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A Study On Translations Of Anti S-Fuzzy Subhemiring Of A Hemiring

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Abstract: *In this paper, we made an attempt to study the algebraic nature of an anti (T, S)-fuzzy normal ideals and translations of anti S-fuzzy subhemiring of a hemiring.*
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I. Introduction:

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R ; + ; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[22], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [11], [12]. In this paper, we introduce the some Theorems in anti (T, S)-fuzzy normal ideal and translations of anti S-fuzzy subhemiring of a hemiring.

1.PRELIMINARIES:

1.1 Definition: A (T, S)-norm is a binary operations T: $[0, 1]\times[0, 1] \to [0, 1]$ and S: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;
(i) T(0, x)= 0, T(1, x) = x (boundary condition)
(ii) T(x, y) = T(y, x) (commutativity)
(iii) T(x, T(y, z))= T (T(x,y), z)(associativity)
(iv) if $x \le y$ and $w \le z$, then $T(x, w) \le T(y, z)$(monotonicity).
(v) S(0, x) = x, S (1, x) = 1 (boundary condition)
(vi) S(x, y) = S (y, x)(commutativity)
(vii) S (x, S(y, z))= S (S(x, y), z) (associativity)
(viii) if $x \le y$ and $w \le z$, then $S(x, w) \le S(y, z)$(monotonicity).

1.2 Definition: Let (R, +, .) be a hemiring. A fuzzy subset A of R is said to be an anti S-fuzzy subhemiring (anti fuzzy subhemiring with respect to S-norm) of R if it satisfies the following conditions:
(i) $\mu_A(x + y) \le S(\mu_A(x), \mu_A(y))$,
(ii) $\mu_A(xy) \le S(\mu_A(x), \mu_A(y))$, for all x and y in R.

1.3 Definition: Let (R, +, .) be a hemiring. A fuzzy subset A of R is said to be an anti (T, S)-fuzzy ideal (anti fuzzy ideal with respect to (T, S)-norm) of R if it satisfies the following conditions:
(i) $\mu_A(x + y) \le S(\mu_A(x), \mu_A(y))$,
(ii) $\mu_A(xy) \le T(\mu_A(x), \mu_A(y))$, for all x and y in R.

1.4 Definition: Let A and B be fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by $A\times B$, is defined as $A\times B = \{\langle (x, y), \mu_{A\times B}(x,y) \rangle$ / for all $x\in G$ and $y\in H \}$, where $\mu_{A\times B}(x, y) = \max\{ \mu_A(x), \mu_B(y) \}$.

1.5 Definition: Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \max\{ \mu_A(x), \mu_A(y) \}$, for all x and y in S.

1.6 Definition: Let R and R^{l} be any two hemirings. Let f : $R \to R^{l}$ be any function and A be an anti (T, S)-fuzzy ideal in R, V be an anti (T, S)-fuzzy ideal in f(R) = R^{l}, defined by $\mu_V(y) = \inf_{x \in f^{-1}(y)} \mu_A(x)$, for all $x \in R$, $y \in R^{l}$. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be a fuzzy subset of X. For α in [0, 1], the lower level subset of A is the set $A_\alpha = \{ x \in X : \mu_A(x) \le \alpha \}$.

1.8 Definition: Let (R , + , .) be a hemiring. An anti (T, S)-fuzzy ideal A of R is said to be an anti (T, S)-fuzzy normal ideal of R if $\mu_A(xy)=\mu_A(yx)$, for all x, y in R.

1.9 Definition: Let A be a fuzzy subset of X and $\alpha \in [\, 0, 1 - \text{Sup}\{ A(x) : x \in X, 0 < A(x) < 1 \}]$. Then T = T_α^A is called a **translation** of A if T(x) = A(x) + α, for all x in X.

II. Properties:

2.1 Theorem: Let (R, +, .) be a hemiring. If A and B are two anti (T, S)-fuzzy normal ideals of R. Then $A \cup B$ is an anti (T, S)-fuzzy normal ideal of R.

Proof: Let x, $y \in R$. Let A $=\{\langle x, \mu_A(x) \rangle / x \in R\}$ and B = $\{\langle x, \mu_B(x)\rangle / x \in R\}$ be anti (T, S)-fuzzy normal ideals of a hemiring R. Let C= $A \cup B$ and C = $\{\langle x, \mu_C(x)\rangle / x \in R\}$, where $\mu_C(x) = \max\{ \mu_A(x), \mu_B(x)\}$. Then, Clearly C is an anti (T, S)-fuzzy ideal of a hemiring R, since A and B are two anti (T, S)-fuzzy ideals of the hemiring R. And, $\mu_C(xy) = \max \{ \mu_A(xy), \mu_B(xy) \}= \max\{\mu_A(yx), \mu_B(yx)\} = \mu_C(yx)$, for all x and y in R. Therefore, $\mu_C(xy) = \mu_C(yx)$, for all x and y in R. Hence $A \cup B$ is an anti (T, S)-fuzzy normal ideal of the hemiring R.

2.2 Theorem: Let (R, +, .) be a hemiring. The union of a family of anti (T, S)-fuzzy normal ideals of R is an anti (T, S)-fuzzy normal ideal of R.

Proof: It is trivial.

2.3 Theorem: Let A and B be anti (T, S)-fuzzy ideals of the hemirings G and H, respectively. If A and B are anti (T, S)-fuzzy normal ideals, then $A \times B$ is an anti (T, S)-fuzzy normal ideal of $G \times H$.

Proof: Let A and B be anti (T, S)-fuzzy normal ideals of the hemirings G and H respectively. Clearly $A \times B$ is an anti (T, S)-fuzzy ideal of $G \times H$. Let x_1, x_2 be in Gand y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in $G \times H$. Now, $\mu_{A\times B} [(x_1, y_1)(x_2, y_2)] = \mu_{A\times B} (x_1x_2, y_1y_2) = \max \{ \mu_A(x_1x_2), \mu_B(y_1y_2) \}= \max \{ \mu_A(x_2x_1), \mu_B(y_2y_1) \}= \mu_{A\times B} (x_2x_1, y_2y_1) = \mu_{A\times B} [(x_2, y_2)(x_1, y_1)]$. Therefore, $\mu_{A\times B} [(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Hence $A \times B$ is an anti (T, S)-fuzzy normal ideal of $G \times H$.

2.4 Theorem: Let A and B be anti (T, S)-fuzzy normal ideal of the hemirings R_1 and R_2 respectively. Suppose that 0 and 0_l are the zero element of R_1 and R_2 respectively. If $A \times B$ is an anti (T, S)-fuzzy normal ideal of $R_1 \times R_2$, then at least one of the following two statements must hold.

(i) $B(0_l) \le A(x)$, for all x in R_1,

(ii) $A(0) \le B(y)$, for all y in R_2.

Proof: It is trivial.

2.5 Theorem: Let A and B be two fuzzy subsets of the hemirings R_1 and R_2 respectively and $A \times B$ is an anti (T, S)-fuzzy normal ideal of $R_1 \times R_2$. Then the following are true:

(i) if $A(x) \ge B(0_l)$, then A is an anti (T, S)-fuzzy normal ideal of R_1.

(ii) if $B(x) \ge A(0)$, then B is an anti (T, S)-fuzzy normal ideal of R_2.

(iii) either A is an anti (T, S)-fuzzy normal ideal of R_1 or B is an anti (T, S)-fuzzy normal ideal of R_2.

Proof: It is trivial.

2.6 Theorem: Let A be a fuzzy subset in a hemiring R and V be the anti-strongest fuzzy relation on R. Then A is an anti (T, S)-fuzzy normal ideal of R if and only if V is an anti (T, S)-fuzzy normal ideal of $R \times R$.

Proof: It is trivial.

2.7 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two hemirings. The homomorphic image of an anti (T, S)-fuzzy normal ideal of R is an anti (T, S)-fuzzy normal ideal of R^{l}.

Proof: Let (R, +, .) and (R^{l}, +, .) be any two hemirings and f : $R \to R^{l}$ be a homomorphism. Then, f(x+y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let V = f(A), where A is an anti (T, S)-fuzzy normal ideal of a hemiring R. We have to prove that V is an anti (T, S)-fuzzy normal ideal of a hemiring R^{l}.Now, for f(x), f(y) in R^{l}, clearly V is an anti (T, S)-fuzzy ideal of a hemiring R^{l}, since A is an anti S-fuzzy ideal of a hemiring R.

Now, $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \leq \mu_A(xy) = \mu_A(yx) \geq \mu_v(f(yx)) = \mu_v(f(y)f(x))$, which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y) f(x))$, for all f(x) and f(y) in R^l. Hence V is an anti (T, S)-fuzzy normal ideal of a hemiring R^l.

2.8 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The homomorphic preimage of an anti (T, S)-fuzzy normal ideal of R^l is an anti (T, S)-fuzzy normal ideal of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and $f : R \to R^l$ be a homomorphism. Then, f(x+y) = f(x)+f(y) and f(xy) = f(x) f(y), for all x and y in R. Let V = f(A), where V is an anti (T, S)-fuzzy normal ideal of a hemiring R^l. We have to prove that A is an anti (T, S)-fuzzy normal ideal of a hemiring R. Let x and y in R. Then, clearly A is an anti (T, S)-fuzzy ideal of a hemiring R, since V is an anti (T, S)-fuzzy ideal of a hemiring R^l. Now, $\mu_A(xy) = \mu_v(f(xy)) =\mu_v(f(x)f(y)) = \mu_v(f(y)f(x)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti (T, S)-fuzzy normal ideal of a hemiring R.

2.9 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The anti-homomorphic image of an anti (T, S)-fuzzy normal ideal of R is an anti (T, S)-fuzzy normal ideal of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and $f : R \to R^l$ be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where A is an anti (T, S)-fuzzy normal ideal of a hemiring R. We have to prove that V is an anti (T, S)-fuzzy normal ideal of a hemiring R^l. Now, for f(x) and f(y) in R^l, clearly V is an anti (T, S)-fuzzy ideal of a hemiring R^l, since A is an anti (T, S)-fuzzy ideal of a hemiring R. Now, $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \leq \mu_A(yx) = \mu_A(xy) \geq \mu_v(f(xy)) = \mu_v(f(y)f(x))$, which implies that $\mu_v(f(x)f(y) = \mu_v(f(y)f(x))$, for all f(x) and f(y) in R^l. Hence V is an anti (T, S)-fuzzy normal ideal of a hemiring R^l.

2.10 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. The anti-homomorphic preimage of an anti (T, S)-fuzzy normal ideal of R^l is an anti (T, S)-fuzzy normal ideal of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and $f : R \to R^l$ be an anti-homomorphism. Then, f (x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an anti (T, S)-fuzzy normal ideal of a hemiring R^l. We have to prove that A is an anti (T, S)-fuzzy normal ideal of a hemiring R.Let x and y in R, then, clearly A is an anti (T, S)-fuzzy ideal of a hemiring R, since V is an anti (T, S)-fuzzy ideal of a hemiring R^l.Now, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) = \mu_v(f(x)f(y)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti (T, S)-fuzzy normal ideal of a hemiring R.

<u>In the following Theorem ◦ is the composition operation of functions:</u>

2.11 Theorem: Let A be an anti (T, S)-fuzzy ideal of a hemiring H and f is an isomorphism from a hemiring R onto H. If A is an anti (T, S)-fuzzy normal ideal of the hemiring H, then A◦f is an anti (T, S)-fuzzy normal ideal of the hemiring R.

Proof: Let x, y in R and A be an anti (T, S)-fuzzy normal ideal of a hemiring H. Then, clearly A◦f is an anti (T, S)-fuzzy ideal of a hemiring R. Now, $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) = \mu_A(f(y)f(x)) = \mu_A(f(yx)) = (\mu_A\circ f)(yx)$, which implies that $(\mu_A\circ f)(xy) = (\mu_A\circ f)(yx)$, for all x and y in R. Hence A◦f is an anti (T, S)-fuzzy normal ideal of a hemiring R.

2.12 Theorem: Let A be an anti (T, S)-fuzzy ideal of a hemiring H and f is an anti-isomorphism from a hemiring R onto H. If A is an anti (T, S)-fuzzy normal ideal of the hemiring H, then A◦f is an anti (T, S)-fuzzy normal ideal of the hemiring R.

Proof: Let x, y in R and A be an anti (T, S)-fuzzy normal ideal of a hemiring H. Then, clearly A◦f is anti (T, S)-fuzzy ideal of the hemiring R. Now, $(\mu_A\circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) = \mu_A(f(x)f(y)) = \mu_A(f(yx)) = (\mu_A\circ f)(yx)$, which implies that $(\mu_A\circ f)(xy) = (\mu_A\circ f)(yx)$, for all x and y in R. Hence A◦f is an anti (T, S)-fuzzy normal ideal of the hemiring R.

2.13 Theorem: The homomorphic image of a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R is a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R^l.

Proof: It is trivial.

2.14 Theorem: The homomorphic pre-image of a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R^l is a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R.

Proof: It is trivial.

2**.15 Theorem:** The anti-homomorphic image of a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R is a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R^l.

Proof: It is trivial.

2.16 Theorem: The anti-homomorphic pre-image of a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R[l] is a lower level ideal of an anti (T, S)-fuzzy normal ideal of a hemiring R.
Proof: It is trivial.

2.17 Theorem: If M and N are two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, .), then their intersection M ∩ N is translation of anti S-fuzzy subhemiring A.

Proof: Let x and y belong to R. Let M = T_α^A = { ⟨x, $\mu_A(x)+\alpha$ ⟩ / x in R} and N = T_γ^A = { ⟨ x, $\mu_A(x)+\gamma$ ⟩ / x in R} be two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, .). Let C = M ∩ N and C = { ⟨ x, $\mu_C(x)$⟩ / x in R}, where $\mu_C(x) = \min\{\mu_A(x)+\alpha, \mu_A(x)+\gamma\}$. **Case (i):** $\alpha \le \gamma$. Now, $\mu_C(x+y) = \min\{\mu_M(x+y), \mu_N(x+y)\} = \min\{\mu_A(x+y)+\alpha, \mu_A(x+y)+\gamma\} = \mu_A(x+y)+\alpha = \mu_M(x+y)$, for all x and y in R. And, $\mu_C(xy) = \min\{\mu_M(xy), \mu_N(xy)\} = \min\{\mu_A(xy)+\alpha, \mu_A(xy)+\gamma\} = \mu_A(xy)+\alpha = \mu_M(xy)$, for all x and y in R. Therefore C = T_α^A = {⟨x, $\mu_A(x)+\alpha$ ⟩ / x in R} is a translation of anti S-fuzzy subhemiring A of the hemiring (R, +, .). **Case (ii):** $\alpha \ge \gamma$. Now, $\mu_C(x+y) = \min\{\mu_M(x+y), \mu_N(x+y)\} = \min\{\mu_A(x+y)+\alpha, \mu_A(x+y)+\gamma\} = \mu_A(x+y)+\gamma = \mu_N(x+y)$, for all x and y in R. And $\mu_C(xy) = \min\{\mu_M(xy), \mu_N(xy)\} = \min\{\mu_A(xy)+\alpha, \mu_A(xy)+\gamma\} = \mu_A(xy)+\gamma = \mu_N(xy)$, for all x and y in R. Therefore C = T_γ^A = { ⟨x, $\mu_A(x)+\gamma$ ⟩ / x in R} is a translation of anti S-fuzzy subhemiring A of the hemiring (R, +, .). Hence all cases, intersection of any two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, ·) is also a translation of anti S-fuzzy subhemiring A.

2.18 Theorem: The intersection of a family of translations of anti S-fuzzy subhemiring A of a hemiring (R, +, ·) is also a translation of anti S-fuzzy subhemiring A.
Proof: It is trivial.

2.19 Theorem: If M and N are two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, ·), then their union M ∪ N is also a translation of anti S-fuzzy subhemiring A.

Proof: Let x and y belong to R. Let M = T_α^A = { ⟨ x, $\mu_A(x)+\alpha$⟩ / x in R} and N= T_γ^A = { ⟨ x, $\mu_A(x)+\gamma$ ⟩ / x in R} be two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, .). Let C = M∪N and C = {⟨x, $\mu_C(x)$ ⟩ / x in R}, where $\mu_C(x) = \max\{\mu_A(x)+\alpha, \mu_A(x)+\gamma\}$. **Case (i):** $\alpha \le \gamma$. Now, $\mu_C(x+y) = \max\{\mu_M(x+y), \mu_N(x+y)\} = \max\{\mu_A(x+y)+\alpha, \mu_A(x+y)+\gamma\} = \mu_A(x+y)+\gamma = \mu_N(x+y)$, for all x and y in R. And, $\mu_C(xy) = \max\{\mu_M(xy), \mu_N(xy)\} = \max\{\mu_A(xy)+\alpha, \mu_A(xy)+\gamma\} = \mu_A(xy)+\gamma = \mu_N(xy)$, for all x and y in R. Therefore C= T_γ^A ={⟨x, $\mu_A(x)+\gamma$⟩ / x in R} is a translation of anti S-fuzzy subhemiring A of a hemiring (R, +, .). **Case (ii):** $\alpha \ge \gamma$. Now, $\mu_C(x+y) = \max\{\mu_M(x+y), \mu_N(x+y)\} = \max\{\mu_A(x+y)+\alpha, \mu_A(x+y)+\gamma\} = \mu_A(x+y)+\alpha = \mu_M(x+y)$, for all x and y in R. And $\mu_C(xy) = \max\{\mu_M(xy), \mu_N(xy)\} = \max\{\mu_A(xy)+\alpha, \mu_A(xy)+\gamma\} = \mu_A(xy)+\alpha = \mu_M(xy)$, for all x and y in R. Therefore C = T_α^A = { ⟨x, $\mu_A(x)+\alpha$ ⟩ / x in R} is a translation of anti S-fuzzy subhemiring A of the hemiring (R, +, .). Hence all cases, union of any two translations of anti S-fuzzy subhemiring A of a hemiring (R, +, ·) is also a translation of anti S-fuzzy subhemiring A.

2.20 Theorem: The union of a family of translations of anti S-fuzzy subhemiring A of a hemiring (R, +, ·) is also a translation of anti S-fuzzy subhemiring A.
Proof: It is trivial.

2.21 Theorem: If T_α^A is a translation of anti S-fuzzy subhemiring A of a hemiring R, then T_α^A is anti S-fuzzy subhemiring of R.

Proof: Assume that T_α^A is a translation of anti S-fuzzy subhemiring A of a hemiring R. Let x and y in R. We have, $T_\alpha^A(x+y) = A(x+y)+\alpha \le S(A(x), A(y))+\alpha \le S(A(x)+\alpha, A(y)+\alpha) = S(T_\alpha^A(x), T_\alpha^A(y))$}. Therefore, $T_\alpha^A(x+y) \le S(T_\alpha^A(x), T_\alpha^A(y))$, for all x and y in R. And, $T_\alpha^A(xy) = A(xy)+\alpha \le S(A(x), A(y))+\alpha \le S(A(x)+\alpha, A(y)+\alpha) = S(T_\alpha^A(x), T_\alpha^A(y))$. Therefore, $T_\alpha^A(xy) \ge S(T_\alpha^A(x), T_\alpha^A(y))$, for all x and y in R. Hence T_α^A is anti S-fuzzy subhemiring of R.

2.22 Theorem: Let (R, +, ,.) and (R^l, +, .) be any two hemirings. If f: R→ R^l is a homomorphism, then the translation of anti S-fuzzy subhemiring A of R under the homomorphic image is anti S-fuzzy subhemiring of f (R) = R^l.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and f : R→ R^l be a homomorphism. That is f(x+y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let T_α^A be a translation of anti S-fuzzy subhemiring A of R. Let V be the homomorphic image of T_α^A under f. We have to prove that V is anti S-fuzzy subhemiring of f(R) = R^l. Now, for f(x) and f(y) in R^l, we have V[f(x)+ f(y)] = V[f(x+y)] ≤ T_α^A (x+y) = A(x+y) + α ≤ S (A(x), A(y)) + α ≤ S(A(x) + α, A(y)+ α) = S (T_α^A (x), T_α^A (y)), which implies that V[f(x)+f(y)] ≤ S (V(f(x)), V(f(y))), for all f(x), f(y)∈R^l. And V[f(x)f(y)] = V [f(xy)] ≤ T_α^A (xy) = A(xy) + α ≤ S(A(x), A(y)) + α ≤ S (A(x) + α, A(y)+ α) = S(T_α^A (x), T_α^A (y)), which implies that V[f(x)f(y)] ≤ S(V(f(x)), V(f(y))), for all f(x) and f(y) in R^l. Therefore, V is an anti S-fuzzy subhemiring of R^l. Hence the homomorphic image of translation of A of R is an anti S-fuzzy subhemiring of R^l.

2.23 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. If f : R→ R^l is a homomorphism, then the translation of an anti S-fuzzy subhemiring V of f (R) = R^l under the homomorphic pre-image is an anti S-fuzzy subhemiring of R.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and f : R→ R^l be a homomorphism. That is f(x+y) = f(x) + f(y) and f(xy) = f(x)f(y), for all x and y in R. Let T_α^V be the translation of anti S-fuzzy subhemiring V of R^l and A be the homomorphic pre-image of T_α^V under f. We have to prove that A is an anti S-fuzzy subhemiring of R. Let x and y be in R. Then, A(x+y) = T_α^V (f(x+y)) = T_α^V (f(x)+f(y)) = V[f(x)+ f(y)] + α ≤ S(V(f(x)), V(f(y)))+ α ≤ S ((V(f(x))+α, V(f(y))+ α) = S(T_α^V (f(x)), T_α^V (f(y))) = S(A(x), A(y)), which implies that A(x+y) ≤ S(A(x), A(y)), for all x, y in R. And, A(xy) = T_α^V (f(xy)) = T_α^V (f(x)f(y)) = V[f(x)f(y)] + α ≤ S(V(f(x)), V(f(y))) + α ≤ S((V(f(x))+α, V(f(y))+ α)) = S (T_α^V (f(x)), T_α^V (f(y))) = S (A(x), A(y)), which implies that, A(xy) ≤ S (A(x), A(y)), for all x and y in R. Therefore, A is anti S-fuzzy subhemiring of R.

2.24 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. If f : R→ R^l is an anti-homomorphism, then the translation of an anti S-fuzzy subhemiring A of R under the anti-homomorphic image is an anti S-fuzzy subhemiring of f (R) = R^l.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and f : R→ R^l be an anti-homomorphism. That is f(x+y) = f(y)+ f(x) and f(xy) = f(y)f(x), for all x and y in R. Let T_α^A be the translation of an anti S-fuzzy subhemiring A of R and V be the anti-homomorphic image of T_α^A under f. We have to prove that V is an anti S-fuzzy subhemiring of f (R) = R^l. Now, for f(x) and f(y) in R^l and q in Q, we have, V[f(x)+f(y)] = V[f(y+x)] ≤ T_α^A (y+x) = A (y+x) + α ≤ S (A(x), A(y)) + α ≤ S (A(x) + α, A(y)+ α) = S (T_α^A (x), T_α^A (y)), which implies that V[f(x) + f(y)] ≤ S(V(f(x)), V(f(y)), for all f(x), f(y)∈R^l. And, V[f(x)f(y)] = V[f(yx)] ≤ T_α^A (yx) = A(yx) + α ≤ S(A(x), A(y)) + α ≤ S(A(x)+α, A(y)+α) = S(T_α^A (x), T_α^A (y)), which implies that V[f(x)f(y)] ≤ S (V(f(x)), V(f(y))), for all f(x) and f(y) in R^l. Therefore, V is an anti S-fuzzy subhemiring of the hemiring R^l. Hence the anti-homomorphic image of translation of A of R is an anti S-fuzzy subhemiring of R^l.

2.25 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. If f: R → R^l is an anti-homomorphism, then the translation of an anti S-fuzzy subhemiring V of f (R) = R^l under the anti-homomorphic pre-image is an anti S-fuzzy subhemiring of R.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and f: R → R^l be an anti-homomorphism. That is f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x), for all x and y in R. Let T_α^V be the translation of an anti S-fuzzy subhemiring V of f(R) = R^l and A be the anti-homomorphic pre-image of T_α^V under f. We have to prove that A is an anti S-fuzzy subhemiring of R. Let x and y be in R. Then, A(x+y) = T_α^V (f(x+y))) = T_α^V [f(y)+f(x)] = V[f

(y)+f(x)] + α ≤ S(V(f(x)), V(f(y))) + α ≤ S(V(f(x))+α, V(f(y))+ α) = S(T_α^V (f(x))), T_α^V (f(y))) = S(A(x), A(y)), which implies that A(x+y) ≤ S (A(x), A(y), for all x and y in R. And, A(xy) = T_α^V (f(xy))) = T_α^V (f(y)f(x)) = V[f(y)f(x)]+α ≤ S(V(f(x)), V(f(y)))+α ≤ S(V(f(x))+α, V(f(y))+ α) = S(T_α^V (f(x))), T_α^V (f(y)))= S(A(x), A(y)), which implies that A(xy) ≤ S(A(x), A(y), for all x and y in R. Therefore, A is an anti S-fuzzy subhemiring of R.

2.26 Theorem: If M and N are two translations of an anti S-fuzzy normal subhemiring A of a hemiring (R, +, .), then their intersection M ∩ N is also a translation of A.
Proof: It is trivial.

2.27 Theorem: The intersection of a family of translations of an anti S-fuzzy normal subhemiring A of a hemiring (R, +, .) is a translation of A.
Proof: It is trivial.

2.28 Theorem: If M and N are two translations of an anti S-fuzzy normal subhemiring A of a hemiring (R, +, .), then their union M ∪ N is also a translation of A.
Proof: It is trivial.

2.29 Theorem: The union of a family of translations of an anti S-fuzzy normal subhemiring A of a hemiring (R, +, .) is also a translation of A.
Proof: It is trivial.

2.30 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. If f : R → R' is a homomorphism, then the translation of an anti S-fuzzy normal subhemiring A of R under the homomorphic image is an anti S-fuzzy normal subhemiring of f (R) = R'.
Proof: Let (R, +, .) and (R', +, .) be any two hemirings and f : R → R' be a homomorphism. That is f(x+y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let T_α^A be the translation of an anti S-fuzzy normal subhemiring A of R and V be the homomorphic image of T_α^A under f. We have to prove that V is an anti S-fuzzy normal subhemiring of R'. Now, for f(x) and f(y) in R', clearly V is an anti S-fuzzy subhemiring of R'. We have V(f(x)f(y)) = V(f(xy)) ≤ T_α^A (xy) = A(xy) + α = A(yx)+α = T_α^A (yx) ≥ V(f(yx)) = V(f(y) f(x)), which implies that V(f(x)f(y)) = V(f(y)f(x)), for all f(x) and f(y) in R'. Therefore, V is an anti S-fuzzy normal subhemiring of the hemiring R'.

2.31 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. If f : R → R' is a homomorphism, then translation of an anti S-fuzzy normal subhemiring V of f(R) = R' under the homomorphic pre-image is an anti S-fuzzy normal subhemiring of R.
Proof: Let (R, +, .) and (R', +, .) be any two hemirings and f : R → R' be a homomorphism. That is f (x+y) = f(x)+f(y) and f(xy) = f(x)f(y), for all x and y in R. Let T_α^V be the translation of an anti S-fuzzy normal subhemiring V of R' and A be the homomorphic pre-image of T_α^V under f. We have to prove that A is an anti S-fuzzy normal subhemiring of R. Let x and y be in R. Then, clearly A is an anti S-fuzzy subhemiring of R, A(xy) = T_α^V (f(xy)) = V(f(xy)) + α = V(f(x)f(y)) + α = V(f(y)f(x)) + α = V(f(yx)) + α = T_α^V (f(yx)) = A(yx), which implies that A(xy) = A(yx), for all x and y in R. Therefore, A is an anti S-fuzzy normal subhemiring of R.

2.32 Theorem: Let (R, +, .) and (R', +, .) be any two hemirings. If f : R → R' is an anti-homomorphism, then the translation of an anti S-fuzzy normal subhemiring A of R under the anti-homomorphic image is an anti S-fuzzy normal subhemiring of R'.
Proof: Let (R, +, .) and (R', +, .) be any two hemirings and f : R → R' be an anti-homomorphism. That is f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x), for all x and y in R. Let T_α^A be the translation of an anti S-fuzzy normal subhemiring A of R and V be the anti-homomorphic image of T_α^A under f. We have to prove that V is an anti S-fuzzy normal subhemiring of f(R) = R'. Now, for f(x) and f(y) in R', clearly V is an anti S-fuzzy subhemiring

of R^l. We have, V(f(x)f(y)) = V(f(yx)) ≤ T_α^A (yx) = A(yx) + α = A(xy)+α = T_α^A (xy) ≥ V(f(xy)) = V(f(y)f(x)), which implies that V(f(x)f(y)) = V(f(y)f(x)), for f(x) and f(y) in R^l. Therefore, V is an anti S-fuzzy normal subhemiring of the hemiring R^l.

2.33 Theorem: Let (R, +, .) and (R^l, +, .) be any two hemirings. If f : R → R^l is an anti-homomorphism, then the translation of an anti S-fuzzy normal subhemiring V of f(R) = R^l under the anti-homomorphic pre-image is an anti S-fuzzy normal subhemiring of R.
Proof: Let (R, +, .) and (R^l, +, .) be any two hemirings and f : R → R^l be an anti-homomorphism. That is f(x + y) = f(y) + f(x) and f(xy) = f(y)f(x), for all x and y in R. Let T_α^V be the translation of an anti S-fuzzy normal subhemiring V of R^l and A be the anti-homomorphic pre-image of T_α^V under f. We have to prove that A is an anti S-fuzzy normal subhemiring of R. Let x and y be in R. Then, clearly A is an anti S-fuzzy subhemiring of R, A(xy) = T_α^V (f(xy)) = V(f(xy)) + α = V(f(y)f(x))+ α = V(f(x)f(y)) + α = V(f(yx)) + α = T_α^V (f(yx)) = A(yx), which implies that A(xy) = A(yx), for all x and y in R. Therefore, A is an anti S-fuzzy normal subhemiring of R.

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