

## A Note on Vorticity of Unsteady MHD Free Convection and Mass Transfer Flow of Visco-elastic fluid through Porous Medium with Constant Suction and Heat flux

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**Abstract:** This paper investigate the study of vorticity of unsteady MHD free convection and mass transfer Flow of Visco-elastic fluid through porous medium with constant Suction and Heat flux. The effects of the important flow parameters such as Magnetic Parameter ( $M$ ), Grashoff number ( $G_r$ ), Modified Grashoff number ( $G_m$ ), Prandtl number ( $P_r$ ) and Schimdt number ( $S_c$ ) on the vorticity of the flow field are analyzed quantitatively with the help of figures and tables.

**Keywords:** MHD flow, free convection, Porous medium, Suction, Heat flux, Mass transfer, Vorticity.

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### NOMENCLATURE

$u$  = Velocity component

$M$  = Magnetic parameter

$E_c$  = Eckert number

$t$  = Time

$G_r$  = Grashof number for heat transfer

$G_m$  = Grashof number for mass transfer

$S_c$  = Schmidt number

$P_r$  = Prandtl number

$v_0$  = Constant suction

$B_0$  = Magnetic induction

$g$  = Acceleration due to gravity

$k_0$  = Porosity parameter

$C_p$  = Specific heat at the constant pressure

$D$  = Concentration diffusivity

$T_\infty$  = Temperature of the fluid in the free steam

$q$  = Constant heat flux per unit area

$C_\infty$  = Concentration at infinite

$m$  = Mass flux per unit area

### GREEK SYMBOLS

$\beta$  = Coefficient of Volume expansion

$\beta^*$  = Coefficient of Concentration expansion

$\rho$  = Density

$\nu$  = Kinematic Viscosity

$\lambda$  = Thermal Conductivity

$\sigma$  = Electrical Conductivity

$\zeta$  = Vorticity

## I. Introduction

The study of free convection MHD flow of a visco-elastic fluid with mass transfer is of general interest in view of its varied applications in the field of Astrophysical and Geophysical Sciences. Such type of problems finds their use in the atmospheric and oceanic circulations as well. Flow through porous media is helpful in the filtration process and also to maintain the temperature of a heated body.

Satyanarayana *et al.* [17] studied current effect on magnetohydro-dynamic free convection flow past a semi-infinite vertical porous plate with mass transfer. Dulal *et al.* [5] studied perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Attia [2] has considered the Hartmann flow with heat transfer of a Visco-elastic fluid considering the Hall Effect. Attia [3] studied the unsteady laminar flow of incompressible viscous fluid and heat transfer two parallel plates in the presence of a uniform suction and injection considering variable properties.

Kesavaiah, D. *et al.* [10] investigate the effect of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Shakya *et al.* [20] studied a Note on vorticity of unsteady hydromagnetic free convective flow of stratified viscous fluid past a porous vertical plate. Acharya *et al.* [1] have analyzed the magnetic field effects on a free convection and mass transfer flow through porous medium with constant suction and heat flux. Chaudhary and Sahoo [4] studied the unsteady free convective flow and mass transfer of a visco-elastic fluid in a rotating porous medium. Das *et al.* [6] studied the effect of free convection and mass transfer on MHD flow of a rotating elastico-viscous fluid past an infinite vertical porous plate through a porous medium with constant suction and heat flux.

Tak, S. S. and Nain, S. [22] discussed the unsteady free convection through a porous medium of fluctuating permeability bounded by porous vertical plate with constant heat and mass flux. Israel-cookey and Sigalo [7] discussed the unsteady MHD free convection and mass transfer flow past an infinite porous vertical plate with time dependent suction. Singh [18] has studied the MHD free convection flow of a fluid past an accelerated vertical porous plate in a rotating fluid. Raptis and Singh [15] studied the effect of rotation on MHD free convection through a vertical channel in a rotating porous medium. Panda *et al.* [14] have investigated the unsteady free convective flow and mass transfer of a rotating elastico-viscous liquid through a porous media past a vertical infinite porous plate. Kumar and Yadav [9] have studied the unsteady MHD free convection flow through porous medium with heat and mass transfer past a porous vertical moving plate with heat source/sink. Varshney *et al.* [23] discussed the Transpiration effect on unsteady MHD free convection flow through porous medium with heat and mass transfer past a porous vertical moving plate with heat source/sink. Soundalgekar [19] studied the unsteady MHD free convection flow past an infinite vertical flat plate with variable suction. Ram [16] discussed the unsteady MHD free convection flow of a viscous rotating fluid through a porous medium with constant heat flux and heat generation. Johari and Singh [8] discussed the effect of thermal diffusion on unsteady MHD free convection flow and mass transfer through porous medium with radiation and variable permeability in slip flow regime. Kothiyal, A. D. *et al.* [11] discussed vorticity of MHD visco-elastic Boundary layer flow through porous medium with free convection past a continuous moving surface.

Sharma *et al.* [21] studied the unsteady free convection oscillatory coquette flow through a porous medium with periodic wall temperature. Kim *et al.* [12] have studied the unsteady convection flow of micropolar fluids past a vertical porous plate embedded in a porous medium. Ogulu and Makinde [13] studied the unsteady hydromagnetic free convection flow of a dissipative and radiating fluid past a vertical plate with constant heat flux.

The purpose of the present investigation is to the study of vorticity of unsteady MHD free convection mass transfer flow of a viscous incompressible electrically conducting visco-elastic fluid through porous medium with constant suction and heat flux.

## II. Mathematical Formulation

We consider unsteady two dimensional motion of visco-elastic fluid through a porous medium occupying semi-infinite region of space bounded by a vertical infinite surface under the action of uniform magnetic field applied normal to the direction of flow. The X-axis is taken along the surface in the upward direction and Y-axis is taken normal to it. The fluid properties are assumed constant except for the influence of density in the body force term. As the bounding surface is infinite in length, all the variables are function of  $y$ . Now, under the usual Bossinesq's approximation, the flow field is governed by the following equations.

**Equation of Continuity:**

$$\frac{\partial v}{\partial y} = 0 \quad \dots(1.1)$$

**Equation of momentum:**

$$\frac{\partial u}{\partial t_0} + v \frac{\partial u}{\partial y} = \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{\kappa_0} u \quad \dots(1.2)$$

**Equation of energy:**

$$\frac{\partial T}{\partial t_0} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p} \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 T}{\partial y^2} \quad \dots(1.3)$$

**Equation of Concentration:**

$$\frac{\partial C}{\partial t_0} + v \frac{\partial C}{\partial y} = D \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 C}{\partial y^2} \quad \dots(1.4)$$

The initial boundary conditions are:

$$\left. \begin{aligned} t_0 = 0, u = 0, v = -v_0 = \text{constant}, \frac{\partial T}{\partial y} = \frac{-q}{\lambda}, \frac{\partial C}{\partial y} = \frac{-m}{D} \text{ at } y = 0 \\ t_0 > 0, u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \dots(1.5)$$

In view of equations (1.5), equations (1.2), (1.3) and (1.4) can be written as

$$\frac{\partial u}{\partial t_0} - v_0 \frac{\partial u}{\partial y} = \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{\kappa_0} u \quad \dots(1.6)$$

$$\frac{\partial T}{\partial t_0} - v_0 \frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p} \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 T}{\partial y^2} \quad \dots(1.7)$$

$$\frac{\partial C}{\partial t_0} - v_0 \frac{\partial C}{\partial y} = D \left( \nu + \beta \frac{\partial}{\partial t_0} \right) \frac{\partial^2 C}{\partial y^2} \quad \dots(1.8)$$

Introducing the following non-dimensional quantities into (1.6), (1.7), (1.8)

$$u = \frac{u}{v_0}, y^* = \frac{v_0 y}{\nu}, S_c = \frac{\nu}{D}$$

$$M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, G_r = g\beta \frac{q \nu^2}{v_0^2 \lambda}, G_m = g\beta^* \frac{m \nu^2}{v_0^4 D}$$

$$P_r = \frac{\mu C_p}{\lambda}, E_c = \frac{\lambda v_0^3}{\nu q C_p}, t^* = \frac{t_0 v_0^2}{\nu}$$

$$\theta = \frac{T - T_\infty}{q \nu} v_0 \lambda, \alpha = \frac{v_0^2 \kappa_0}{\nu^2}, \phi = \frac{C - C_\infty}{m \nu} v_0 D$$

Equations (1.2), (1.3) and (1.4) after dropping the asterisks (\*) can be written as

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + \beta_0 \frac{\partial^3 u}{\partial t \partial y^2} = \frac{\partial u}{\partial t} + u \left( M + \frac{I}{\alpha} \right) + G_r \theta - G_m \phi \quad \dots(2.0)$$

$$\nu \frac{\partial^2 \theta}{\partial y^2} + P_r \frac{\partial \theta}{\partial y} + \beta_1 P_r \frac{\partial^3 \theta}{\partial t \partial y^2} = P_r \frac{\partial \theta}{\partial t} \quad \dots(2.1)$$

$$\nu \frac{\partial^2 \phi}{\partial y^2} + S_c \frac{\partial \phi}{\partial y} + \beta_2 S_c \frac{\partial^3 \phi}{\partial t \partial y^2} = S_c \frac{\partial \phi}{\partial t} \quad \dots(2.2)$$

The corresponding initial and boundary conditions in non-dimensional form are:

$$\left. \begin{aligned} u = 0, \theta' = -1, \phi' = -1 \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad \dots(2.3)$$

### III. Method Of Solution

Let us choose the solutions of equations (2.0), (2.1) and (2.2) respectively as:

$$\left. \begin{aligned} u(y,t) &= u_0(y) \cdot e^{-nt} \\ \theta(y,t) &= \theta_0(y) \cdot e^{-nt} \\ \phi(y,t) &= \phi_0(y) \cdot e^{-nt} \end{aligned} \right\} \quad \dots(2.4)$$

Substituting expressions (2.4) into equations (2.0) to (2.2), we obtain following equations

$$(1 - n\beta_0)u_0'' + u_0' - \left( M + \frac{1}{\alpha} - n \right) u_0 = G_r \theta_0 - G_m \phi_0 \quad \dots(2.5)$$

$$(\nu - \beta_1 P_r)\theta_0'' + P_r \theta_0' + n P_r \theta_0 = 0 \quad \dots(2.6)$$

$$(\nu - \beta_2 S_c)\phi_0'' + S_c \phi_0' + n S_c \phi_0 = 0 \quad \dots(2.7)$$

The corresponding boundary conditions can be written as

$$\left. \begin{aligned} u_0 = 0 \text{ at } y = 0 \\ u_0 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad \dots(2.8)$$

Solving equation (2.5) under the boundary conditions (2.8), we obtain the velocity distribution in the boundary layer as

$$u(y,t) = \left[ K_1 G_r e^{-K_2 y} + K_2 G_m e^{-K_3 y} - K_1 G_r e^{-K_4 y} - K_2 G_m e^{-K_5 y} \right] \cdot e^{-nt} \quad \dots(2.9)$$

The vorticity of the equation (2.9) will be

$$\zeta = \left[ K_1 K_4 e^{-K_4 y} + K_2 K_5 e^{-K_5 y} - K_2 (K_1 G_r e^{-K_2 y} + K_3 G_m e^{-K_3 y}) \right] e^{-nt}$$

Where,

$$K_1 = \frac{1}{K_4 \left[ (1-n\beta_0)K_4^2 + K_4 - \left( M + \frac{1}{\alpha} - n \right) \right]}, K_2 = \frac{1}{K_5 \left[ (1-n\beta_0)K_5^2 + K_5 - \left( M + \frac{1}{\alpha} - n \right) \right]}$$

$$K_3 = \frac{1 + \sqrt{1 + 4(1-n\beta_0) \left( M + \frac{1}{\alpha} - n \right)}}{2(1-n\beta_0)}, K_4 = \frac{P_r + \sqrt{P_r^2 - 4n(\nu - \beta_1 P_r)P_r}}{2(\nu - \beta_1 P_r)}$$

$$K_5 = \frac{S_c + \sqrt{S_c^2 - 4n(\nu - \beta_2 S_c)S_c}}{2(\nu - \beta_2 S_c)}$$

#### IV. Numericals Results And Discussion

**Table-1**  $P_r = 0.71, G_r = 2, G_m = 2, S_c = 0.6, R_e = 1$

Y	0	1	2	3	4	5
M = 1	0.4736	0.2516	0.1920	0.1434	0.0997	0.0655
M = 2	0.1041	0.1063	0.1020	0.0800	0.0561	0.0368
M = 3	-0.3268	0.0328	0.0671	0.0550	0.0391	0.0256
M = 4	-0.7103	-0.0041	0.0501	0.0426	0.0300	0.0196

**Table-2**  $P_r = 0.71, M = 2, G_m = 2, S_c = 0.6, R_e = 1$

Y	0	1	2	3	4	5
$G_r = 2$	0.1041	0.1063	0.1020	0.0800	0.0561	0.0368
$G_r = 4$	-3.4673	1.8381	1.5520	0.9431	0.5391	0.3031
$G_r = 6$	-9.4197	4.7242	3.9686	2.3824	1.3441	0.7470
$G_r = 8$	-17.7530	8.7649	7.3520	4.3970	2.4711	1.3684

**Table-3**  $M = 2, G_r = 2, G_m = 2, S_c = 0.6, R_e = 1$

Y	0	1	2	3	4	5
$P_r = 0.71$	0.1041	0.1063	0.1020	0.0800	0.0561	0.0368
$P_r = 1.42$	-2.0265	0.1609	0.3537	0.2525	0.1519	0.0865
$P_r = 2.13$	-2.5905	0.2256	0.4165	0.2765	0.1592	0.0885
$P_r = 2.84$	-2.8237	0.2608	0.4349	0.2809	0.1600	0.0886

**Table-4**  $M = 2, G_r = 2, G_m = 2, P_r = 0.71, R_e = 1$

Y	0	1	2	3	4	5
$S_c = 0.6$	0.1041	0.1063	0.1020	0.0800	0.0561	0.0368
$S_c = 1.0$	1.2946	-0.0058	-0.1473	-0.1132	-0.0689	-0.0368
$S_c = 1.4$	2.0883	-0.0544	-0.2488	-0.1710	-0.0953	-0.0495
$S_c = 1.8$	4.4692	-0.1728	-0.2589	-0.1854	-0.1009	-0.0512

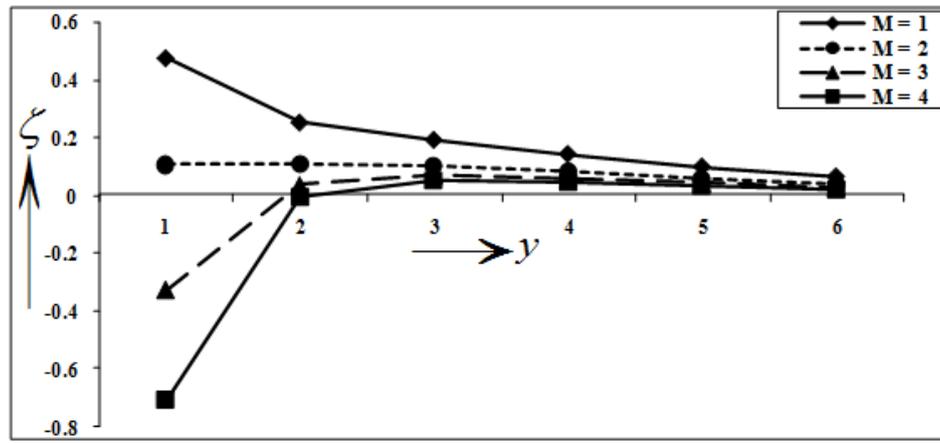


Figure 1. Vorticity profiles against  $y$  for different values of  $M$  with  $P_r = 0.71, G_r = 2, G_m = 2, S_c = 0.6$  and  $R_e = 1$

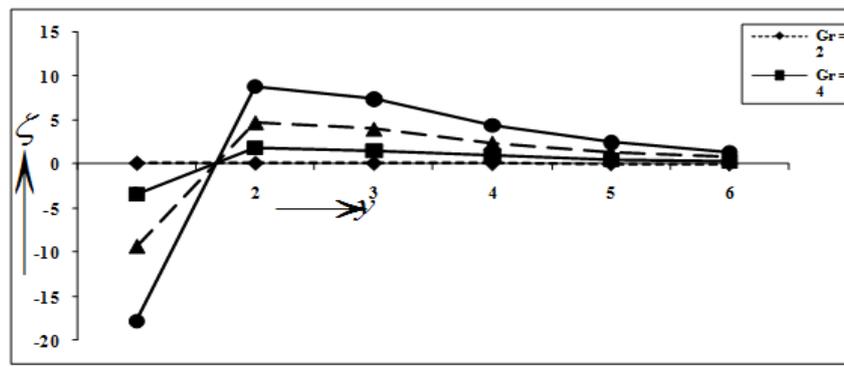


Figure 2. Vorticity profiles against  $y$  for different values of  $G_r$  with  $M = 2, P_r = 0.71, G_m = 2, S_c = 0.6$  and  $R_e = 1$

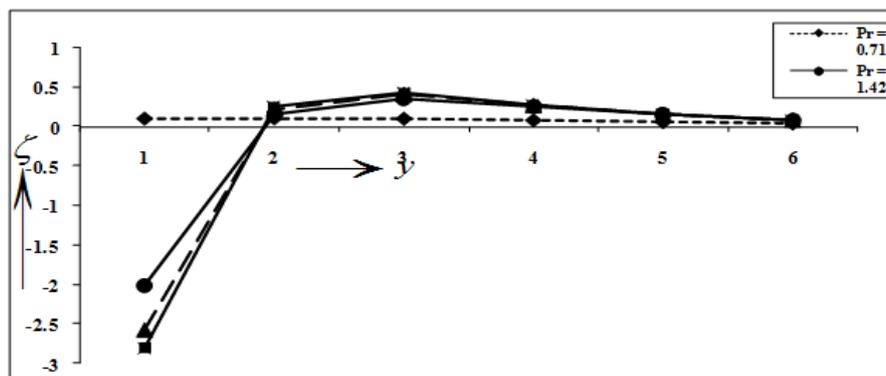


Figure 3. Vorticity profiles against  $y$  for different values of  $P_r$  with  $M = 2, G_r = 2, G_m = 2, S_c = 0.6$  and  $R_e = 1$

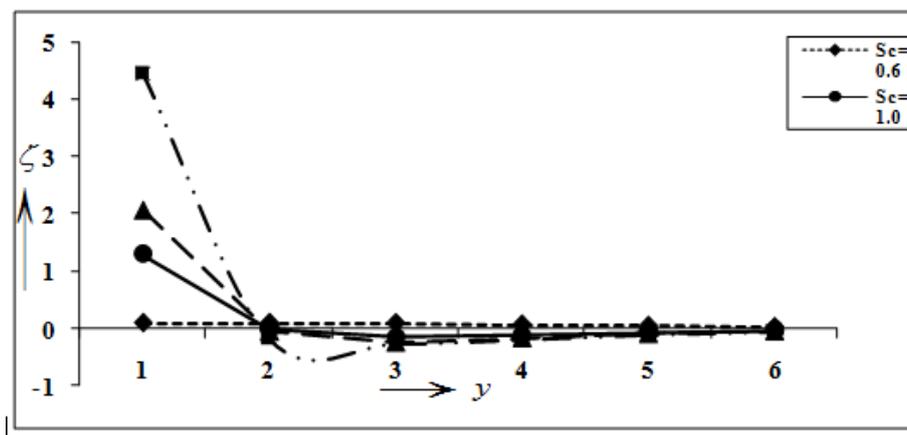


Figure 4. Vorticity profiles against  $y$  for different values of  $S_c$  with  $M = 2, G_r = 2, G_m = 2, P_r = 0.71$  and  $R_e = 1$

The vorticity distribution for different values of magnetic parameter  $M$  ( $M = 1, 2, 3, 4$ ) with  $P_r = 0.71, G_r = 2, G_m = 2, S_c = 0.6, R_e = 1$  are shown in table (1) and Figure (1). Separately it is found that the vorticity increases sharply till  $y = 0$ , after its vorticity decreases continuously with increases in  $y$ . It is observed that vorticity distribution decreases due to increasing the value of Magnetic parameter  $M$  at  $y = 0$ .

The vorticity distribution for different values of Grashoff number  $G_r$  ( $G_r = 2, 4, 6, 8$ ) with  $P_r = 0.71, M = 2, G_m = 2, S_c = 0.6, R_e = 1$  are shown in table (2) and figure (2). Separately it is found that the vorticity increases sharply  $y = 0$  to  $y = 2$ , after its vorticity decreases continuously with increases in  $y$ . It is observed that vorticity distribution decreases due to increases the value of Grashoff number  $G_r$  at  $y = 0$ .

The vorticity distribution for different values of  $P_r$  ( $P_r = 0.71, 1.42, 2.13, 2.83$ ) with  $M = 2, G_r = 2, G_m = 2, S_c = 0.6, R_e = 1$  are shown in table (3) and figure (3), Separately it is found that the vorticity decreases continuously with increases in  $y$ . It is observed that vorticity decreases with increasing Prandtl number  $P_r$ .

The vorticity distribution for different values of  $S_c$  ( $S_c = 0.6, 1.0, 1.4, 1.8$ ) with  $M = 2, G_r = 2, G_m = 2, P_r = 0.71, R_e = 1$  are shown in table (4) and figure (4), separately it is found that the vorticity decrease with increases in  $y$ . It is observed that vorticity increases with increasing Schmidt number  $S_c$  at  $y = 0$ .

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