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Higher Dimensional Dark Energy and Phantom Energy Dilation Brans-Dicke Multiverse with Positive Cosmological Constant

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Abstract: *The present study deals with higher dimensional dilaton-Brans-Dicke cosmology where the multiverse is filled with dark energy or phantom energy having a positive cosmological constant and which contains infinite singularities with eternally Big Rip. It is the extension of 4-dimensional cosmological model discussed by El-Nabulsi Ahmad Rami (2010).*

Keywords: *Anisotropic Generalised Chaplygin gas, Dilation Brans –Dicke scalar field, Dynamical cosmological constant. Dark energy. Phantom energy. Big Rip. Multiverse. Higher dimensions. Dark Energy, Perfect Fluid, Bianchi type-IX Universe.*

I. Introduction:

The Observations [1-6] claims that the universe at large scale is homogeneous and isotropic. Many cosmological observations like Ia Supernova (SNe Ia) and results from Wilkinson Microwave Anisotropic Probe (WMAP) [7-8] and Chandra X-ray observatory [9] etc. have conclude that universe is in accelerated expansion phase. It is believed that this expansion of universe is due to the presence of dark energy. The universe density consists of 4% visible matter, 76% dark energy and 20% dark matter .

In 1998, from the experiments of two different teams of researchers, as the universe is accelerating indicating that the cosmological constant wasn't just zero as expected but had to have a very slight positive value. With this concept of positive cosmological constant, the theory of dark energy has introduced. There are other causes of accelerating universe such as quintessence but the cosmological constant is in most respects the simplest solution. The unusual large value of cosmological constant arising out of the spontaneously broken field theoretic vacuum leads to the search for alternative dynamical dark energy models. To solve the quintessence trouble, lot of works have been done and a new class of cosmological models characterized by exotic type of scalar field have been conjectured for modeling the dark energy in terms of sub-negative pressure. Various models are studied in this respect specially Viscous fluid, K-essence, Chaplygin gas with equation of state $p=-K\rho^{-1}$, $\rho>0$, $K\in R$ [10], exotic matters with generalized equation of state $p=A\rho-B\rho^{a}$, $A,B,a\in R$ [11], generalized Chaplygin gas model with equation of state is $p=-K\rho^{-a}$, $\rho>0$, $0<a\leq 1$ [12], Holographic dark energy and so on [13-17].

The unification of gravity with other fundamental forces in nature is still challenging problem today. Modern developments of superstring theory [18] and Young Hiper Mills super gravity in its field theory limit need higher dimensional space times. Hence in recent years, researchers are interested in theories with higher dimensional space times. The experimental detection of time variation of fundamental constant could provides strong evidence for existence of extra dimensions, was suggested by Alvarez *et. al.* [19] Randjbar-Daemi *et. al.* [20], Marciano [21]. The theory in which gravitation and electromagnetism could be unified in single geometrical structure was proposed by Kaluza [22] and Klein[23]. Thiry[24] and Jordan [25] generalized the Kaluza Klein idea to consider the coefficient of fifth coordinate constant. Number of authors [26-34] have studied the physics of the universe in higher dimensional space times.

Brans-Dicke [35] suggested a scalar tensor theory of gravitation which is developed by various modifications in general relativity called as Brans – Dicke theory of gravitation. This theory is the scalar tensor theory in which the gravitational interaction involves as scalar field ф and the metric tensor g_{ij}. The scalar field ϕ has the dimension of the inverse of the gravitational constant ` G ', where ϕ is accepted to satisfy a scalar wave equation whose source is all the matter in universe. The varying gravitational constant, the non-minimal coupling between the scalar field and geometry, compatibility with equivalence principle, Mach's principle and Dirac's large number hypothesis are some important characteristics of this theory [36]. Naria [37], Belinskii and Khalantnikov [38], Reddy and Rao [39], Banergee and Santos [40], Singh *et.al.* [41], Shriram [42], Shriram and Singh [43], Beram *et. al.* [44], Reddy [45], Reddy *et.al.* [46], Adhav *et. al.* [47], Rao *et. al.* [48] are some authors who have investigated several aspects of this theory.

Recently El-Nabulsi [49-57] introduced the equation of state in a particular Brans-Dicke cosmology controlled by generalized chaplygin gas is given by,

$$p = 3\eta a^m \rho^n - \rho, \quad (\eta, m, n) \in R \tag{1}$$

The cosmological constant and dark energy models provide a possible mechanism to allow for an accelerating universe. Restricting ourselves to constant ω, which treats as a flourishing conjecture. The mathematical forms of the cosmological constant and the dilation scalar field should be fixed as follows :

(1) The cosmological constant Λ is considered as a function of matter density ρ and scalar field ϕ given as $\Lambda\phi = 4\pi\varepsilon\rho$, $\varepsilon \in R$. It is noted that the effective gravitational constant in the theory is $|G| \alpha \phi^{-1}$, which implies $\Lambda \alpha G\rho$, which is obtained in many cosmological theories exploring the decaying vacuum energy densities [59]

(2) We conjecture that $\phi = \xi a,^{\alpha}$ $(\xi, \alpha) \in R$, as for dilation scalar field.

The corresponding modified Einstein field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = \Lambda g_{ij} - 8\pi G T_{ij} - \frac{\omega}{\phi}\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,p}\phi^{,p}\right) - \frac{1}{\phi}(\phi_{i;j} - g_{ij}\,\mathrm{p}\,\phi) \tag{2}$$

$$\mathrm{p}\,\phi = \phi_{;i}^{;i} = \frac{8\pi T^{(matter)}}{2\omega + 3} \tag{3}$$

$$T_{;i}^{ij} = 0, \tag{4}$$

where R_{ij} is a Ricci tensor, R is a Ricci scalar, T_{ij} is an energy momentum tensor.

In this paper, dark energy and phantom energy dilaton Brans-Dicke 4-dimmensional multiverse with positive cosmological constant studied by El-Nabulsi (2010) [58] is extended in higher dimensional spacetimes.

Field Equations:

The higher dimensional FRW metric for homogeneous and isotropic flat universe is given by,

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2 + dv^2), \tag{5}$$

where $a(t)$ is scale factor and t represents the cosmic time, v is the fifth dimension in the space time. Here *0, 1, 2, 3, 4* represent the variables t, x, y, z, v respectively.

The energy momentum tensor in the early universe is given by,

$$T_{ij} = (\rho + p)u_i u_i + g_{ij} p, \tag{6}$$

where ρ is the energy density, g_{ij} is the metric tensor, u_i is the four velocity of fluid which satisfy the condition $u^i u_i = 0$ for $i = 1,2,3,4$ and $u^0 u_o = -1$. (7)

Using equations (5) and (6), the matter tensor is given by

$$T_j^i = diag.(-\rho, p, p, p, p). \tag{8}$$

Using (6), (7), (8) for the component (0,0), equation (2) takes the form

$$H^2 + \frac{2}{3} H \frac{\dot{\phi}}{\phi} = \frac{\omega}{12}\frac{\dot{\phi}^2}{\phi^2} + \frac{8\pi\rho}{6\phi} + \frac{\Lambda}{6}, \tag{9}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and ρ is the matter density.

Adopting the equation of state

$$p = (\gamma - 1)\rho, \text{ where } \gamma \text{ is constant,} \tag{10}$$

the modified Klein-Gordon equation is then

$$\ddot{\phi} + 3H\dot{\phi} + \frac{8\pi}{2\omega + 3}\left[(4p - \rho) - \frac{\phi\Lambda}{4\pi}\right] = 0. \tag{11}$$

The conservation equation of stress-energy tensor $T_{;i}^{ij} = 0$ becomes

$$\dot{\rho} + 4H\gamma\rho = 0$$

which on simplification gives

$$\rho=\left[K-12(1-n)\eta\frac{a^{m}}{m}\right]^{\frac{1}{1-n}}, \tag{12}$$

where K is the constant of integration.

At $t=0$, $a=a_0=1$, $\rho=\rho_0=1$, equation (12) gives $\rho=\left[1+\frac{12(1-n)\eta}{m}-\frac{12(1-n)\eta a^{m}}{m}\right]^{\frac{1}{1-n}}$. (13)

From equation (9), putting $\phi=\xi a,^{\alpha}$ $(\xi,\alpha)\in R$ we get

$$\left[1+\frac{2}{3}\alpha-\frac{\alpha^2}{12}\omega\right]H^2=\frac{8\pi+4\pi\varepsilon}{6\xi a^{\alpha}}\left[1+\frac{12(1-n)\eta}{m}-\frac{12(1-n)\eta}{m}a^{m}\right]^{\frac{1}{1-n}}. \tag{14}$$

On simplification of (11) we get,

$$\omega=\frac{-4\left[4p-\rho(1+\xi)\right]}{\xi\alpha\left[(\alpha+2)\dot{a}^2\,a^{\alpha-2}+a^{\alpha-1}\ddot{a}\right]}-\frac{3}{2}. \tag{15}$$

To get reasonable solution put $n=0$ and $m=\alpha$ in the equation (14)

$$H^2=\frac{(8\pi+4\pi\varepsilon)}{6\xi}\frac{\left(1+\frac{12\eta}{\alpha}\right)}{\left(1+\frac{2\alpha}{3}-\frac{\alpha^2}{12}\omega\right)}a^{-\alpha}-\frac{(8\pi+4\pi\varepsilon)}{\xi\alpha}\frac{2\eta}{\left(1+\frac{2\alpha}{3}-\frac{\alpha^2}{12}\omega\right)}$$

$$=C\,a^{-\alpha}-\Psi, \tag{16}$$

where

$$C=\frac{(8\pi+4\pi\varepsilon)}{6\xi}\frac{\left(1+\frac{12\eta}{\alpha}\right)}{\left(1+\frac{2\alpha}{3}-\frac{\alpha^2}{12}\omega\right)} \tag{17}$$

and

$$\Psi=\frac{(8\pi+4\pi\varepsilon)}{\xi\alpha}\frac{2\eta}{\left(1+\frac{2\alpha}{3}-\frac{\alpha^2}{12}\omega\right)}. \tag{18}$$

Integrating equation (16) with $\alpha>0$, $\Psi>0$

$$a(t)=a_0\left[\cos\left(\left|\frac{\alpha}{2}\right|\sqrt{\psi}(t-t_0)\right)+\sqrt{\frac{C}{\Psi}a_0^{-\alpha}-1}.\sin\left(\left|\frac{\alpha}{2}\right|\sqrt{\Psi}(t-t_0)\right)\right]^{\frac{2}{\alpha}} \tag{19}$$

and for $\alpha<0$, $\Psi>0$

$$a(t)=a_0\left[Cos\left(\left|\frac{\alpha}{2}\right|\sqrt{\Psi}(t-t_0)\right)-\sqrt{\frac{C}{\psi}a_0^{-\alpha}-1}\,Sin\left(\left|\frac{\alpha}{2}\right|\sqrt{\Psi}(t-t_0)\right)\right]^{\frac{2}{|\alpha|}} \tag{20}$$

The scale factor diverges continuously an infinite number of times. For each divergence we get Big Rip singularity which takes place at

$$T = t_0 + \frac{2}{|\alpha|\sqrt{\Psi}} \arctan\left(\frac{C}{\Psi} a_0^{-\alpha} - 1\right)^{\frac{-1}{2}} + \frac{2n\pi}{|\alpha|} \tag{21}$$

with life time given by

$$T_L = \frac{2\pi}{|\alpha|\sqrt{\Psi}} . \tag{22}$$

The scale factor can be written as

$$a(\tau) = a_{\min} (\cos)^{\frac{-2}{|\alpha|}}, \tag{23}$$

with the new time τ covering the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, where each cycling universe reaches the initial and final big rip at the extrema.

The dilation scalar field behaves accordingly like:

$$\phi(\tau) \alpha \, \xi (\cos\tau)^2 . \tag{24}$$

This scenario is similar to dark energy multiverse models [60] but with the major difference that in our scenario, universe is controlled by the periodic equation of state [58]

$$\begin{aligned} p(\tau) &= 3\eta a^{\alpha}(\tau) - \rho(\tau) \\ &= 3\eta\left(1 + \frac{4}{\alpha}\right) a_{\min}^{\alpha} (\cos\tau)^2 - \left(1 + \frac{12\eta}{\alpha}\right) \end{aligned} \tag{25}$$

In order to satisfy restriction that Brans-Dicke parameter is time dependent, it is required to have from equation (15), the equation of state $4p = (1+\in)\rho$. This yields $w = -\frac{3}{2}$, which is the same value obtained in conformal relativity action within the framework of Brans-Dicke cosmology, in compare to the reduced (gravity-dilaton) low-energy-effective superstring action which corresponds to a Brand-Dicke action with Brans-Dicke parameter $\omega \approx -1$. In fact, Brans-Dicke parameter $\omega \approx -\frac{3}{2}$ gives a margin between a standard scalar field and a ghost.

Comparing (21) and the constraint $4p = (1+\varepsilon)\rho$, we obtain

$$12\eta = -\alpha = 5 + \varepsilon \quad \text{and} \quad \omega = -\frac{4+\alpha}{4} = \frac{1+\varepsilon}{4} .$$

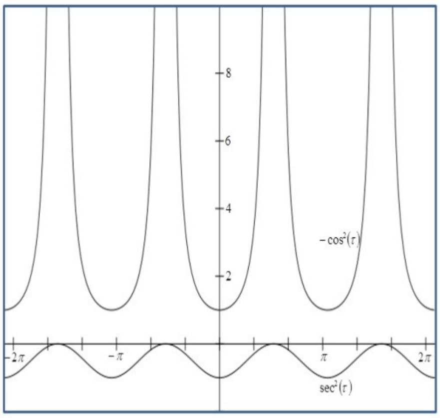

Fig. 1. Variation of the scale factor and the scalar field with τ

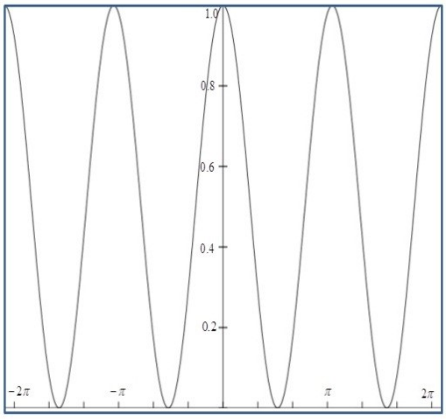

Fig. 2. Variation of the cosmological constant with τ

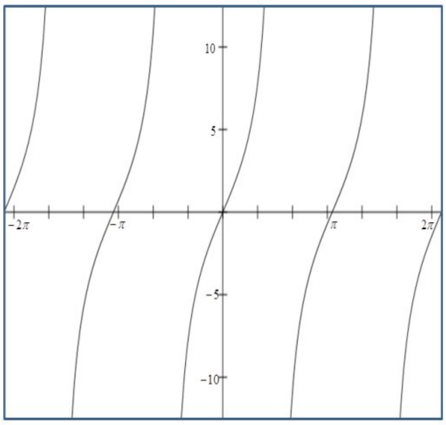

Fig. 3. Variation of the Hubble parameter H = 2tanτ with τ for ε = -3

Therefor for $\varepsilon < -1,$ the universe is dominated by dark energy and cosmological constant $\Lambda = \dfrac{4\pi\varepsilon\rho\, a^{-\alpha}}{\xi}$ is positive unless $\varepsilon < 0,$ i.e. negative scalar field. To illustrate, choose $\varepsilon = -3$ and $\xi = -1$: therefore $\omega = \dfrac{-1}{2}$. The scenario corresponds then to a dark energy multiverse with positive cosmological constant and eternally repeating Big Rip. The scale factor and scalar field vary respectively like ; $a(\tau) = a_{\min}(\cos)^{-2}$ and $\phi(\tau)\ \alpha - (\cos\tau)^2$. We may plot Figures 1, 2 and 3.

If for instance, choose $\varepsilon = -7$ and $\xi = -1$ then $\omega = -\dfrac{3}{2} < -1$ and multiverse is then dominated by phantom energy. For this particular choice, we find $\alpha = 3$. The scenario corresponds in that case to a phantom energy multiverse with positive cosmological constant and eternally repeating Big Rip. The scale factor varies like $a(\tau) = a_{\min}(\cos)^{\frac{-2}{3}}$ whereas the scalar field behaves like $\phi(\tau)\ \alpha - (\cos\tau)^2$. The Hubble parameter is $H = 2\tan\dfrac{\tau}{3}$ and is as well periodic.

II. Conclusion:

A dark energy and phantom energy dilation Brans-Dicke multiverse with positive cosmological constant has been investigated by El Nabulsi (2010), whose work has been extended and studied in five dimensions. An attempt has been made to retain El-Nabulsi (2010) form of the various quantities. The model is controlled by generalized equation of state $p = 3\eta a^{\alpha} - \rho$, a cosmological constant behaving like $\Lambda\phi = 4\pi\varepsilon\rho$ and a dilation scalar field behaving like $\phi = \xi a^{\alpha}$, where $12\eta = -\alpha = (\varepsilon + 5)$ and $\varepsilon < 0$. It is quite interesting to obtain a higher dimensional cosmological multiverse model in which the lambda is positive and not negative, in contrast to those models that rise from string, superstring and supergravity models where the vacuum has negative energy density and hence describe by an AdS space time. As we have $|G|\alpha\phi^{-1}$, then obviously the gravitational coupling constant is well oscillatory in our scenario. We have note that all the results of El Nabulsi (2010) can be obtained from our results by assuming appropriate values of the functions concerned.

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