Fuzzy Semi-Pre-Generalized Super Closed Sets

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Abstract: In this paper, a new class of sets called fuzzy semi-pre-generalized super closed sets is introduced and its properties are studied and explore some of its properties.

Keywords: Fuzzy topology, fuzzy super closure, fuzzy super interior, fuzzy super closed set, fuzzy super open set, fuzzy super closed set, fuzzy super generalized closed set.

1. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family {Aα: α∈A} of fuzzy sets of X is defined by to be the mapping sup Aα (resp. inf Aα). A fuzzy set A of X is a fuzzy set defined by x β ∈ X such that A(x) + B(x) > 1 .A ≤ B if and only if [A_oB].

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy super open subsets of A.

Definition 1.1[5]:- Let (X,τ) fuzzy topological space and A⊆X then

1. Fuzzy Super closure scl(A)={x∈X:cl(U)∩A≠ϕ}
2. Fuzzy Super interior sint(A)={x∈X:cl(U)≤A≠ϕ}

Definition 1.2[5]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a) Fuzzy super closed if scl(A) ≤ A.
(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space (X,τ), then A ⊔ B is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space (X,τ) may not be fuzzy super closed.

Definition 1.3[1,5,6,7]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:
(a) fuzzy semi super open if there exists a super open set O such that O≤ A ≤ cl(O).
(b) fuzzy semi super closed if its complement 1-A is fuzzy semi super open.

Remark 1.4[1,5,7]:- Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true.

Definition 1.5[4]:- The intersection of all fuzzy super closed sets which contains A is called the semi super closure of a fuzzy set A of a fuzzy topological space (X,τ). It is denoted by scl(A).

Definition 1.5[3,8,9,10, 11]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:
1. fuzzy g- super closed if cl(A) ≤ G whenever A ≤ G and G is super open.
2. fuzzy g- super open if its complement 1-A is fuzzy g- super closed.
3. fuzzy sg- super closed if scl(A) ≤ O whenever A ≤ O and O is fuzzy semi super open.
4. fuzzy sg- super open if its complement 1-A is sg- super closed.
5. fuzzy gs- super closed if scl(A) ≤ O whenever A ≤ O and O is fuzzy super open.
6. fuzzy gs- super open if its complement 1-A is gs- super closed.

Remark 1.5[10,11]:- Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs- super open) but the converses may not be true.
Remark 1.6[10,11]:- Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed
(resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super
closed (resp. gs-super open) but the converses may not be true.

Definition 1.6[3,8,9,10,11] A fuzzy set A of (X, τ) is called:
(1) Fuzzy semi super open (briefly, Fs-super open) if A \subseteq \text{cl}(\text{int}(A)) and a fuzzy semi super closed (briefly, Fs-super closed) if \text{int}(\text{cl}(A)) \subseteq A.
(2) Fuzzy pre super open (briefly, Fp-super open) if A \subseteq \text{int}(\text{cl}(A)) and a fuzzy pre super closed (briefly, Fp-super closed) if \text{cl}(\text{int}(A)) \subseteq A.
(3) Fuzzy α super open (briefly, Fa-super open) if A \subseteq \text{IntCl}(\text{Int}(A)) and a fuzzy α-super closed (Briefly, Fa-super closed) if \text{cl}(\text{int}(\text{cl}(A))) \subseteq A.
(4) Fuzzy semi-pre super open (briefly, Fspg-super open) if A \subseteq \text{cl}(\text{int}(\text{cl}(A))) and a fuzzy semi-pre super closed (briefly, Fspg-super closed) if \text{int}(\text{cl}(\text{int}(A))) \subseteq A. By FSPGc (X, τ), we denote the family of all fuzzy semi-pre super open sets of fts X.

The semi closure (resp. α-super closure, semi-pre super closure of a fuzzy set A of (X, τ) is the intersection of all Fs-super closed (resp. Fa-super closed, Fspg-super closed) sets that contain A and is denoted by \text{sc}(A) (resp. α cl(A) and sccl(A)).

Definition 1.7. [3,8,9,10,11]:- A fuzzy set A of (X, τ) is called:
(1) Fuzzy generalized super closed (briefly, Fg-super closed) if \text{cl}(A) \subseteq H, whenever A \subseteq H and H is fuzzy super open set in X;
(2) Generalized fuzzy semi super closed (briefly, gFs-super closed) if \text{sc}(A) \subseteq H, whenever A \subseteq H and H is Fspg-super open set in X.
(3) Fuzzy generalized semi super closed (briefly, Fgs-super closed) if \text{sc}(A) \subseteq H, whenever A \subseteq H and H is fuzzy super open set in X;
(4) Fuzzy α generalized super closed (briefly, Fag-super closed) if α cl(A)\subseteq H, whenever A \subseteq H and H is fuzzy super open set in X;
(5) Fuzzy generalized α-super closed (briefly, Fgα-super closed) if α cl(A) \subseteq H, whenever A \subseteq H and H is Fa-super open set in X;
(6) Fuzzy generalized semi-pre super closed (briefly, Fgs-p-super closed) if sccl(A) \subseteq H, whenever A \subseteq H and H is fuzzy semi-pre super open set in X.

Definition 1.8. [3,8,9,10,11]:- A fuzzy topology space (X, τ) is said to be fuzzy semi connected (briefly, Fs-connected) iff the only fuzzy sets which are both Fs-super open and Fs-super closed sets are 0 and 1.

II. Fspg-Super closed sets

Definition 2.1.- A fuzzy set A of (X, τ) is called fuzzy semi-pre-generalized super closed (briefly, Fspg-super closed) if \text{scCl}(A) \subseteq H, whenever A \subseteq H and H is Fspg-super open in X. By FSPGC (X, τ), we denote the family of all fuzzy semi-pre-generalized super closed sets of fts X.

Lemma 2.1.- Every Fp-super closed, gFs-super closed, Fspg-super closed sets are Fspg-super closed and every Fspg-super closed set is Fspg-super closed but the converse may not be true in general. For,

Example 2.1.- Let X = \{a, b\} and Y = \{x, y, z\} and fuzzy sets A, B, E, H, K and M be defined by; A(a) = 0.3, A(b) = 0.4; B(a) = 0.4, B(b) = 0.5; E(a) = 0.3, E(b) = 0.7; H(a) = 0.7, H(b) = 0.6; K(x) = 0.1, K(y) = 0.2, K(z) = 0.7; M(x) = 0.9, M(y) = 0.2, M(z) = 0.5. Let τ = \{\emptyset, A, 1\}, σ = \{0, E, 1\} and γ = \{0, K, 1\}. Then B is Fspg-super closed in (X, τ) but not Fp-super closed; M is Fspg-super closed in (Y, τ) but not gFs-super closed because:
If we consider a fuzzy set T(x) = 0.9, T(y) = 0.2, T(z) = 0.7, then clearly scCl(M) \nsubseteq T where as M \subseteq T and T is fuzzy semi super open in (Y, σ) and H is Fspg-super closed in (X, τ) but neither Fspg-super closed because:
If we consider a fuzzy set L(a) = 0.8, L(b) = 0.7, then clearly scCl(H) \nsubseteq L where as H \subseteq L and L is fuzzy semi super open in (X, τ) nor Fspg-super closed because int(cl(cl(H))) \nsubseteq H.

Theorem 2.1.- If A is fuzzy semi super open and Fspg-super closed in (X, τ), then A is a Fsp super closed in (X, τ).
Proof.- Since A \subseteq A and A is fuzzy semi super open and Fspg-super closed, then scCl(A) \subseteq A. Since A \subseteq scCl(A), we have A = scCl(A) and thus A is a Fsp-super closed set in X.

Theorem 2.2.- A fuzzy set A of (X, τ) is Fspg-super closed if AqE \Rightarrow scCl(A)qE, for every Fsp-super closed set E of X.
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Proof. (Necessity.):- Let E be a Fs- super closed set of X an AqE. Then A ≤ 1 − E and 1 − E is Fs-open in X which implies that spcl(A) ≤ 1 − E as A is Fs-pg super closed. Hence, spcl(A)qE.

(Sufficiency.):- Let H be a Fs- super open set of X such that A ≤ H. Then Aq(1 − H)
and 1−H is Fs- super closed in X. By hypothesis, spcl(A)q(1−H) implies spcl(A) ≤ H. Hence, A is Fs-pg super closed in X.

Theorem 2.3.- Let A be a Fs-pg super closed set of (X, τ) and xp be a fuzzy point of X such that xpq spcl(A) then spcl(xp)qA.

Proof.- If spcl(xp)qA then A ≤ spcl(xp) and so spcl(A) ≤ 1−spcl(xp) ≤ 1−xp because 1−spcl(xp) is Fs- super open and A is Fs-pg super closed in X. Hence, xpq spcl(A), a contradiction.

Theorem 2.4.- If A is a Fs-pg super closed set of (X, τ) and A ≤ B⊆spcl(A), then B is a Fs-pg super closed set of (X, τ).

Proof:- Let H be a Fs- super open set of (X, τ ) such that B ≤ H. Then A ≤ H. Since A is Fs-pg super closed, it follows that spcl(A) ≤ H. Now, B ≤ spcl(A) implies spcl(B)⊆spcl(spcl(A)) = spcl(A). Thus, spcl(B) ≤ H. This proves that B is also a Fs-pg super closed set of (X, τ).

Definition 2.2.- A fuzzy set A of (X, τ ) is called fuzzy semi- pre-generalized super open (briefly, Fs-pg super open) iff (1−A) is Fs-pg super closed in X. That is, A is Fs-pg super open iff E ≤ sp Int(A) whenever E ≤ A and E is a Fs- super closed set in X. By FSPGO (X, τ), we denote the family of all fuzzy semi-pre-generalized super open sets of its X.

Lemma2.2.- Every Fp- super open, gFs- super open, Fsp- super open sets are Fs-pg super open and every Fspg super open set is Fs-pg super open but not conversely.

Theorem 2.5.- FSPSO(X, τ) ≤ FSPGSO(X, τ).

Proof.- Let A be any fuzzy semi-pre super open set in X. Then, 1 − A is Fs-p super closed and hence Fs-pg super closed by Lemma.1. This implies that A is Fs-pg super open. Hence, FSPSO (X, τ ) ≤ FSPGSO(X, τ).

Theorem 2.6.- Let A be Fs-pg super open in X and sp Int(A) ≤ B ≤ A, then B is Fs-pg super open.

Proof.- Suppose A is Fs-pg super open in X and sp Int(A) ≤ B ≤ A. Then 1 − A is Fs-pg super closed and 1−A≤1−B≤spcl(1−A). Then 1−B is Fs-pg super closed set by Theorem 2.4. Hence, B is Fs-pg super open set in X.

Proof:- Obvious

References