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The General formula to find the sum of first n K^{th} Dimensional S sided polygonal numbers and a simple way to find the n-th term of polynomial sequences

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***Abstract**: Here a particular method is made to generate a A Single Formula to find the n^{th} term and sum of n terms of first n K^{th} dimensional S sided Polygonal numbers.*
***Keywords**: Dimensional Polygonal Numbers, Polygonal Numbers,Square Numbers, Triangular Numbers, 3Dimensional Polygonal Numbers,*

I. Introduction

In mathematics, a polygonal number is a number represented as dots or pebbles arranged in the shape of a regular polygon. The dots were thought of as alphas (units).

A triangular number or triangle number, numbers the objects that can form an equilateral triangle. The n^{th} term of Triangle number is the number of dots in a triangle with n dots on a side The sequence of triangular numbers are 1,3,6,10,15,21,28,36,45,55,....

A Square number , numbers the objects that can form a square. The n^{th} square number is the number of dots in a square with n dots on a side.1,4,9,16,25,36,49,64,81....

A Pentagonal number , numbers the objects that can form a regular pentagon. The n^{th} pentagonal number is the number of dots in a pentagon with n dots on a side. 1,5,12,22,35,51,70,92.... Etc

3Dimension: A tetrahedral number ,or triangular pyramidal number, is a number that represents pyramid with triangular base and three sides, called tetrahedron. The n^{th} term of tetrahedral number is the sum of the first n triangular numbers. 1,4,10,20,35,56,84,120,165,....

A Square pyramidal number, is a number that represents a pyramid with a square base and 4 sides. The n^{th} term of Square pyramidal number is the sum of the first n Square numbers. 1,5,14,30,55,91,140,204.... Etc

4Dimension: The n^{th} term of 4D triangular number is the sum of the first n triangular pyramidal numbers. 1,5,15,35....
The n^{th} term of 4D square number is the sum of the first n square pyramidal numbers. 1,6,20,50....
The n^{th} term of 4D S-gonal number is the sum of the first n S-gonal pyramidal numbers. Etc
5Dimension: The n^{th} term of 5D triangular number is the sum of the first n 4D triangular numbers. 1,6,21,56....
The n^{th} term of 5D square number is the sum of the first n 4D square numbers. 1,7,27,77.....
The n^{th} term of 5D S-gonal number is the sum of the first n 4D S-gonal numbers.

The n^{th} term of KD S-gonal numbers is the sum of the first n (k-1)D S-gonal numbers. ,.
Here I am going to introduce a new method, Using this method we can find the n^{th} term and sum of n terms of any different kinds of polynomial sequences.
It is worth noting that using this method we can invent The General Formula to find the k^{th} Dimensional S sided Polygonal numbers n^{th} term and Sum of n terms
This is a formula generated by myself in the above manner

II. The general method to find the n^{th} term and sum of n terms of any polynomial sequences using Polynomial Difference Theorem

Polynomial Difference Theorem
Suppose the n^{th} term of a sequence is a polynomial in n of degree m i.e.$p(n) = a_1n^m + a_2n^{m-1} + a_3n^{m-2} + a_4n^{m-3} + \dots + a_{m+1}$
Then its m^{th} difference will be equal and $(m + 1)^{th}$ difference will be zero.

2.1 Case1:- (When m=1)
Consider a polynomial sequence of power 1, i.e. p(n)=an+b
First term= p(1)=a+b
Second term=P(2)=2a+b

Third term=p(3)=3a+b

	First Term P (1)	Second Term P(2)	Third Term P(3)
	a+b	2a+b	3a+b
First Difference	a	a	
Second Difference	0		

2.2 Case2:- (When m=2)
Consider a polynomial sequence of power 2, i.e. $p(n) = an^2 + bn + c$
First term= p(1)=a+b+c
Second term=P(2)=4a+2b+c
Third term=p(3)=9a+3b+c
Fourth term=p(4)=16a+4b+c

	First Term P(1)	Second Term P(2)	Third Term P(3)	Fourth Term P(4)
	a+b+c	4a+2b+c	9a+3b+c	16a + 4b + c
First Difference	3a+b	5a+b	7a+b	
Second Difference	2a	2a		
Third Difference	0			

2.3 Case3:- (When m=3)
Consider a polynomial sequence of power 3, i.e. $p(n) = an^3 + bn^2 + cn + d$
First term= p(1)=a+b+c+d
Second term=P(2)=8a+4b+2c+d
Third term=p(3)=27a+9b+3c+d
Fourth term=p(4)=64a+16b+4c+d
Fifth term=p(5)=125a+25b+5c+d

	First Term P(1)	Second Term P(2)	Third Term P(3)	Fourth Term P(4)	Fifth Term P(5)
	a+b+c+d	8a+4b+2c+d	27a+9b+3c+d	64a+16b+4c+d	125a+25b+5c+d
D1	7a+3b+c	19a+5b+c	37a+7b+c	61a+9b+c	
D2	12a+2b	18a+2b	24a+2b		
D3	6a	6a			
D4	0				

From the previous experience, the basic knowledge of the polynomial difference theorem can be put in another way.

III. Inverse Polynomial Differnce theorem !

If the m^{th} difference of a polynomial sequence is equal then its n^{th} term will be a polynomial in n of degree m.

First Difference is the difference between two consecutive terms. Second difference is the difference between two neighboring first differences. So on

3.1 Problem 1: Find the n^{th} term of the triangular number sequence1,3,6,10,15,21,28,36,45,55...
Solution
First term=$t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$, $t_5 = 15$ Using polynomial difference theorem, we can find the n^{th} term,

	FirstTerm t_1=P(1)	SecondTerm t_2=P(2)	ThirdTerm t_3=P(3)	FourthTerm t_4=P(4)	FifthTerm t_5
	1	3	6	10	15
D1	2	3	4	5	
D2	1	1	1		
D3	0	0			

Here Second difference is Equal, which means we can represent its n^{th} term as a polynomial of degree 2, i.e. the general representation $t_n = p(n) =an^2 + bn + c$

$t_1 = p(1) = a + b + c = 1$
$t_2 = p(2) = 4a + 2b + c = 3$
$t_3 = p(3) = 9a + 3b + c = 6$
Solving this we get value of a=12; b= 12; c=0;

n^{th} term=p(n) = $\frac{n(n+1)}{2}$

3.2 Problem 2: the n^{th} term of the number sequence $\sum k^4$
solution

Taking $t_n = \sum k^4$

	t_1	t_2	t_3	t_4	t_5	t_6	t_7
	1	17	98	354	979	2275	4676
D1	16	81	256	625	1296	2401	
D2	65	175	369	671	1105		
D3	110	194	302	434			
D4	84	108	132				
D5	24	24					

Let p(n) be the n^{th} term , Since 5^{th} difference is equal, the degree of n^{th} term is 5

i.e.$t_n = p(n) = an^5 + bn^4 + cn^3 + dn^2 + en + f$

t_1=p(1) = a+b+c+d+e+f =1
t_2=p(2) = 32a+16b+8c+4d+2e+f =17
t_3=p(3) = 243a+81b+27c+9d+3e+f =98
t_4=p(4) = 1024a+256b+64c+16d+4e+f=354
t_5=p(5) = 3125a+625b+125c+25d+5e+f=979
t_6=p(6)=7776a+1296b+216c+36d+6e+f=2275
p(2)-p(1)=3a+b=2—-(1)
p(3)-p(2)=5a+b=3—-(2)
(2)-(1)=2a=1
a=12

Solving this we get $p(n) = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

$$\sum_{k=1}^{n} k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

IV. Polygonal Numbers

In mathematics, a polygonal number is a number represented as dots or pebbles arranged in the shape of a regular polygon. The dots were thought of as alphas (units).

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3Dimension
A tetrahedral number, or triangular pyramidal number, is a number that represents pyramid with triangular base and three sides, called tetrahedron. The n^{th} term of tetrahedral number is the sum of the first n triangular numbers. 1,4,10,20,35,56,84,120,165,....

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4Dimension
The n^{th} term of 4D triangular number is the sum of the first n triangular pyramidal numbers. 1,5,15,35....
The n^{th} term of 4D square number is the sum of the first n square pyramidal numbers.1,6,20,50....
The n^{th} term of 4D S-gonal number is the sum of the first n S-gonal
pyramidal numbers. Etc

5Dimension
The n^{th} term of 5D triangular number is the sum of the first n 4D triangular numbers. 1,6,21,56....
The n^{th} term of 5D square number is the sum of the first n 4D square numbers. 1,7,27,77.....
The n^{th} term of 5D S-gonal number is the sum of the first n 4D S-gonal numbers.
The n^{th} term of KD S-gonal numbers is the sum of the first n (k-1)D S-gonal numbers.

4.1 2D Polygonal Numbers
4.1.1 Triangular numbers:-1,3,6,10,15,21,28,36,45
Refer section 3.1

4.1.2 Square Numbers:1,4,9,16,25,36,49,64,81...
t_1=1; t_2=4, t_3=9 , t_4=16 , t_5=25, t_6=36
First term=t_1 = 1, t_2 = 4, t_3 = 9 , t_4 = 16 , t_5 = 25 Using polynomial difference theorem, we can find the n^{th} term,

	t_1	t_2	t_3	t_4	t_5
	1	4	9	16	25
D1	3	5	7	9	
D2	2	2	2		
D3	0				

Here Second difference is Equal, which means we can represent its n^{th} term as a polynomial of degree 2, i.e. the general representation $t_n = p(n) = an^2 + bn + c$
$t_1 = p(1) = a + b + c = 1$
$t_2 = p(2) = 4a + 2b + c = 4$
$t_3 = p(3) = 9a + 3b + c = 9$
Solving this we get n^{th} term= $p(n) = n^2$

4.1.3 Pentagonal Numbers: 1,5,12,22...
t_1=1; t_2=5, t_3=12 , t_4=22 , t_5=35,
First term=t_1 = 1 t_2 = 5, t_3 = 12 , t_4 = 22 , t_5 = 35 Using polynomial difference theorem, we can find the n^{th} term,

	t_1	t_2	t_3	t_4	t_5
	1	5	12	22	35
D1	4	7	10	13	
D2	3	3	3		
D3	0	0			

Here Second difference is Equal, which means we can represent its n^{th} term as a polynomial of degree 2, i.e. the general representation $t_n = p(n) = an^2 + bn + c$
Solving this we get n^{th} term=$p(n) = \frac{n(3n-1)}{2}$

4.1.4 In similar way we can find Equations of n^{th} term of other polygonal Numbers

Triangular Numbers:-1,3,6,10,15,21,28,36,45,55...	$\frac{n(n+1)}{2}$
Square Numbers:-1,4,9,16,25,36,49,64,81,100...	n^2
Pentagonal Numbers:-1,5,12,22,35,51,70,92,117,145...	$\frac{n(3n-1)}{2}$
Hexagonal Numbers:-1,6,15,28,45,66,91,120,153,190...	$n(2n-1)$
Heptagonal Numbers:-1,7,18,34,55,81,112,148,189,235...	$\frac{n(5n-3)}{2}$
Octagonal Numbers:-1,8,21,40,65,96,133,176,225,280...	$n(3n-2)$
Nonagonal Numbers:-1,9,24,46,75,111,154,204,261,325...	$\frac{n(7n-5)}{2}$
Decagonal Numbers:-1,10,27,52,85,126,175,232,297,370...	$n(4n-3)$

Now we can move to another look,**Equation's Equation**
Considering p(3) as 3 sided polygonal numbers (Triangular numbers)
P(4) as 4 sided polygonal numbers (Square Numbers)

P(5) as 5 sided polygonal numbers (Pentagonal numbers)
P(10) as 10 sided polygonal number(Decagonal Numbers)
i.e. p(s) will be s sided polygonal number.

	P(3)	**P(4)**	**P(5)**	**P(6)**
	$\frac{n(n+1)}{2}$	n^2	$\frac{n(3n-1)}{2}$	$n(2n-1)$
First Difference D1	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$	$\frac{n(n-1)}{2}$
Second Difference D2	0	0	0	

Therefore P(s)=as+b
p(3)=3a+b = $\frac{n(n+1)}{2}$
p(4)=4a+b= n^2
p(5)=5a+b = $\frac{n(3n-1)}{2}$
Solving we get a = $\frac{n(n-1)}{2}$
b=n(2 − n)
Therefore $P(s) = \frac{n}{2}[(n-1)s + 4 - 2n]$ **In similar way (As shown above) we can find the curresponding equations of n^{th} term in any dimension !**

4.2 3D Polygonal Numbers (3 Dimensional Polygonal Numbers)

Tetrahedral Numbers:-1,4,10,20,35,56,84...	$\frac{n(n+1)(n+2)}{6}$
Square Pyramidal Numbers 1,5,14,30,55,91,......	$\frac{n(n+1)(2n+1)}{6}$
Pentagonal Pyramidal Numbers 1,6,18,40,75,126,196,288...	$\frac{n^2(n+1)}{6}$
Hexagonal Pyramidal Numbers 1,7,22,50,95,161,252,372...	$\frac{n(n+1)(4n-1)}{6}$
Heptagonal Pyramidal Numbers 1,8,26,60,115,196,308...	$\frac{n(n+1)(5n-2)}{6}$
Octagonal Pyramidal Numbers 1,9,30,70,135,231,364...	$\frac{n(n+1)(2n-1)}{2}$
Nonagonal Pyramidal Numbers 1,10,34,80,155,266,420,624...	$\frac{n(n+1)(7n-4)}{6}$
DecagonalPyramidal Numbers 1,11,38,90,175,301,...	$\frac{n(n+1)(8n-5)}{6}$
S-gonal Pyramidal Numbers	$\frac{n(n+1)}{6}[(n-1)s+5-2n]$

4.3 4D Polygonal Numbers (4 Dimensional Polygonal Numbers)

4D Triangular Numbers 1,5,15,35,70,126,...	$\frac{n(n+1)(n+2)(n+3)}{24}$
4D Square Numbers 1,6,20,50,105,196,......	$\frac{n(n+1)^2(n+2)}{12}$
4D Pentagonal Numbers 1,7,25,65,140,266,......	$\frac{n(n+1)(n+2)(3n+1)}{24}$
4D Hexagonal Numbers 1,8,30,80,175,336...	$\frac{n^2(n+1)(n+2)}{6}$
4D Heptagonal Numbers 1,9,35,95,210,406...	$\frac{n(n+1)(n+2)(5n-1)}{24}$
4D Octagonal Numbers 1,10,40,110,245,476,...	$\frac{n(n+1)(n+2)(3n-1)}{12}$
4D 9-gonal Numbers 1,11,45,125,280,546...	$\frac{n(n+1)(n+2)(7n-3)}{24}$
4D Decagonal Numbers 1,12,50,140,315,616,...	$\frac{n(n+1)(n+2)(2n-1)}{6}$
4D S-gonal Numbers	$\frac{n(n+1)(n+2)}{24}[(n-1)s+6-2n]$

4.4 5D Polygonal Numbers (5 Dimensional Polygonal Numbers)

5D Triangular Numbers1,6,21,56,126,252,462...	$\frac{n}{120}(n+1)(n+2)(n+3)(n+4)$
5D Square Numbers1,7,27,77,182,378,714,.......	$\frac{n}{120}(n+1)(n+2)(n+3)(2n+3)$
5D Pentagonal Numbers1,8,33,98,238,504,966,...	$\frac{n}{120}(n+1)(n+2)(n+3)(3n+2)$
5D Hexagonal Numbers1,9,39,119,294,630,1218...	$\frac{n}{120}(n+1)(n+2)(n+3)(4n+1)$
5D Heptagonal Numbers1,10,45,140,350,756,1470...	$\frac{n}{120}(n+1)(n+2)(n+3)(5n)$
5D Octagonal Numbers1,11,51,161,406,882,1722,...	$\frac{n}{120}(n+1)(n+2)(n+3)(6n-1)$
5D 9-gonal Numbers1,12,57,182,462,1008,1974,...	$\frac{n}{120}(n+1)(n+2)(n+3)(7n-2)$
5D Decagonal Numbers 1,13,63,203,518,1134,2226,...	$\frac{n}{120}(n+1)(n+2)(n+3)(8n-3)$
5D S-gonal Numbers	$\frac{n(n+1)(n+2)(n+3)}{120}[(n-1)s+7-2n]$

In similar strategy we can formulate the general equation for 3D S-gonal,4D S-gonal,5D S-gonal , 6D S-gonal,7D-sgonal, 8D S-gonal, 9D S-gonal, n^{th} term and sum of n terms.

Can we make a most Generalization formula , that is valid for all dimensions all sides and every term ?
Let us take a look on that.

V. Dimensional Polygonal Numbers And Its N^{th} Term!

Dimension	n^{th}term
2D S-GONAL NUMBERS	$\frac{n}{2!}[(n-1)s+4-2n]$
3D S-GONAL NUMBERS	$\frac{n(n+1)}{3!}[(n-1)s+5-2n]$
4D S-GONAL NUMBERS	$\frac{n(n+1)(n+2)}{4!}[(n-1)s+6-2n]$
5D S-GONAL NUMBERS	$\frac{n(n+1)(n+2)(n+3)}{5!}[(n-1)s+7-2n]$
6D S-GONAL NUMBERS	$\frac{n(n+1)(n+2)(n+3)(n+4)}{6!}[(n-1)s+8-2n]$
......	......
......	......
......	......
......	......
KD S-GONAL NUMBERS	$\frac{n(n+1)(n+2)(n+3)(n+4)\dots(n+k-2)}{k!}[(n-1)s+k+2-2n]$

The General formula of k^{th} Dimensional s sided polygonal numbers n^{th} term is

$$P(k,s,n)=\frac{n(n+1)(n+2)(n+3)(n+4)\dots(n+k-2)}{k!}[(n-1)s+k+2-2n]$$

Where
k denotes dimension
s denotes no.of sides
n denotes no.of terms.
The n^{th} term of kD S-gonal numbers is the sum of the first n (k-1)D S-gonal numbers.

So replacing k by k+1 gives the equation of the summation

The General formula to find the sum of first n k^{th} Dimensional s sided polygonal numbers is

$$\sum P(k,s,n)=\frac{n(n+1)(n+2)(n+3)(n+4)\dots(n+k-1)}{(k+1)!}[(n-1)s+k+3-2n]$$

OR

$$P(k,s,n)=\frac{(n+k-2)!}{(n-1)!\ k!}[(n-1)s+k+2-2n]$$

$$\sum P(k,s,n) = \frac{(n+k-1)!}{(n-1)!\ (k+1)!}[(n-1)s+k+3-2n]$$

VI. Some Properties of polygonal Numbers

6.1 2 Dimension

Name	t_1	t_2	t_3	t_4	t_5	t_6	n^{th}Term
Triangular	1	3	6	10	15	21	$\frac{n(n+1)}{2}$
Square	1	4	9	16	25	36	n^2
Pentagonal	1	5	12	22	35	51	$\frac{n(3n-1)}{2}$
Hexagonal	1	6	15	28	45	66	$n(2n-1)$
Heptagonal	1	7	18	31	50	81	$\frac{n(5n-3)}{2}$
Octagonal	1	8	21	40	65	96	$n(3n-2)$

Column Difference	0	1	3	6	10	12	$\frac{n(n-1)}{2}$

Here the difference between two rows is same as the sequence of triangular numbers in such a way that the n^{th} triangular number is the $(n+1)^{th}$ difference.
A diagonal Relation ship
Which means the difference between the equations will be in such a way that
n^{th} term of s-gonal = n^{th} term of triangular number+(s-3)d = $\frac{n}{2}[(n-1)s+4-2n]$

6.2 3 Dimension

Name	t_1	t_2	t_3	t_4	t_5	t_6	n^{th} Term
Triangular	1	4	10	20	35	56	$\frac{n(n+1)(n+2)}{6}$
Square	1	5	14	30	55	91	$\frac{n(n+1)(2n+1)}{6}$
Pentagonal	1	6	18	40	75	126	$\frac{n^2(n+1)}{6}$
Hexagonal	1	7	22	50	95	161	$\frac{n(n+1)(4n-1)}{6}$
Heptagonal	1	8	26	60	115	196	$\frac{n(n+1)(5n-2)}{6}$
Octagonal	1	9	30	70	135	231	$\frac{n(n+1)(2n-1)}{2}$

Column Difference	0	1	4	10	20	35	$\frac{n(n+1)(n-1)}{6}$

The diagonal relationship repeats here also, So n^{th} term of s-gonal =
n^{th} term of tetrahedral number+(s-3)d $= \frac{n(n+1)}{6}[(n-1)s+5-2n]$

6.3 4 Dimension

Name	t_1	t_2	t_3	t_4	t_5	t_6	n^{th} Term
Triangular	1	5	15	35	70	126	$\frac{n(n+1)(n+2)(n+3)}{24}$
Square	1	6	20	50	105	196	$\frac{n(n+1)^2(n+2)}{12}$
Pentagonal	1	7	25	65	140	266	$\frac{n(n+1)(n+2)(3n+1)}{24}$
Hexagonal	1	8	30	80	175	336	$\frac{n^2(n+1)(n+2)}{6}$
Heptagonal	1	9	35	95	210	406	$\frac{n(n+1)(n+2)(5n-1)}{24}$

Column Difference	0	1	5	15	35	70	$\frac{n(n+1)(n+2)(n-1)}{24}$

Same effect repeats here Therefore n^{th} term of s-gonal = n^{th} term of 4d triangular number+(s-3)d= $\frac{n(n+1)(n+2)}{24}[(n-1)s+6-2n]$

6.4 5 Dimension

Name	t_1	t_2	t_3	t_4	t_5	t_6	n^{th} Term
Triangular	1	6	21	56	126	252	$\frac{n}{120}(n+1)(n+2)(n+3)(n+4)$
Square	1	7	27	77	182	378	$\frac{n}{120}(n+1)(n+2)(n+3)(2n+3)$
Pentagonal	1	8	33	98	238	504	$\frac{n}{120}(n+1)(n+2)(n+3)(3n+2)$
Hexagonal	1	9	39	119	294	630	$\frac{n}{120}(n+1)(n+2)(n+3)(4n+1)$
Heptagonal	1	10	45	140	350	756	$\frac{n}{120}(n+1)(n+2)(n+3)(5n)$

Column Difference	0	1	6	21	56	126	$\frac{n}{120}(n+1)(n+2)(n+3)(n-1)$

Same effect repeats here Therefore n^{th} term of s-gonal = n^{th} term of 5d triangular number+(s-3)d $=\frac{n(n+1)(n+2)(n+3)}{5!}[(n-1)s+7-2n]$

Therefore n^{th} term of 5 Dimensional s gonal number is= $\frac{n(n+1)(n+2)(n+3)}{5!}[(n-1)s+7-2n]$

VII. More Sequences

7.1 m Times Sum of Sequences

Numbers	Sum	Notation
1,2,3,4,.....	$\frac{n(n+1)}{2}$	$\sum n$
1,1+2,1+2+3,1+2+3+4,.... ie, 1,3,6,10...etc	$\frac{n(n+1)(n+2)}{3!}$	$\sum(\sum n) = \Sigma^2 n$
1,1+3,1+3+6,1+3+6+10,.. ie, 1,4,10,20,...etc	$\frac{n(n+1)(n+2)(n+3)}{4!}$	$\sum\sum\sum n = \Sigma^3 n$

In general $\Sigma^m n = \frac{n(n+1)(n+2)(n+3)....(n+m)}{(m+1)!}$

Numbers	Sum	Notation
$1^2, 2^2, 3^2, 4^2, 5^2$…..	$\frac{n(n+1)(2n+1)}{6}$	$\sum n^2$
$1^2, 1^2+2^2, 1^2+2^2+3^2, 1^2+2^2+3^2+4^2$... 1,5,14,30,55,91,140,...	$\frac{n(n+1)(n+2)(2n+2)}{4!}$	$\sum(\sum n^2) = \Sigma^2 n^2$
1,1+5,1+5+14,1+5+14+30,... 1,6,20,50,105,196,336....	$\frac{n(n+1)(n+2)(n+3)(2n+3)}{5!}$	$\Sigma\Sigma\Sigma n^2 = \Sigma^3 n^2$
1,7,27,77,182,378,714,...	$\frac{n(n+1)(n+2)(n+3)(n+4)(2n+4)}{6!}$	$\Sigma\Sigma\Sigma\Sigma n^2 = \Sigma^4 n^2$

In general $\Sigma^m n^2 = \frac{n(n+1)(n+2)...(n+m)(2n+m)}{(m+2)!}$

$$\Sigma^m n = \frac{n(n+1)(n+2)(n+3)\dots.(n+m)}{(m+1)!}$$

$$\Sigma^m n^2 = \frac{n(n+1)(n+2)(n+3)\dots.(n+m)(2n+m)}{(m+2)!}$$

$$\Sigma^m n^3 = \frac{n(n+1)(n+2)(n+3)\dots.(n+m)(6n^2+6mn+m^2-m)}{(m+3)!}$$

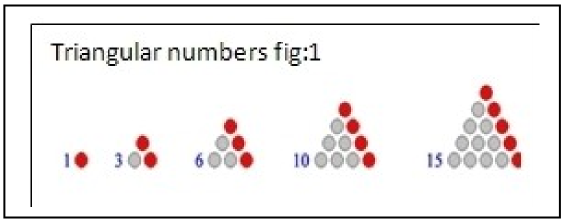
$$\Sigma^m n^4 = \frac{n(n+1)(n+2)(n+3)\dots.(n+m)(24n^3+36mn^2+14m^2n-10mn-5m^2+m^3)}{(m+4)!}$$

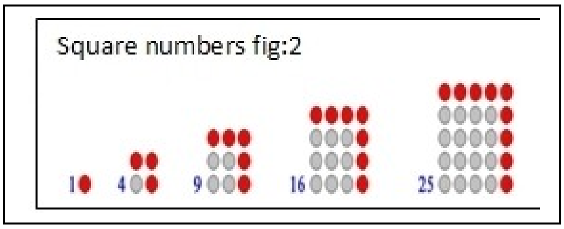
$$\Sigma^m n^5 = \frac{n(n+1)(n+2)(n+3)\ldots.(n+m)}{(m+5)!}$$

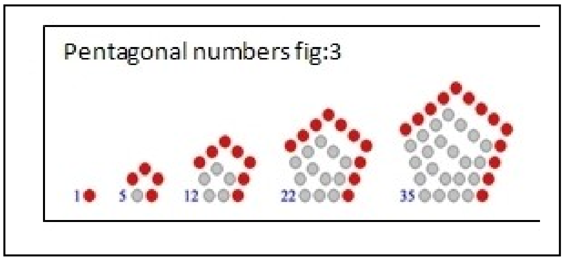
$$x$$

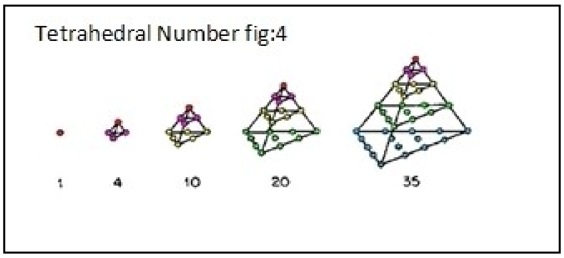
$$(120n^4 + 240mn^3 - 150m^2n^2 - 90mn^2 - 90m^2n + 30m^3n + 4m + 11m^2 - 16m^3 + m^4)$$

VIII. Figures









IX. Conclusion

We have shown that, through this special type method we can find the n^{th} term and sum of n terms of any different kinds of polynomial sequences.
It is worth noting that using this process we can find The General Formula to find the kth Dimensional S sided Polygonal numbers n^{th} term and Sum of n terms

The General formula of k^{th} Dimensional s sided polygonal numbers n^{th} term is

$$P(k,s,n) = \frac{n(n+1)(n+2)(n+3)(n+4)\ldots(n+k-2)}{k!}[(n-1)s+k+2-2n]$$

Where
k denotes dimension
s denotes no.of sides
n denotes no.of terms.

The n^{th} term of kD S-gonal numbers is the sum of the first n (k-1)D S-gonal numbers.
So replacing k by k+1 gives the equation of the summation

The General formula to find the sum of first n k^{th} Dimensional S sided polygonal numbers is

$$\sum P(k,s,n) = \frac{n(n+1)(n+2)(n+3)(n+4)\ldots(n+k-1)}{(k+1)!}[(n-1)s+k+3-2n]$$

OR

$$P(k,s,n) = \frac{(n+k-2)!}{(n-1)!\ k!}[(n-1)s+k+2-2n]$$

$$\sum P(k,s,n) = \frac{(n+k-1)!}{(n-1)!\ (k+1)!}[(n-1)s+k+3-2n]$$

Similarly in addition to the above equation I have generated many polynomial sequence's n^{th} term. Among them some equations generated are

$$\Sigma^m n = \frac{n(n+1)(n+2)(n+3)\ldots.(n+m)}{(m+1)!}$$

$$\Sigma^m n^2 = \frac{n(n+1)(n+2)(n+3)\ldots.(n+m)(2n+m)}{(m+2)!}$$

$$\Sigma^m n^3 = \frac{n(n+1)(n+2)(n+3)\ldots.(n+m)(6n^2+6mn+m^2-m)}{(m+3)!}$$

$$\Sigma^m n^4 = \frac{n(n+1)(n+2)(n+3)\ldots.(n+m)(24n^3+36mn^2+14m^2n-10mn-5m^2+m^3)}{(m+4)!}$$

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