $\overline{A}$ be an IVFGCS in $X$. If a mapping $g : X \rightarrow Y$ from an IVFTS $X$ onto an IVFTS $Y$ is both IVF continuous and an IVFSPC, then $g(\overline{A})$ is an IVFGSPCS in $Y$.
Proof. Let $g(\overline{A}) \subseteq \overline{U}$ where $\overline{U}$ is an IVFOS in $Y$. Then $\overline{A} \subseteq g^{-1}(g(\overline{A})) \subseteq g^{-1}(\overline{U})$, where $g^{-1}(\overline{U})$ is an IVFOS in $X$, by hypothesis. Since $\overline{A}$ is an IVFGCS, $\text{ivfcl}(\overline{A}) \subseteq g^{-1}(\overline{U})$ in $X$. This implies $g(\text{ivfcl}(\overline{A})) \subseteq g^{-1}(\overline{U}) = \overline{U}$. But $g(\text{ivfcl}(\overline{A}))$ is an IVFSPCS in $Y$, since $\text{ivfcl}(\overline{A})$ is an IVFCS in $X$ and by hypothesis. Therefore $\text{ivfspcl}(g(\text{ivfcl}(\overline{A}))) \subseteq \overline{U}$. Now $\text{ivfspcl}(g(\overline{A})) \subseteq \text{ivfspcl}(g(\text{ivfcl}(\overline{A}))) \subseteq \overline{U}$. Hence $g(\overline{A})$ is an IVFGSPCS in $Y$. 

Theorem 3.11 A bijective mapping \( g : X \to Y \) from an \( IVFTS \ X \) into an \( IVFTS \ Y \) is an \( IVFGSPC \) mapping if and only if for every \( IVFS \ \overline{B} \) of \( Y \) and for every \( IVFOS \ \overline{U} \) containing \( g^{-1}(\overline{B}) \), there is an \( IVFGSPOS \overline{A} \) of \( Y \) such that \( \overline{B} \subseteq \overline{A} \) and \( g^{-1}(\overline{A}) \subseteq \overline{U} \).

Proof. Necessity: Let \( \overline{B} \) be any \( IVFS \) in \( Y \). Let \( \overline{U} \) be an \( IVFOS \) in \( X \) such that \( g^{-1}(\overline{B}) \subseteq \overline{U} \). Then \( \overline{U}^c \) is an \( IVFCS \) in \( X \). By hypothesis \( g(\overline{U}^c) \) is an \( IVFGSPCS \) in \( Y \). Let \( \overline{A} = (g(\overline{U}^c))^c \). Then \( \overline{A} \) is an \( IVFGSPOS \) in \( Y \) and \( \overline{B} \subseteq \overline{A} \). Now \( g^{-1}(\overline{A}) = g^{-1}(g(\overline{U}^c))^c = (g^{-1}(g(\overline{U}^c)))^c \subseteq \overline{U} \).

Sufficiency: Let \( \overline{A} \) be any \( IVFCS \) in \( X \). Then \( \overline{A}^c \) is an \( IVFOS \) in \( X \) and \( g^{-1}(g(\overline{A}^c))) \subseteq \overline{A}^c \) where \( g(\overline{A}) \) an \( IVFS \) in \( Y \). By hypothesis, there exists an \( IVFGSPOS \ \overline{B} \) in \( Y \) such that \( g(\overline{A}^c) \subseteq \overline{B} \) and \( g^{-1}(\overline{B}) \subseteq \overline{A}^c \). Therefore \( \overline{A} \subseteq (g^{-1}(\overline{B}))^c \). Hence \( \overline{B}^c \subseteq g(\overline{A}) \subseteq g(g^{-1}(\overline{B}))^c \subseteq \overline{B}^c \). This implies that \( g(\overline{A}) = \overline{B}^c \). Since \( \overline{B}^c \) is an \( IVFGSPCS \) in \( Y \), \( g(\overline{A}) \) is an \( IVFGSPCS \) in \( Y \). Hence \( g \) is an \( IVFGPC \) mapping.

Theorem 3.12 Let \( X \), \( Y \) and \( Z \) be \( IVFTS \)s. If \( g : X \to Y \) is an \( IVFC \) mapping and \( h : Y \to Z \) is an \( IVFGSPC \) mapping, then \( h \circ g \) is an \( IVFGSPC \) mapping.

Proof. Let \( \overline{A} \) be an \( IVFCS \) in \( X \). Then \( g(\overline{A}) \) is an \( IVFCS \) in \( Y \), by hypothesis. Since \( h \) is an \( IVFGSPC \) mapping, \( h(g(\overline{A})) \) is an \( IVFGSPCS \) in \( Z \). Therefore \( h \circ g \) is an \( IVFGSPC \) mapping.

Theorem 3.13 Let \( g : X \to Y \) be a bijection from an \( IVFTS \ X \) to an \( IVFSPT_{1/2} \) space \( Y \). Then the following statements are equivalent:

(i) \( g \) is an \( IVFGSPC \) mapping,

(ii) \( ivfspcl\left(g(\overline{A})\right) \subseteq g(ivfcl(\overline{A})) \) for each \( IVFS \ \overline{A} \) of \( X \).

(iii) \( g^{-1}(ivfspcl(\overline{B})) \subseteq ivfcl\left(g^{-1}(\overline{B})\right) \) for every \( IVFS \ \overline{B} \) of \( Y \).

Proof. (i) \( \Rightarrow \) (ii) Let \( \overline{A} \) be an \( IVFS \) in \( X \). Then \( ivfcl(\overline{A}) \) is an \( IVFCS \) in \( X \). (i) implies that \( g(ivfcl(\overline{A})) \) is an \( IVFGSPC \) in \( Y \). Since \( Y \) is an \( IVFSPT_{1/2} \) space, \( g(ivfcl(\overline{A})) \) is an \( IVFSPCS \) in \( Y \). Therefore \( ivfspcl(g(ivfcl(\overline{A}))) = g(ivfcl(\overline{A})) \). Now \( ivfspcl(g(\overline{A})) \subseteq ivfspcl(g(ivfcl(\overline{A}))) = g(ivfcl(\overline{A})). \) Hence \( ivfspcl(g(\overline{A})) \subseteq g(ivfcl(\overline{A})) \) for each \( IVFS \ \overline{A} \) of \( X \).

(ii) \( \Rightarrow \) (i) Let \( \overline{A} \) be any \( IVFCS \) in \( X \). Then \( ivfcl\left(\overline{A}\right) = \overline{A} \). (ii) implies that \( ivfspcl\left(g(\overline{A})\right) \subseteq g(ivfcl(\overline{A})) = g(\overline{A}). \) But \( g(\overline{A}) \subseteq ivfspcl(g(\overline{A})). \) Therefore \( ivfspcl(g(\overline{A})) = g(\overline{A}). \) This implies \( g(\overline{A}) \) is an \( IVFSPC \) in \( Y \). Since every \( IVFSPCS \) is an \( IVFGSPCS \), \( g(\overline{A}) \) is an \( IVFGSPCS \) in \( Y \). Hence \( g \) is an \( IVFGSPC \) mapping.

(iii) \( \Rightarrow \) (ii) Let \( \overline{B} \) be an \( IVFS \) in \( Y \). Then \( g^{-1}(\overline{B}) \) is an \( IVFS \) in \( X \). Since \( g \) is onto, \( ivfspcl(\overline{B}) = ivfspcl(g(g^{-1}(\overline{B}))) \) and (ii) implies \( ivfspcl\left(g(g^{-1}(\overline{B}))\right) \subseteq g(ivfcl(g^{-1}(\overline{B}))). \) Therefore we have \( ivfspcl(\overline{B}) \subseteq g(ivfcl\left(g^{-1}(\overline{B})\right)). \) Now \( g^{-1}(ivfspcl(\overline{B})) \subseteq g^{-1}(g(ivfcl(g^{-1}(\overline{B})))) = ivfcl\left(g^{-1}(\overline{B})\right), \) since \( g \) is one to one. Hence \( g^{-1}(ivfspcl(\overline{B})) \subseteq ivfcl\left(g^{-1}(\overline{B})\right). \)

(iii) \( \Rightarrow \) (ii) Let \( \overline{A} \) be any \( IVFS \) in \( X \). Then \( g(\overline{A}) \) is an \( IVFS \) in \( Y \). Since \( f \) is one to one,
(iii) implies that \( g^{-1}(ivfspcl(g(\overline{A}))) \subseteq ivfcl(g^{-1}(g(\overline{A}))) = ivfcl(\overline{A}) \). Therefore \( g(g^{-1}(ivfspcl(g(\overline{A})))) \subseteq g(ivfcl(\overline{A})) \). Since \( g \) is onto \( ivfspcl(g(\overline{A})) = g(g^{-1}(ivfspcl(g(\overline{A})))) \) \( \subseteq g(ivfcl(\overline{A})) \).

**Theorem 3.14** Let \( g : X \rightarrow Y \) be bijective mapping, where \( X \) is an IVFTS and \( Y \) is an IVFSPT\(_{1/2} \) space. Then the following statements are equivalent:

(i) \( g \) is an IVFGSPC mapping,

(ii) \( g \) is an IVFGSPO mapping,

(iii) \( g(ivfint(\overline{B})) \subseteq ivfcl(ivfint(ivfcl(g(\overline{B})))) \) for every IVFS \( \overline{B} \) in \( X \).

**Proof.** (i) \( \Leftrightarrow \) (ii) is obvious.

(ii) \( \Rightarrow \) (iii) Let \( \overline{B} \) be an IVFS in \( X \). Then \( ivfint(\overline{B}) \) is an IVFOS in \( X \). By hypothesis \( g(ivfint(\overline{B})) \) is an IVFGSPO in \( Y \). Since \( Y \) is an IVFSPT\(_{1/2} \) space, \( g(ivfint(\overline{B})) \) is an IVFGSPO in \( Y \). Therefore \( g(ivfint(\overline{B})) \subseteq ivfcl(ivfint(ivfcl(g(ivfint(\overline{B})))))) \subseteq ivfcl(ivfint(ivfcl(g(\overline{B})))) \).

(iii) \( \Rightarrow \) (i) Let \( \overline{A} \) be an IVFCS in \( X \). Then \( \overline{A}^c \) is an IVFOS in \( X \). By hypothesis, \( g(ivfint(\overline{A}^c)) = g(\overline{A}^c) \subseteq ivfcl(ivfint(ivfcl(g(\overline{A}^c))))) \). That is \( ivfint(ivfcl(ivfint(g(\overline{A}^c)))) \subseteq g(\overline{A}) \). This implies \( g(\overline{A}) \) is an IVFB\( CS \) in \( Y \) and hence an IVFGSPCS in \( Y \). Therefore \( g \) is an IVFGSPC mapping.

**Theorem 3.15** Let \( g : X \rightarrow Y \) be bijective mapping, where \( X \) is an IVFTS and \( Y \) is an IVFSPT\(_{1/2} \) space. Then the following statements are equivalent:

(i) \( g \) is an IVFGSPC mapping,

(ii) \( g(\overline{B}) \) is an IVFGSPCS in \( Y \) for every IVFS \( \overline{B} \) in \( X \),

(iii) \( ivfint(ivfcl(ivfint(g(\overline{B})))) \subseteq g(ivfcl(\overline{B})) \) for every IVFS \( \overline{B} \) in \( X \).

**Proof.** (i) \( \Leftrightarrow \) (ii) is obvious.

(ii) \( \Rightarrow \) (iii) Let \( \overline{B} \) be an IVFS in \( X \). Then \( ivfcl(\overline{B}) \) is an IVFCS in \( X \). By hypothesis \( g(ivfcl(\overline{B})) \) is an IVFGSPCS in \( Y \). Since \( Y \) is an IVFSPT\(_{1/2} \) space, \( g(ivfcl(\overline{B})) \) is an IVFGSPCS in \( Y \). Therefore \( g(ivfcl(\overline{B})) \supseteq ivfint(ivfcl(ivfint(g(ivfcl(\overline{B})))))) \supseteq ivfint(ivfcl(ivfint(g(\overline{B})))) \).

(iii) \( \Rightarrow \) (i) Let \( \overline{A} \) be an IVFCS in \( X \). By hypothesis, \( g(ivfcl(\overline{A})) = g(\overline{A}) \supseteq ivfint(ivfcl(ivfint(g(\overline{A})))) \). This implies \( g(\overline{A}) \) is an IVFB\( CS \) in \( Y \) and hence an IVFGSPCS in \( Y \). Therefore \( g \) is an IVFGSPC mapping.

**Theorem 3.16** Let \( X \) and \( Y \) be IVFTSSs. A mapping \( g : X \rightarrow Y \) is an IVFGSPC mapping if \( g(ivfspint(\overline{A})) \subseteq ivfspint(g(\overline{A})) \) for every \( \overline{A} \) in \( X \).

**Proof.** Let \( \overline{A} \) be an IVFOS in \( X \). Then \( ivfint(\overline{A}) = \overline{A} \). Now \( g(\overline{A}) = g(ivfint(\overline{A})) \subseteq g(ivfspint(\overline{A})) \subseteq ivfspint(g(\overline{A})) \), by hypothesis. But \( ivfspint(g(\overline{A})) \subseteq g(\overline{A}) \). Therefore \( g(\overline{A}) \) is an IVFSPOS in \( Y \). That is \( g(\overline{A}) \) is an IVFGSPOS in \( Y \). Hence \( g \) is an IVFGSPC mapping, by
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Theorem 3.17 Let \( X \) be an IVFTS and \( Y \) be an IVFSPT\(_{1/2} \) space. Let \( g : X \to Y \) be bijection. Then the following statements are equivalent:

(i) \( g \) is an IVFGSPC mapping.

(ii) \( g(ivfint(A)) \subseteq ivfspint(g(A)) \) for each IVFS \( A \) of \( X \).

(iii) \( ivfint(g^{-1}(B)) \subseteq g^{-1}(ivfspint(B)) \) for every IVFS \( B \) of \( Y \).

Proof. (i) \( \Rightarrow \) (ii) Let \( g \) be an IVFGSPC mapping. Let \( A \) be any IVFS in \( X \). Then \( ivfint(A) \) is an IVFOS in \( X \). Now \( g(ivfint(A)) \) is an IVFGSPOS in \( Y \), by theorem 3.14. Since \( Y \) is an IVFSPT\(_{1/2} \) space, \( g(ivfint(A)) \) is an IVFPOS in \( Y \). Therefore \( ivfspint(g(ivfint(A))) = g(ivfint(A)) \). Now \( g(ivfint(A)) \subseteq ivfspint(g(ivfint(A))) \subseteq ivfspint(g(A)) \).

(ii) \( \Rightarrow \) (iii) Let \( B \) be an IVFS in \( Y \). Then \( g^{-1}(B) \) is an IVFS in \( X \). By (ii), \( g(ivfint(g^{-1}(B))) \subseteq ivfspint(g(g^{-1}(B))) \subseteq ivfspint(B) \). Now \( ivfint(g^{-1}(B)) \subseteq g^{-1}(ivfspint(g^{-1}(B))) \subseteq ivfspint(B) \).

(iii) \( \Rightarrow \) (i) Let \( A \) be an IVFOS in \( X \). Then \( ivfint(A) = A \) and \( g(A) \) is an IVFS in \( Y \). By (iii), \( ivfint(g^{-1}(A)) \subseteq g^{-1}(ivfspint(g(A))) \). Now \( \overline{A} = ivfint(A) \subseteq ivfspint(g^{-1}(g(A))) \subseteq ivfspint(g(A)) \). Therefore \( g(A) \subseteq g^{-1}(ivfspint(g(A))) \subseteq ivfspint(g(A)) \subseteq g(A) \). Therefore \( ivfspint(g(A)) = g(A) \) is an IVFPOS in \( Y \) and hence an IVFGSPOS in \( Y \). Thus \( g \) is an IVFGSPC mapping, by theorem 3.14.

References