IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 8, Issue 3 (Sep. - Oct. 2013), PP 11-14
www.iosrjournals.org

MV- Optimality of Nearest Neighbour Balanced Block Designs using Second order correlated Models for Five Treatments.

B.Ruban Raja[1] , C. Santharam[2]
[1] Department f Statistics, Loyola College, India
[2] Department f Statistics, Loyola College, India

Abstract: *Block designs for observations correlated in one dimension are investigated. Santharam and Ponnusamy (1995, 1996) investigated the universal optimality on Nearest Neighbor Balanced Block Designs (NNBD) using first order and second order correlated models (AR(1), MA(1) , ARMA (1,1) and AR (2), MA(2)). Ruban raja and santharam (2013) investigated the MV-optimality of Nearest Neighbour Balanced Block Designs using AR(1), MA(1) and ARMA (1,1) (First order Auto Regressive, First order Moving Average and First order Auto Regressive Moving average) model for five treatments. In this paper we have investigated MV-optimality of Nearest Neighbour Balanced Block Designs using AR(2) and MA(2) (Second order Auto Regressive and Second order Moving Average) models for five treatments*
Key words: *Auto-regressive Model, Moving Average model, MV-optimality, Optimal experimental design.*

I. Introduction

Serology is a branch of Biometrics which is concerned with the study of virus and viral preparations. Many studies concerned with viral preparation require the arrangement of antigens in a plate so that each antigen has two other antigens as its neighbours. In analysis of such experiment the classical statistical design may not perform efficiently. Therefore REES (1967) introduced neighbouring structure. The following is the experiment considered by REES (1967) for the use of Nearest Neighbour Balanced Block Designs. If the observations available are correlated, the usual assumptions like independence of observations in the analysis of classical comparative experiment may not perform valid. Therefore there is a necessity for the use of NNBD.

In biometrical sciences we can cite many areas where this kind of correlated structure exists. Now consider the viral preparations. Let there be v kinds of antigens to be arranged on b plates, each containing k antigens. Each antigen appears r times (but not necessarily on r different plates) and is a neighbour of every other antigen exactly λ times.
Rees used circular neighbouring block design and he was used incomplete neighbor design ($k<v$) in his experiment.

The parameters of the design are
$v = 9$, b = 5, $k = 5$, $r = 5$, $\lambda = 1$ and the 9 plates are
$P_1 = (5, 6, 4, 1)$, $P_2 = (6, 7, 5, 2)$, $P_3 = (7, 8, 6, 3)$
$P_4 = (8, 9, 7, 4)$, $P_5 = (9, 1, 8, 5)$, $P_6 = (1, 2, 9, 6)$
$P_7 = (2, 3, 1, 7)$, $P_8 = (3, 4, 2, 8)$, $P_9 = (4, 5, 3, 9)$
In the present paper we have taken complete NNBD ($k = v$) with the parametric structures. $v = 5$, b = 5, $k = 5$ $r = 5$, $\lambda = 2$ and investigated the optimality of NNBD (for $\rho_1 and\, \rho_2 = 0.1$;$\rho_1 and\, \rho_2 = 0.2$;…$\rho_1 and\, \rho_2 = 0.9$ where ρ_1 and $\rho_2$is the correlation coefficients) when the errors behaving according to AR(2) and MA(2) models.

II. AR(2) And Ma(2) In Complete Nnbd Design

REES (1967) introduced neighbour design in serology and defined it as a collection of circular blocks in which any two distinct treatments appear as neighbour equally often. UDDIN, N., (2008) has constructed MV – optimality of block design for 3 treatments in $b = 3n \pm 1$ block of each size and under the assumption that blocks behave independently but there is correlation among the observations within the same block according to first order auto regressive process. Let Δ be a class of unary block design for t treatments in which each treatment applied to r plots being arranged in b blocks of size t. let Y be a rt x 1 random vector corresponding to the observations.
We assuming the following model

$$Y_d = Z\beta + X_d\tau + \in + \eta \qquad (1)$$

Where X is the observation-treatment incidence matrix of order rt x t, Z is the observation block incidence matrix of order rt x b, τ and β and vector of treatment and block effects respectively, $\in$ is the random error vector representing local variation in soil fertility with E($\in$) = 0 and Var ($\in$) = $\sigma^2\Sigma$, where Σ is the correlation matrix, η is the additional error vector with E(η) = 0 and Var(η) = $\sigma_\eta^2 I$, representing other sources of variability in plots which are independent of local fertility. The model (1) is called an error-in variables model (BESAG, 1977) and is closely related to the smooth trend plus error model of WILKINSON et al.(1983). This is a general model which gives a better fit in situations where the error structure is non stationary (BESAG, 1977; WILKINSON et al. 1983; PATERSON, 1983). GILL AND SHUKLA, (1985) studied universal optimality of NNBD using AR(1) and MA(1) models for ρ = 0.2, 0.45 and 0.9.

SANTHARAM and PONNUSAMY,(1997) introduced ARMA(1,1) model along with AR(1) and MA(1) and explored the performance of NNBD for ρ = 0.1(0.1)0.9. In the present paper we have investigated MV – Optimality of NNBD using AR(2) and MA(2) models for ρ_1 and ρ_2 = 0.1,0.2,…, 0.9. The two correlation models considered for the error vector $\in$ in (1) are the Second order autoregressive model AR(2) and the Second order moving average model MA(2).

If the errors within a block follow an autoregressive model AR (2) then $\Omega = I_b \otimes M_k$ where M_k is the k $\otimes$ k matrix is given by

$$M_k = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{k-1} \\ r_1 & r_0 & r_1 & \cdots & r_{k-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ r_{k-1} & r_{k-2} & r_{k-3} & \cdots & r_0 \end{bmatrix}$$

The element of M_k are

$$r_0 = (1-\rho_2)/(1-\rho_2)\{(1-\rho_2)^2 - {\rho_1}^2\}$$
$$r_1 = \{{\rho_1}^2/(1-\rho_2)\}\, r_0$$
$$r_2 = \{{\rho_1}^2/(1-\rho_2) + \rho_2\}\, r_0$$

for

$$k \geq 3, \quad r_k = \{\rho_1 r_{k-1} + \rho_2 r_{k-2}\}\, r_0$$

If the errors within a block follow second order moving average model, MA(2) then $\Omega = I_b \otimes N_k$, where N_k is the k x k matrix

$$N_k = \begin{bmatrix} 1+{\rho_1}^2+{\rho_2}^2 & \rho_1+\rho_1\rho_2 & \rho_2 & 0 & \cdots & 0 \\ \rho_1+\rho_1\rho_2 & 1+{\rho_1}^2+{\rho_2}^2 & \rho_1+\rho_1\rho_2 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1+{\rho_1}^2+{\rho_2}^2 \end{bmatrix}$$

III. Information Matrix

In our investigation of MV- optimal design under model (1) we take this correlation into consideration via the following generalized least squares information matrix.

$$C_d = X'_d\Sigma^{-1}X_d - X'_d\Sigma^{-1} Z\,(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}X_d \qquad (2)$$

The above matrix is utilized by several authors (e.g. Martin and Eccleston, 1991; Jin and Morgan,2008; Gill and Shukla, 1985; Kunert,1987;Santharam and Ponnnuswamy,1995,1996,1997; Uddin, 2008a, 2008b) in their investigation of various optimal and highly efficient design.

IV. Variance Of The Generalized Least Squares Estimates Of Treatment Differences

Let C_{dij}denote the (i, j) th element of C_d.

For any $d \in D_b$, the following inequalities hold (see Lee and Jacroux, 1987):

$$\mathrm{Var}_d(\hat{\tau}_i - \hat{\tau}_j) \geq \frac{C_{dii} + C_{djj} + 2C_{dij}}{C_{dii}\, C_{djj} - C_{dij}^2}$$

V. MV – Optimal Designs

A design $d^* \in D$ is said to be MV - Optimal iff

$$\max_{\substack{1 \le i \le s \\ s+1 \le j \le t}} \{Var_{d^*}(\hat{T}_i - \hat{T}_j)\} \le \max_{\substack{1 \le i \le s \\ s+1 \le j \le t}} \{Var_d(\hat{T}_i - \hat{T}_j)\}$$

VI. Mv- Optimality Of Nearest Neighbour Balanced Complete Block Design Using Second Order Correlated Models

Case1 For AR(2)model ($v = 5,\ b = 5, k = 5\ and\ \lambda\ = 2$)

Table 6.1

ρ_1	ρ_2	D_1	D_1^*
0.1	0.1	0.2579927	0.241101
0.2	0.2	0.1492259	0.127459
0.3	0.3	0.0701440	0.053028
0.4	0.4	0.0186203	0.013838
0.5	0.5	0.0013426	0.000515
0.6	0.6	0.1227659	0.035735
0.7	0.7	0.1194538	2.785182
0.8	0.8	0.5545687	0.018352
0.9	0.9	2.4368210	0.121509

Case2 For MA (2) model ($v = 5,\ b = 5, k = 5\ and\ \lambda\ = 2$)

Table 6.2

ρ_1	ρ_2	D_1	D_1^*
0.1	0.1	0.406857	0.387600
0.2	0.2	0.426515	0.380274
0.3	0.3	0.457149	0.375312
0.4	0.4	0.512643	0.370840
0.5	0.5	0.605250	0.366000
0.6	0.6	0.747165	0.361073
0.7	0.7	0.967496	0.357257
0.8	0.8	1.314361	0.355966
0.9	0.9	1.860294	0.358072

VII. Conclusion

From table 6.1 we conclude that the variance of the treatment differences for D_1^* is less than D_1 for $\rho_1 = 0.1, 0.2, 0.3, \dots, 0.9\ and\ \rho_2 = 0.1, 0.2, 0.3, \dots, 0.9$ under AR (2) model, so we conclude that the design D_1^* is MV- Optimal comparing with D_1 .

From table 6.2 we conclude that the variance of the treatment differences for D_1^* is less than D_1 for $\rho_1 = 0.1, 0.2, 0.3, \dots, 0.9\ and\ \rho_2 = 0.1, 0.2, 0.3, \dots, 0.9$ under MA (2) model, so we conclude that the design D_1^* is MV- Optimal comparing with D_1 .

References

[1]. Besag, J., 1977 "Errror-in-variables estimation for Gaussian schemes. Journal Royal Statistical Society, B 39, 73-78.

[2]. Gill, P.S., and Shukla, G.K., 1985 "Efficiency of Nearest Neighbour Balanced Block Designs for Correlated Observations," *Biometrika*. 72, 639 -644.

[3]. Kunert, J., 1987, "Neighbour Balanced Block Designs for Correlated Errors. *Biometrika*. 74, 4, 717 – 724.

[4]. Jin, B. and Morgan, J.P. (2008) " Optimal Saturated Block Designs when observations are correlated, J. Statist. Plann.Infernce, in press, doi:10.1016/j.2006.06.048.

[5]. Lee, K.Y., And Jacroux, M., 1987,"Some Sufficient Conditions Fore-Optimality and Mv-Optimality of Block Designs having Blocks of Unequal Size,"*Ann.Inst.Statist.Math*. 39, 385 – 397.

[6]. Martin, R. J. and Eccleston, J.A. (1991), " Optimal incomplete Block Designs for General Dependence Structures. J. Statist. Plann.Inference., 28, 68 -81.

[7]. Patterson, H. D., 1983, "Discussion of paper by G.N. Wilkinson et al," Journal of Royal Statistical Society, B 45, 175 -180.

[8]. Ruban Raja B., Santharam C., and Ramesh kumar, 2012, "MV – Optimality of Nearest Neighbour Balanced Block Designs using First order and Second order Correlated models," International Journal of Statistika and Mathematika(ISSN: 2277 – 2790 E-ISSN: 2249 – 8605)

[9]. Ruban Raja B., Santharam C., 2013, "MV – Optimality of Nearest Neighbour Balanced Block Designs using First order Correlated models," International Journal of Statistics Analysis, ISSN 2248-9959 Vol.3, pp . 379 -389.

[10]. Santharam, C., Ponnusamy, K.N and Chandrasekar, B., 1996,"Universal Optimality Of Nearest Neighbour Balanced Block Designs using ARMA Model,". *Biometrical.J*. 38, 725 – 730.

[11]. Santharam, C., And Ponnusamy, K.N., 1997, "On the Efficiencies Of Nearest Neighbour Balanced Block Designs with Correlated Error Structure,"*Biometrical J*. 39, 85 – 89.
[12]. Uddin, N., 2008,"Mv – Optimal Designs for Three treatments When Observations are Correlated. The Indian Journal of Statistics," 70-B, 113 – 120.
[13]. Uddin, N., 2008a,"Optimal Block Designs for Three Treatments When Observations Are Correlated. *J.Statist.Plann.Inference*. 138, 1960 – 1966.
[14]. Uddin, N., 2008 b, "MV- Optimal designs for Correlated Errors," Statist Probab. Lett., in press, doi:10.10 (16/j.spl.2008.04.017.
[15]. Wilkinson, G. N., Eckert, S.R., Hancock, T. W., and Mayo, O., 1983, "Nearest neighbor analysis of field experiments," (with discussions), Journal Royal Statistical Society, B 45, 151-211.