

Development of An inventory model with Volume flexibility, random deterioration and increasing exponentially demand rate

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Abstract: A volume flexible inventory model is developed. This model is developed for deteriorating item by assuming that the deterioration rate is depend on a parameter and time. The production rate is variable, production cost become a function of production rate, unit cost depending upon material cost, labour cost and tool cost. The demand rate is increasing exponentially shortages are allowed. Cost minimization technique used to find out optimal values for different inventory variables.

Key words: volume flexibility, deterioration

I. Introduction:

This study presents a production inventory model with a time dependent random deterioration function and increasing exponentially demand over a fix time horizon with the finite. The shortages allow and excess demand is backlogged. Expressions for optimal parameter are obtained. We also obtained Production scheduling period, maximum inventory level and total average cost.

An inventory system the effect of deterioration plays an important role. Deterioration is derived as decay or damage such that the item cannot be used for its original propose. Foods, pharmaceuticals, chemicals, blood, drugs are a few examples of such items in which sufficient deterioration can take place during the storage period of the units and the importance of this loss must be taken into account when analyzing the system.

In this paper we present a realistic inventory model in which the production rate is variable and demand is an exponentially increasing function time and deterioration is random function says that deterioration of an item depends upon the fluctuation of humidity, temperature, etc. It would be more reasonable and realistic if we assume the deterioration function θ to depend upon a parameter " α " in addition to time t .

When describing optimum policies for deteriorating items **Ghare and Schrader (1963)** proposed a constant rate of deterioration and constant rate demand. In recent year, inventory problem for deterioration items have been widely studied after **Ghare and Schrader (1963)**, **Covert and Philip (1973)** formulated the model for variable deterioration rate with two parameters Weibull disturbance **Goswami and Chaudhuri (1991)**, **Bose et al (1995)** assumed either instantaneous or finite production with different assumption on the pattern of deterioration.

Balkhi and Benkheroot (1996) considered a production a production lot size inventory model with arbitrary production and demand rate depends on the time function.

Bhunja and Maiti's (1977) model to formulate a production inventory model. **Chang and Deve (1999)** investigated an EOQ model allow shortage and backlogging. It is assumed that the backlogging rate is variable and dependent on the length of waiting time for the next replenishment. Recently, many researchers have modified inventory policies by considering the "time proportional partial backlogging rate" such as **Wang (2002)**, **Perumal (2002)**, **Teng et al (2003)**, **Skouri and Papachristos (2003)** and **Kun-Shan et al (2005)** etc.

In the Classical Economic Production Lot Size (EPLS) model, the production rate of a machine is regarded to be pre-determined and inflexible. **Alder and Nanda**, **Sule**, **Axsater** and **Elmaghraby**, **Muth** and **Spearmann** extended the EPLS model to situations where learning effects would induce an increase in the production rate. **Proteus**, **Rosenblat** and **Lee** and **Cheng** considered the EPLS model in an imperfect production process in which the demand would exceed the supply. **Schweitzer** and **Seidmann** adopted, for the first time, the concept of flexibility in the machine production rate and discussed optimization of processing rates for a FMS (flexible manufacturing system). Obviously, the machine production rate is a decision variable in the case of a FMS and then the unit production cost becomes a function of the production rate. **Khouja** and **Mehrez** and **Khouja** extended the EPLS model to an imperfect production process with a flexible production rate. **Silver**, **Moon**, **Gallego** and **Simchi Levi** discussed the effects of slowing down production in the context of a manufacturing equipment of a family of items, assuming a common cycle for all the items. **Gallego** extended this model by removing the stipulation of a common cycle for all the items. But the above studies did not consider the demand rate to be variable. It is a common belief that large piles of goods displayed in a supermarket lead the customers to buy more. Volume flexibility is a major component in a FMS. The

manufacturing flexibility which is capable of adjusting the production rate with the variability in the market demand is known as volume flexibility.

S. Sana and K. S. Chaudhuri, consider a volume flexible manufacturing system for a deteriorating item with an inventory-level-dependent demand rate. They find that the demand rate remains stock-dependent for some time and then becomes a constant after the stock falls down to a certain level. Several factors like limited number of potential customers and their goodwill, price and quality of the goods, locality of shop, etc. can be accounted for the change in the demand pattern

This model is developed for deteriorating item by assuming that the deterioration rate is depend on a parameter and the production rate is variable . The demand rate is increasing exponentially. Cost minimization technique used to find out optimal values for different inventory variables.

II. Assumption and Notation:

The mathematical model of the production inventory problem considered herein is developed on the basic of the following assumptions:-

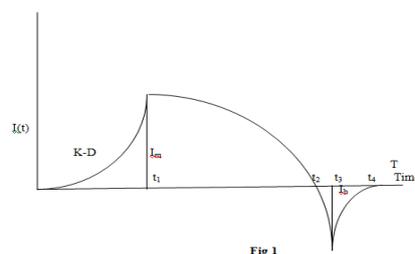
- (a) A single item is considered over a prescribed period of T units of time, which is subject to a time dependent Random deterioration rate.
- (b) Demand rate D (t) is known and increasing exponentially $D(t) = Ae^{\lambda t}$, $t \geq 0$, A is initial demand, λ is a constant governing the increasing rate of demand.
- (c) Production rate K is variable.
- (d) Deterioration of the units is considered only after they have been received into the inventory.
- (e) No replacement or repair of deteriorated item is made during given cycle.
- (f) Shortages are allowed and backlogged.

Notation :

- (1) $I(t)$ = Inventory level at any time t, $t \geq 0$
- (2) $\theta = \theta(\alpha)t$, the items deterioration rate is random.
- (3) I_m = Maximum inventory level.
- (4) I_b = Unfilled order backlog.
- (5) $\eta(K) = R + \frac{G}{K} + HK$ = Unit production cost, $[\eta(K) = R + \frac{G}{K} + HK]$ is depends upon the production rate, where R, G and H are the material cost, labour cost and tool or dye cost respectively.
- (6) C= Set up cost
- (7) C_D = The cost of a deteriorated item.
- (8) C_h = Inventory carrying cost per unit per month.
- (9) C_s = Shortage cost per unit.
- (10) $T = (t_1 + t_2 + t_3 + t_4)$ is the cycle time.
- (11) C_T = The total average cost of system.

III. Mathematical Model :

Initially, the inventory level is start with zero. The production inventory level starts at time t=0 and it reaches at maximum inventory level I_m after t_1 time unit .At that time production is stopped and the inventory level is decreasing continuously and reaches zero at time t_2 ,at this time shortages start developing at time t_3 it reaches to maximum shortage level I_b . This time fresh production start to remove backlog by the time t_4 . Here our aim is to find out the optimal values of t_1, t_2, t_3, t_4, I_m & I_b that minimize the total average cost (C_T) over the time horizon (0,T).



The differential equation governing the stock status during the period $0 \leq t \leq T$ can be written as

$$\frac{dI(t)}{dt} + \theta(\alpha)tI(t) = K - Ae^{\lambda t}, \quad 0 \leq t \leq t_1 \quad \dots (1)$$

$$\frac{dI(t)}{dt} + \theta(\alpha)tI(t) = -Ae^{\lambda t}, \quad 0 \leq t \leq t_2 \quad \dots(2)$$

$$\frac{dI(t)}{dt} = -Ae^{\lambda t}, \quad 0 \leq t \leq t_3 \quad \dots(3)$$

$$\frac{dI(t)}{dt} = K - Ae^{\lambda t}, \quad 0 \leq t \leq t_4 \quad \dots (4)$$

Using the boundary condition

$$I(t) = 0 \text{ at } t = 0, T \text{ and } t_1 + t_2 \quad \dots$$

(5)

$$I(t_2) = I_m \text{ and } -I(t_3) = I_b \quad \dots (6)$$

Solution of equation (1), (2), (3) and (4) by adjusting the constant of integration using boundary condition are given by

$$I(t) = e^{-\theta(\alpha)t^2/2} \left[K \left\{ t + \frac{\theta(\alpha)t^3}{6} \right\} - \frac{Ae^{\lambda t}}{\lambda} \left\{ 1 + \frac{\theta(\alpha)t^2}{2} - \frac{\theta(\alpha)t}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} + \frac{A}{\lambda} \left\{ 1 + \frac{\theta(\alpha)}{\lambda^2} \right\} \right] \quad 0 \leq t \leq t_1 \quad \dots(7)$$

$$I(t) = A \left[\frac{e^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left\{ 1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} - \frac{e^{\lambda t}}{\lambda} e^{-\theta(\alpha)t^2/2} \left\{ 1 + \frac{\theta(\alpha)t^2}{2} - \frac{\theta(\alpha)t}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} \right], 0 \leq t \leq t_2 \quad \dots (8)$$

$$I(t) = \frac{A}{\lambda} (1 - e^{\lambda t}), 0 \leq t \leq t_3 \quad \dots$$

(9)

$$I(t) = K(t - t_4) - \frac{A}{\lambda} (e^{\lambda t} - e^{\lambda t_4}), 0 \leq t \leq t_4 \quad \dots (10)$$

The inventory level of production start initially at time unit $t = 0$ to $t = t_1$ at maximum level I_m is obtained using equation (5), (7) and (8).

$$I_m = e^{-\theta(\alpha)t_1^2/2} \left[K \left\{ t_1 + \frac{\theta(\alpha)t_1^3}{6} \right\} - \frac{Ae^{\lambda t_1}}{\lambda} \left\{ 1 + \frac{\theta(\alpha)t_1^2}{2} - \frac{\theta(\alpha)t_1}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} + \frac{A}{\lambda} \left\{ 1 + \frac{\theta(\alpha)}{\lambda^2} \right\} \right] \\ = A \left[\frac{e^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left\{ 1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} - \frac{1}{\lambda} \left\{ 1 + \frac{\theta(\alpha)}{\lambda^2} \right\} \right] \quad \dots (11)$$

and after time t_1 the production is stopped and stock level is decreasing continuously and become zero at time $t=t_2$ at that time shortages are develop and reaching to I_b at time $t = t_3$ is obtained by equation (6)

$$I_b = \frac{A}{\lambda} (1 - e^{\lambda t_3}) = -Kt_4 - \frac{A}{\lambda} (1 - e^{\lambda t_4}) \quad \dots (12)$$

Thus by equation (11) we observed that t_1 and t_2 are dependent so they are related by the equation

$$t_2 = f(t_1) \quad \dots (13)$$

and by equation (12) t_3 and t_4 are dependent to each other so related by the equation

$$t_3 = g(t_4) \quad \dots (14)$$

Total amount of deteriorated units (I_D) during the period $(0, T)$ is given by

$$\begin{aligned}
 I_D &= \int_0^{t_1} \theta(\alpha)I(t)dt + \int_0^{t_2} \theta(\alpha)I(t)dt \\
 &= \int_0^{t_1} \theta(\alpha)te^{-\theta(\alpha)t^2/2} \left[K \left\{ t + \frac{\theta(\alpha)t^3}{6} \right\} - \frac{Ae^{\lambda t}}{\lambda} \left\{ 1 + \frac{\theta(\alpha)t^2}{2} - \frac{\theta(\alpha)t}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} \right. \\
 &\quad \left. + \frac{1}{\lambda} \left\{ 1 + \frac{\theta(\alpha)}{\lambda^2} \right\} \right] dt + \\
 &\quad \int_0^{t_2} \theta(\alpha)te^{-\theta(\alpha)t^2/2} \left[\frac{Ae^{\lambda t_2}}{\lambda} \left\{ 1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} \right. \\
 &\quad \left. - \frac{Ae^{\lambda t}}{\lambda} \left\{ 1 + \frac{\theta(\alpha)t^2}{2} - \frac{\theta(\alpha)t}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} \right] dt \\
 &= \left[\left\{ K\theta(\alpha) \left(\frac{t_1^3}{6} - \frac{\theta(\alpha)t_1^5}{15} - \frac{\theta^2(\alpha)t_1^7}{84} \right) \right\} + \frac{A\theta(\alpha)}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left(\frac{t_1^2}{2} - \frac{\theta t_1^4}{8} \right) \right. \\
 &\quad \left. - \frac{A\theta(\alpha)e^{\lambda t_1}}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left[\frac{t_1}{\lambda} - \frac{1}{\lambda^2} - \frac{\theta(\alpha)}{2} \left(\frac{t_1^3}{\lambda} - \frac{3t_1^2}{\lambda^2} + \frac{6t_1}{\lambda^3} - \frac{6}{\lambda^4} \right) \right] \right. \\
 &\quad \left. - \frac{A\theta^2(\alpha)e^{\lambda t_1}}{2\lambda} \left[\begin{aligned} &t_1 \left(\frac{t_1^3}{\lambda} - \frac{t_1^2}{\lambda^2} + \frac{6t_1}{\lambda^3} - \frac{6}{\lambda^4} \right) - \frac{\theta(\alpha)}{2} \\ &\left(\frac{t_1^5}{\lambda} - \frac{5t_1^4}{\lambda^2} + \frac{20t_1^3}{\lambda^3} - \frac{60t_1^2}{\lambda^4} + \frac{120t_1}{\lambda^5} - \frac{120}{\lambda^6} \right) \\ &- \frac{2}{\lambda} \left(\frac{t_1^2}{\lambda} - \frac{2t_1}{\lambda^2} + \frac{2}{\lambda^3} \right) + \frac{\theta(\alpha)}{\lambda} \left(\frac{t_1^4}{\lambda} - \frac{4t_1^3}{\lambda^2} + \frac{12t_1^2}{\lambda^3} - \frac{24t_1}{\lambda^4} + \frac{24}{\lambda^5} \right) \end{aligned} \right] \right. \\
 &\quad \left. + \left[\frac{\theta(\alpha)Ae^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left(1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} - \frac{\theta(\alpha)}{\lambda} \right) \frac{t_2^2}{2} \right] \right. \\
 &\quad \left. - \frac{A\theta(\alpha)e^{\lambda t_2}}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left[\frac{t_2}{\lambda} - \frac{1}{\lambda^2} - \frac{\theta(\alpha)}{2} \left(\frac{t_2^3}{\lambda} - \frac{3t_2^2}{\lambda^2} + \frac{6t_2}{\lambda^3} - \frac{6}{\lambda^4} \right) \right] \right. \\
 &\quad \left. - \frac{A\theta^2(\alpha)e^{\lambda t_2}}{2\lambda} \left[\begin{aligned} &\left(\frac{t_2^3}{\lambda} - \frac{3t_2^2}{\lambda^2} + \frac{6t_2}{\lambda^3} - \frac{6}{\lambda^4} \right) - \frac{\theta(\alpha)}{2} \left(\frac{t_2^5}{\lambda} - \frac{5t_2^4}{\lambda^2} + \frac{20t_2^3}{\lambda^3} \right. \\ &\quad \left. - \frac{60t_2^2}{\lambda^4} + \frac{120t_2}{\lambda^5} - \frac{120}{\lambda^6} \right) \\ &- \frac{2}{\lambda} \left(\frac{t_2^2}{\lambda} - \frac{2t_2}{\lambda^2} + \frac{2}{\lambda^3} \right) + \frac{\theta(\alpha)}{\lambda} \left(\frac{t_2^4}{\lambda} - \frac{4t_2^3}{\lambda^2} + \frac{12t_2^2}{\lambda^3} - \frac{24t_2}{\lambda^4} + \frac{24}{\lambda^5} \right) \end{aligned} \right] \right. \\
 &\quad \left. \dots \right] \tag{15}
 \end{aligned}$$

During period $(0, T)$ total inventory carrying (I_H) can be obtained as

$$\begin{aligned}
 I_H &= \int_0^{t_1} I(t)dt + \int_0^{t_2} I(t)dt \\
 &= K \left(\frac{t_1^2}{2} - \frac{\theta(\alpha)t_1^4}{12} - \frac{\theta^2(\alpha)t_1^6}{72} \right) + \frac{A}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left(t_1 - \frac{\theta t_1^3}{6} \right) - \frac{Ae^{\lambda t_1}}{\lambda^2} \\
 &\quad \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) + \theta^2(\alpha) \left(-\frac{10}{\lambda^5} + \frac{10t_1}{\lambda^4} - \frac{5t_1^2}{\lambda^3} + \frac{3t_1^3}{2\lambda^2} - \frac{t_1^4}{4\lambda} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{Ae^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left\{ 1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} t_2 - \\
 & \frac{Ae^{\lambda t_2}}{\lambda^2} \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_2}{\lambda} - \frac{1}{\lambda^2} \right) + \theta^2(\alpha) \left(\begin{aligned} & -\frac{10}{\lambda^5} + \frac{10t_2}{\lambda^4} - \frac{5t_2^2}{\lambda^3} \\ & + \frac{3t_2^3}{2\lambda^2} - \frac{t_2^4}{4\lambda} \end{aligned} \right) \right]
 \end{aligned}
 \tag{16}$$

Total amount of shortage units (I_s) during the period (0, T) is given by

$$I_s = -\int_0^{t_3} I(t)dt + \int_0^{t_4} I(t)dt = \left[\frac{A}{\lambda} \left\{ \left(\frac{e^{\lambda t_3}}{\lambda} - t_3 \right) - \frac{1}{\lambda} \right\} - \frac{Kt_4^2}{2} \right]
 \tag{17}$$

Total production $K_T = \int_0^{t_1} Kdt + \int_0^{t_4} Kdt$
 $= K(t_1 + t_4)$

Production cost $= K\eta(K)(t_1 + t_4)$
 $= (RK + G + HK^2)(t_1 + t_4)$

Hence the total average cost of the inventory system is

$C_T =$ Set up cost + Production cost + deterioration cost + inventory carrying cost + shortage cost

$$= \frac{1}{T} [C + \eta(K)K_T + C_D I_D + C_h I_H + C_S I_S]
 \tag{18}$$

and putting the value of I_D, I_H and I_S we getting the total average cost of the inventory system.

IV. The Approximation Solution Procedure

$$\begin{aligned}
 C_1 = & \frac{C + (RK + G + HK^2)(t_1 + t_4)}{T} + \frac{C_D}{T} \left[\left\{ K\theta(\alpha) \left(\frac{t_1^3}{6} - \frac{\theta(\alpha)t_1^5}{15} - \frac{\theta^2(\alpha)t_1^7}{84} \right) \right\} + \right. \\
 & \left. \frac{A\theta(\alpha)}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left(\frac{t_1^2}{2} - \frac{\theta t_1^4}{8} \right) \right. \\
 & - \frac{C_D A\theta(\alpha) e^{\lambda t_1}}{\lambda T} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left[\frac{t_1}{\lambda} - \frac{1}{\lambda^2} - \frac{\theta(\alpha)}{2} \left(\frac{t_1^3}{\lambda} - \frac{3t_1^2}{\lambda^2} + \frac{6t_1}{\lambda^3} - \frac{6}{\lambda^4} \right) \right] \\
 & - \frac{C_D A\theta^2(\alpha) e^{2\lambda t_1}}{2\lambda T} \left[\begin{aligned} & t_1 \left(\frac{t_1^3}{\lambda} - \frac{t_1^2}{\lambda^2} + \frac{6t_1}{\lambda^3} - \frac{6}{\lambda^4} \right) - \frac{\theta(\alpha)}{2} \left(\frac{t_1^5}{\lambda} - \frac{5t_1^4}{\lambda^2} + \frac{20t_1^3}{\lambda^3} \right. \\ & \left. - \frac{60t_1^2}{\lambda^4} + \frac{120t_1}{\lambda^5} - \frac{120}{\lambda^6} \right) \\ & - \frac{2}{\lambda} \left(\frac{t_1^2}{\lambda} - \frac{2t_1}{\lambda^2} + \frac{2}{\lambda^3} \right) + \frac{\theta(\alpha)}{\lambda} \left(\frac{t_1^4}{\lambda} - \frac{4t_1^3}{\lambda^2} + \right. \\ & \left. \left. \frac{12t_1^3}{\lambda^3} - \frac{24t_1}{\lambda^4} + \frac{24}{\lambda^5} \right) \right] \\
 & + \frac{C_D}{T} \left[\frac{\theta(\alpha)Ae^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left(1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} - \frac{\theta(\alpha)}{\lambda} \right) \frac{t_2^2}{2} \right. \\
 & \left. - \frac{C_D A\theta(\alpha) e^{\lambda t_2}}{\lambda T} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left[\frac{t_2}{\lambda} - \frac{1}{\lambda^2} - \frac{\theta(\alpha)}{2} \left(\frac{t_2^3}{\lambda} - \frac{3t_2^2}{\lambda^2} + \frac{6t_2}{\lambda^3} - \frac{6}{\lambda^4} \right) \right] \right]
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{C_D A \theta^2(\alpha) e^{\lambda t_2}}{2\lambda T} \left[\begin{aligned} & \left(\frac{t_2^3}{\lambda} - \frac{3t_2^2}{\lambda^2} + \frac{6t_2}{\lambda^3} - \frac{6}{\lambda^4} \right) - \frac{\theta(\alpha)}{2} \left(\frac{t_2^5}{\lambda} - \frac{5t_2^4}{\lambda^2} + \frac{20t_2^3}{\lambda^3} \right) \\ & \left(-\frac{60t_2^2}{\lambda^4} + \frac{120t_2}{\lambda^5} - \frac{120}{\lambda^6} \right) \\ & -\frac{2}{\lambda} \left(\frac{t_2^2}{\lambda} - \frac{2t_2}{\lambda^2} + \frac{2}{\lambda^3} \right) + \frac{\theta(\alpha)}{\lambda} \left(\frac{t_2^4}{\lambda} - \frac{4t_2^3}{\lambda^2} + \frac{12t_2^2}{\lambda^3} - \frac{24t_2}{\lambda^4} + \frac{24}{\lambda^5} \right) \end{aligned} \right] \\
 & \frac{C_h}{T} K \left(\frac{t_1^2}{2} - \frac{\theta(\alpha)t_1^4}{12} - \frac{\theta^2(\alpha)t_1^6}{72} \right) + \frac{C_h}{T} \frac{A}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) \left(t_1 - \frac{\theta t_1^3}{6} \right) \\
 & -\frac{Ae^{\lambda t_1}}{\lambda^2} \frac{C_h}{T} \left[\begin{aligned} & \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) + \right. \\ & \left. \theta^2(\alpha) \left(-\frac{10}{\lambda^5} + \frac{10t_1}{\lambda^4} \right) \right] \\ & \left(-\frac{5t_1^2}{\lambda^3} + \frac{3t_1^3}{2\lambda^2} - \frac{t_1^4}{4\lambda} \right) \end{aligned} \right] \\
 & + \frac{C_h}{T} \frac{Ae^{\lambda t_2}}{\lambda} e^{-\theta(\alpha)t_2^2/2} \left\{ 1 + \frac{\theta(\alpha)t_2^2}{2} - \frac{\theta(\alpha)t_2}{\lambda} + \frac{\theta(\alpha)}{\lambda^2} \right\} t_2 - \\
 & \frac{C_h}{T} \frac{Ae^{\lambda t_2}}{\lambda^2} \left[\begin{aligned} & \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_2}{\lambda} - \frac{1}{\lambda^2} \right) + \right. \\ & \left. \theta^2(\alpha) \left(-\frac{10}{\lambda^5} + \frac{10t_2}{\lambda^4} - \frac{5t_2^2}{\lambda^3} + \frac{3t_2^3}{2\lambda^2} - \frac{t_2^4}{4\lambda} \right) \right] \end{aligned} \right] + \frac{C_S}{T} \left[\frac{A}{\lambda} \left(\frac{e^{\lambda t_3}}{\lambda} - t_3 \right) - \frac{1}{\lambda} \right] - \frac{Kt_4^2}{2}
 \end{aligned}$$

In many cases λ and $\theta_0(\alpha)$ are extremely small hence to use Maclaurin series for approximation

$$e^{\lambda t} = 1 + \lambda t + \frac{\lambda^2 t^2}{2} \dots (19)$$

By using equation (19) the total average cost of system

$$\begin{aligned}
 C_1 = & \frac{C + (RK + G + HK^2)(t_1 + t_4)}{T} + \frac{C_D K \theta(\alpha) t_1^3}{6T} + \frac{A \theta(\alpha) t_1^2}{2\lambda} \\
 & - \frac{C_D A \theta(\alpha) (1 + \lambda t_1 + \frac{\lambda^2 t_1^2}{2})}{\lambda T} \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) + \frac{C_D \theta(\alpha) A (1 + \lambda t_2 + \frac{\lambda^2 t_2^2}{2}) t_2^2}{2\lambda T} \\
 & - \frac{C_D A \theta(\alpha) (1 + \lambda t_2 + \frac{\lambda^2 t_2^2}{2})}{\lambda T} \left[\frac{t_2}{\lambda} - \frac{1}{\lambda^2} \right] \left[\frac{C_h}{T} K \left(\frac{t_1^2}{2} - \frac{\theta(\alpha)t_1^4}{12} \right) + \frac{C_h}{T} \frac{A t_1}{\lambda} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \right. \\
 & \left. \frac{A (1 + \lambda t_1 + \frac{\lambda^2 t_1^2}{2})}{\lambda^2} \frac{C_h}{T} \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) \right] \right] \\
 & + \frac{C_h}{T} \frac{A (1 + \lambda t_2 + \frac{\lambda^2 t_2^2}{2}) \theta(\alpha)}{\lambda} \left\{ -\frac{t_2}{\lambda} + \frac{1}{\lambda^2} \right\} t_2 - \\
 & \frac{C_h}{T} \frac{A (1 + \lambda t_2 + \frac{\lambda^2 t_2^2}{2})}{\lambda^2} \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_2}{\lambda} - \frac{1}{\lambda^2} \right) \right] + \frac{C_S}{T} \left[A \left(\frac{1}{\lambda^2} - \frac{t_3^2}{2} \right) - \frac{1}{\lambda} - \frac{Kt_4^2}{2} \right] \\
 & \dots (20)
 \end{aligned}$$

According to equation (20) contain five variables K, t_1, t_2, t_3 and t_4 and these are dependent variable and related by equation (11) and (12). Also we have $C_T > 0$, hence the optimum value of t_1, t_4 and K which minimize total average cost are the solutions of the equations

$$\frac{\partial C_T}{\partial t_1} = 0, \frac{\partial C_T}{\partial t_4} = 0 \text{ and } \frac{\partial C_T}{\partial K} = 0 \quad \dots (21)$$

Provided that satisfy the conditions

$$\begin{aligned} \frac{\partial^2 C_T}{\partial t_1^2} > 0, \frac{\partial^2 C_T}{\partial t_4^2} > 0 \text{ and } \frac{\partial^2 C_T}{\partial t_1^2} \cdot \frac{\partial^2 C_T}{\partial t_4^2} - \frac{\partial^2 C_T}{\partial t_1 \partial t_4} > 0, \\ \frac{\partial^2 C_T}{\partial K^2} > 0, \frac{\partial^2 C_T}{\partial t_4^2} > 0 \text{ and } \frac{\partial^2 C_T}{\partial K^2} \cdot \frac{\partial^2 C_T}{\partial t_4^2} - \frac{\partial^2 C_T}{\partial K \partial t_4} > 0, \\ \frac{\partial^2 C_T}{\partial t_1^2} > 0, \frac{\partial^2 C_T}{\partial K^2} > 0 \text{ and } \frac{\partial^2 C_T}{\partial t_1^2} \cdot \frac{\partial^2 C_T}{\partial K^2} - \frac{\partial^2 C_T}{\partial t_1 \partial K} > 0. \end{aligned}$$

Now, differentiating (20) with respect to t_1, t_4 and K we get

$$\begin{aligned} \frac{\partial C_T}{\partial t_1} &= \frac{(RK + G + HK^2)}{T} + \frac{C_D K \theta(\alpha) t_1^2}{2T} + \frac{A\theta(\alpha) t_1}{\lambda} \\ &\quad - \frac{C_D A \theta(\alpha) (1 + \lambda t_1) \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) - \frac{C_D A \theta(\alpha) (1 + \lambda t_1 + \frac{\lambda^2 t_1^2}{2})}{\lambda^2 T}}{T} \\ &\quad + \frac{C_D \theta(\alpha) A (2f(t_1) + 3\lambda f(t_1)^2 + 2\lambda^2 f(t_1)^3) f'(t_1)}{2\lambda T} \\ &\quad - \frac{C_D A \theta(\alpha) (1 + \lambda f(t_1)) f'(t_1) \left(\frac{f(t_1)}{\lambda} - \frac{1}{\lambda^2} \right) - \frac{C_D A \theta(\alpha) (1 + \lambda f(t_1) + \frac{\lambda^2 f(t_1)^2}{2}) f'(t_1)}{\lambda^2 T}}{T} \\ &\quad + \frac{C_h K \left(t_1 - \frac{\theta(\alpha) t_1^3}{3} \right) + \frac{C_h A}{T} \left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) + \frac{\theta(\alpha) A (1 + \lambda t_1 + \frac{\lambda^2 t_1^2}{2}) C_h}{\lambda^3}}{T} \\ &\quad - \frac{C_K A (1 + \lambda t_1) \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{t_1}{\lambda} - \frac{1}{\lambda^2} \right) \right]}{T \lambda} \\ &\quad + \frac{A C_h}{T} \frac{(1 + \lambda f(t_1) + \frac{\lambda^2 f(t_1)^2}{2}) \theta(\alpha) f'(t_1) \left\{ -\frac{2f(t_1)}{\lambda} + \frac{1}{\lambda^2} \right\} + \frac{\theta(\alpha) A (1 + \lambda f(t_1) + \frac{\lambda^2 f(t_1)^2}{2}) f'(t_1) C_h}{\lambda^3}}{\lambda} \\ &\quad - \frac{A C_h}{T} (1 + \lambda f(t_1)) \theta(\alpha) \left\{ -\frac{f(t_1)^2}{\lambda} + \frac{f(t_1)}{\lambda^2} \right\} f'(t_1) - \frac{C_K A (1 + \lambda f(t_1)) f'(t_1) \left[\left(1 + \frac{\theta(\alpha)}{\lambda^2} \right) - \theta(\alpha) \left(\frac{f(t_1)}{\lambda} - \frac{1}{\lambda^2} \right) \right]}{T \lambda} = 0. \end{aligned}$$

... (22)

$$\frac{\partial C_T}{\partial t_4} = \frac{(RK + G + HK^2)}{T} + \frac{C_S}{T} [-Ag(t_4)g'(t_4) - Kt_4] = 0. \quad \dots (23)$$

and

$$\begin{aligned} \frac{C_D}{T} \left[\left\{ \theta(\alpha) \left(\frac{t_1^3}{6} - \frac{\theta(\alpha) t_1^5}{15} - \frac{\theta^2(\alpha) t_1^7}{84} \right) \right\} \right] + \frac{C_h}{T} \left(\frac{t_1^2}{2} - \frac{\theta(\alpha) t_1^4}{12} - \frac{\theta^2(\alpha) t_1^6}{72} \right) \\ - \frac{C_S K t_4^2}{2T} + \frac{(R + 2HK)(t_1 + t_4)}{T} = 0. \end{aligned}$$

If $\theta(\alpha) \ll 1$ then

$$K = - \frac{\theta(\alpha) [R + C_D t_1^3 + 3C_h f^2(t_1) - 3C_S t_4^2]}{12H(t_1 + t_4)} \quad \dots (24)$$

And optimal production rate,

$$K^* = - \frac{\theta(\alpha) [R + C_D t_1^{*3} + 3C_h f^2(t_1^*) - 3C_S t_4^{*2}]}{12H(t_1^* + t_4^*)} \quad \dots (25)$$

Here we obtain three simultaneous non-linear equation in t_1, t_4 and K . Optimal value of t_1, t_4 and K can be find out by using some suitable computational numerical method and the optimum value of t_2, t_3, I_m, I_b and minimum total average cost ' C_T ' can be obtained from the above equation.

V. Special Cases:

Case i : If $\theta_0(\alpha) = 0$ then the discussed model reduces to production inventory model with out deterioration

Case ii : If $\lambda = 0$, the model reduce to constant demand.

VI. Conclusion :

In the proposed model a production inventory model is formulated for random deteriorating item with a increasing market demand rate with time and production rate is variable. Result in this study can provide a valuable reference for decision markers in planning the production and controlling the inventory. The model proposed here in is resolved by using maclaurin series and cost minimization technique is used to get the approximate expression for total average cost and other parameters & some special cases of model are also discussed. A future study will incorporate more realistic assumption in the proposed model.

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