IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 8, Issue 3 (Sep. - Oct. 2013), PP 40-42
www.iosrjournals.org

Elementary Proof for Fermat' S Last Theorem 01-01-2010

S. Haridasan,
Kadayil House, Pampuram, Kalluvathukkal P.O, Kollam, Kerala,India.Pin:691578 .

I. Introduction

The famous Fermat's Last Theorem was proved, after three and a half centuries, by Prof: Andrew Wiles and his associate Prof: Richard Taylor in 1994. It is highly advanced. There is search for a simple proof. Congruence modulo addition and multiplication theorems, which are common text book matters, comes to our help. This proof, if found valid, offers very simple one that can be understood by UG students as well.

II. Fermat's last theorem .

There is no solution for $a^n + b^n = c^n$ for $n > 2$ and a, b, c integers > 0

III. Proof

First we may try to prove single digit solutions using congruence relations for nonzero positive integers.

3.1 Congruence Modulo addition

Let $A \equiv r_1 \pmod p$, $0 \le r_1 < p$, And $A = a_1 + a_2$

Also $a_1 \equiv b_1 \pmod p$

$a_2 \equiv b_2 \pmod p$

$a_1 + a_2 \equiv (b_1 + b_2) \pmod p$

Or $A \equiv (b_1 + b_2) \pmod p$

$\equiv r_2 \pmod p$, $0 \le r_2 < p$

$r_1 \pmod p \equiv r_2 \pmod p$, That means $r_1 = r_2$

Least Residue before expansion is equal to Least Residue after expansion in Congruence modulo addition for the same (mod p).

Ex: (i)

$35 = 25 + 10$

$35 \equiv 3 \pmod 8$

$25 \equiv 1 \pmod 8$

$10 \equiv 2 \pmod 8$

$25 + 10 \equiv (1 + 2) \pmod 8$

$35 \equiv 3 \pmod 8$

$3 \pmod 8 \equiv 3 \pmod 8$

Therefore the Least Residues are equal

3.2 Congruence modulo Multiplication

Let $A \equiv r_1 \pmod p$, $0 \le r_1 < p$

And $A = a_1 . a2$

Also $a_1 \equiv b_1 \pmod p$, $0 \le b_1 < p$

$a_2 \equiv b_2 \pmod p$, $0 \le b_2 < p$

$a_1 . a_2 \equiv (b_1.b_2) \pmod p$

$A \equiv r_2 \pmod p$, $0 \le r_2 < p$

$r_1 \pmod p \equiv r_2 \pmod p$, So $r_1 = r_2$

For Congruence modulo Multiplication also Least Residue before expansion is equal to Least Residue after expansion for same (mod p)

Ex :(ii) $143 = 13 \times 11$

$143 \equiv 7 \pmod 8$

$13 \equiv 5 \pmod 8$

$11 \equiv 3 \pmod 8$

$13 \times 11 \equiv (5 \times 3) \pmod 8$

$143 \equiv 15 \pmod 8$

$\equiv 7 \pmod 8$

$7 \pmod 8 \equiv 7 \pmod 8$

Therefore the Least Residues are equal

3.3 General case for single digits

$3^3 \equiv 3 \pmod 8$

$3^2 \equiv 1 \pmod 8$

$3^3 \times 3^2 \equiv (3 \times 1) \pmod 8$

$3^5 \equiv 3 \pmod 8$

Again $3^5 \times 3^2 \equiv (3 \times 1) \pmod 8$

$3^7 \equiv 3 \pmod 8$

⇨ ***$x^n \equiv x \pmod 8$ for all x odd integers , n odd integers ≥ 3***

Also $4^3 \equiv 0 \pmod 8$

$4^2 \equiv 0 \pmod 8$

$4^3 \times 4^2 \equiv (0 \times 0) \pmod 8$

$4^5 \equiv 0 \pmod 8$

$4^5 \times 4^2 \equiv (0 \times 0) \pmod 8$

$4^7 \equiv 0 \pmod 8$

⇨ ***$y^n \equiv 0 \pmod 8$, for all y even integers , n odd integers ≥ 3.***

Now for $n = 3, 5, 7, 9$ ….. odd powers

$1^n \equiv 1 \pmod 8$

$2^n \equiv 0 \pmod 8$

$3^n \equiv 3 \pmod 8$

$4^n \equiv 0 \pmod 8$

$5^n \equiv 5 \pmod 8$

$6^n \equiv 0 \pmod 8$

$7^n \equiv 7 \pmod 8$

$8^n \equiv 0 \pmod 8$

$9^n \equiv 1 \pmod 8$

Hence the TABLE [1] below

3.4 Proposition I

For any triplet of the form $a^2 + b^2 = c^2$ this congruence modulo test satisfies. It can be (mod 9), (mod 8), (mod 7) etc.

Ex: (iii) $25^2 = 24^2 + 7^2$

$25^2 \equiv 1 \pmod 8$

$24^2 \equiv 0 \pmod 8$

$7^2 \equiv 1 \pmod 8$

By 3.1

$24^2 + 7^2 \equiv (0 + 1) \pmod 8$

$25^2 \equiv 1 \pmod 8$

$1 \pmod 8 \equiv 1 \pmod 8$ **Here the Least Residue parts are equal**

If $a^2 + b^2 = c^2$ is a solution for a, b, c > 0 and co-prime , we have

$a^2 \equiv a_1 \pmod p$

$b^2 \equiv b_1 \pmod p$

$c^2 \equiv c_1 \pmod p$

Then $(a_1 + b_1) \pmod p \equiv c_1 \pmod p$

$r_1 \pmod p \equiv c_1 \pmod p$

That means $r_1 = c_1$ [by 3.1]

r_1, c_1 being the Least Residues

In general if $a^n + b^n = c^n$ has a solution for n > 2 then it should satisfy the congruence modulo test.

3.5 Proposition II

If $x = y + z$ where $x, y, z \neq 0$

$x^2 = (y + z)^2 = y^2 + 2yz + z^2$, then $x^2 \neq y^2 + z^2$

Similarly when $x = y + z$, then $x^n \neq y^n + z^n$

3.6 Proposition III

For non zero integers x, y

$$1^n + x^n \neq y^n$$

With the help of these propositions we can verify, from the table [1] that for any a, b, c non zero integers and co-prime, $a^n + b^n = c^n$ has no single digit solutions for n= 3, 5, 7, 9, ….. odd powers. Verification from TABLE [1], 1st and 3rd rows:

Numbers	Residues	Solutions	Reason
$1^n + 2^n$ etc		NIL	Prop. III
$2^n + 3^n$	$0 + 3 \Rightarrow 3$	NIL	
$2^n + 4^n$	$0 + 0 \Rightarrow 0$	NIL	Co-prime
$2^n + 5^n$	$0 + 5 \Rightarrow 5$	NIL	
$2^n + 6^n$	$0 + 0 \Rightarrow 0$	NIL	Co-prime
$2^n + 7^n$	$0 + 7 \Rightarrow 7$	NIL	
$2^n + 8^n$	$0 + 0 \Rightarrow 0$	NIL	Co-prime
$2^n + 9^n$	$0 + 1 \Rightarrow 1$	NIL	
$3^n + 4^n$	$3 + 0 \Rightarrow 3$	NIL	
$3^n + 5^n$	$3 + 5 \Rightarrow 8 \Rightarrow 0$	NIL	Prop II
$3^n + 6^n$	$3 + 0 \Rightarrow 3$	NIL	
$3^n + 7^n$	$3 + 7 \Rightarrow 10 \Rightarrow 2$	NIL	
$3^n + 8^n$	$3 + 0 \Rightarrow 3$	NIL	
$3^n + 9^n$	$3 + 1 \Rightarrow 4$	NIL	
$4^n + 5^n$	$0 + 5 \Rightarrow 5$	NIL	
$4^n + 6^n$	$0 + 0 \Rightarrow 0$	NIL	Co-prime
$4^n + 7^n$	$0 + 7 \Rightarrow 7$	NIL	
$4^n + 8^n$	$0 + 0 \Rightarrow 0$	NIL	
$4^n + 9^n$	$0 + 1 \Rightarrow 1$	NIL	
$5^n + 6^n$	$5 + 0 \Rightarrow 5$	NIL	
$5^n + 7^n$	$5 + 7 \Rightarrow 12 \Rightarrow 4$	NIL	
$5^n + 8^n$	$5 + 0 \Rightarrow 5$	NIL	
$5^n + 9^n$	$5 + 1 \Rightarrow 6$	NIL	
$6^n + 7^n$	$0 + 7 \Rightarrow 7$	NIL	
$6^n + 8^n$	$0 + 0 \Rightarrow 0$	NIL	Co-prime
$6^n + 9^n$	$0 + 1 \Rightarrow 1$	NIL	
$7^n + 8^n$	$7 + 0 \Rightarrow 7$	NIL	
$7^n + 9^n$	$7 + 1 \Rightarrow 8 \Rightarrow 0$	NIL	
$8^n + 9^n$	$0 + 1 \Rightarrow 1$	NIL	

Hence there is no single digit solution for $a^n + b^n = c^n$, n = 3, 5, 7, 9,… odd powers. It is enough to prove for prime exponents greater than 2.

Acknowledgement

I am extremely grateful to Dr.Thrivikraman, Former Professor and Head of Department of Mathematics, Cochin University (CUSAT) for the valuable suggestions and guidance and Dr.M.I.Jinnah , visiting Professor IISER Thiruvananthapuram for the precise comments that helped me to present this article.

Table [1]

NUMBERS	1	2	3	4	5	6	7	8	9		P	0	W	E	R	S
	1	4	1	0	1	4	1	0	1		2					
RESIDUES	1	0	3	0	5	0	7	0	1		3	5	7	9	11	…
	1	0	1	0	1	0	1	0	1		4	6	8	10	12	…