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On ∅- Concircurlarly Symmetric Para Sasakian Manifold

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Abstract : *The present paper deals with the study of ∅- concircurlarly symmetric Para Sasakian manifold and have study locally and globally ∅- concircurlarly symmetric Para Sasakian manifold.further we have shown that globally symmetry and globally ∅- concircurlarly symmetric are equivalent.Next we study 3 – dimensional locally ∅- concircurlarly symmetric Para Sasakian manifold.*
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I. Introduction:

A transformation of an n-dimensional Riemannian manifold M, which transform every geodesic circle of M in to a geodesic circle, is called a concircular transformation. A concircular transformation is always a conformal transformation. Here geodesic circle means a curve in M whose first curvature is constant and second curvature is identically zero. Thus the geometry of concircular transformations, that is, the concircular geometry,is a generalization of inversive geometry in the sence that the change of metric is more general than that induced by a circle preserving diffeomorphism. An interesting invariant of a concircular transformation is the concircular curavture tensor. A concircular transformation is always a conformal transformation . an interesting invariant of a concircular transformation is the concircular curvature tensor. A (1,3) type of tensor $C(X,Y)Z$ which remains invariant under concircular transformation for n dimensional Riemanniam manifold is given by [4]

(1.1) $C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]$

Where R is the Riemannian curvature tensor ,r is the scalar curvature tensor.
from (1.1) we obtain

(1.2) $(D_W C)(X,Y)Z = (D_W R)(X,Y)Z - \frac{dr(w)}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]$

In this paper we study of ∅- concircurlarly symmetric Para Sasakian manifold.we have also study locally and globally ∅- concircurlarly symmetric Para Sasakian manifold and shown that globally symmetry and globally ∅- concircurlarly symmetric are equivalent.Next we have study 3 – dimensional locally ∅- concircurlarly symmetric Para Sasakian manifold and shown some interesting result.

A contact manifold is said locally ∅- concircularly symmetric if the concircular curvature tensor C satisfies

(1.3) $\emptyset^2\big((D_X C)(Y,Z,W)\big) = 0$

For horizontal vector fields $X,Y,Z \in X_n M$holds on M. If X,Y,Z and W are arbitrary vector fields the manifold is called Globally ∅- concircurlarly symmetric.

II. Preliminaries:

Let $M^n(\emptyset,\xi,\eta,g)$ be an almost contact Riemannian manifold ,where ∅ is a tensor field of type (1,1) ,ξ is the structure vector field ,η is a 1-form and g is the Riemannian metric which satisfy

(2.1) $\emptyset^2 X = X - \eta(X)\xi$

(2.2) $(a)\ \eta(\xi) = 1\ \ (b)\ \ g(X,\xi) = \eta(X)\ \ \ (c)\ \ \ \eta(\emptyset X) = 0\ \ (d)\ \ \ \emptyset\xi = 0$

(2.3) $g(\emptyset X, \emptyset Y) = -g(X.Y) - \eta(X)\eta(Y)$

(2.4) $(D_X\emptyset)(Y) = -G(X,Y) - \eta(Y)X + 2\eta(X)\eta(Y)\xi$

(2.5) $(a)\ D_X\xi = \emptyset X,\ (b)\ (D_X\eta)(Y) = g(\emptyset X, Y),\ (c)\ \ d\eta = 0$

For all vector field X,Y,Z where D denotes the operator of covariant differentiation with respect to g , the manifold $M^n(\emptyset,\xi,\eta,g)$ is called a Para –Sasakian manifold or briefly a P – Sasakian manifold. [2-8].
In a Para Sasakian manifold , the following relation holds [1,2,6]

(2.6) $\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X)$

(2.7) $R(X,Y)\xi = \eta(X)Y - \eta(Y)X$

(2.8) $S(\emptyset X, \emptyset Y) = S(X,Y) + (n-1)\eta(X)\eta(Y)$

(2.9) $S(X,\xi) = -(n-1)\eta(X)$

For all vector fields X, Y, Z, where S is the Ricci Tensor of type (0,2) and R is the Riemannian curvature tensor of the manifold.

For a 3 – dimensional Para Sasakian manifold,we have [4]

(2.10) $R(X,Y)Z = \left(\frac{r+4}{2}\right)[g(Y,Z)X - g(X,Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta Y\eta ZX - \eta X\eta ZY$

(2.11) $S(X,Y) = \frac{1}{2}[(r+2)g(X,Y) - (r+6)\eta(X)\eta(Y)]$

For all vector fields X, Y, Z, where S is the Ricci Tensor of type (0,2) and R is the curvature tensor of the type (1,3) and r is scalar curvature of the manifolds.

If the Ricci tensor S of the manifold is of the form $S(X,Y) = \lambda g(X,Y)$, where λ is a constant and X,Y ∈ $X_n M$, then the manifold is Einstein manifold.

III. Globally ∅- concircularly symmetric para sasakian manifold:

Definition (3.1): A Para Saskian manifold M is said to be Globally ∅- concircularly symmetric if the concircular curvature tensor C satisfies

(3.1) $\emptyset^2\big((D_X C)(Y,Z,W)\big) = 0$

For all vector fields $X, Y, Z \in X_n M$.

Let us suppose that M is globally ∅- concircularly symmetric Para Sasakian manifold then by the def (3.1) , we have

$$\emptyset^2\big((D_W C)(X,Y,Z)\big) = 0$$

Using (2.1) , we have

$$(D_W C)(X,Y)Z - \eta\big(((D_W C)(X,Y)Z\big)\xi = 0$$

From it follows that

$$g\big((D_W C)(X,Y)Z, U\big) - \eta\big(((D_W C)(X,Y)Z\big)\eta(U) = 0$$

Using (1.4) in above we get

(3.2) $g\big((D_W R)(X,Y)Z, U\big) - \frac{dr(w)}{n(n-1)}[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] - \eta\big(((D_W R)(X,Y)Z\big)\eta(U) + \frac{dr(W)}{n(n-1)}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\eta(U) = 0$

Let $(e_i), i = 1,2,3 \ldots\ldots n$ be orthnormal basis of the tangent space at any point of the manifold the putting $X = U = e_i$ in (3.2) and taking summation over i, we get

(3.3)

$$0 = (D_w S)(Y,Z) - \frac{dr(W)}{n}g(Y,Z) - \eta\big(((D_W R)(e_i,Y)Z\big)\eta(e_i) + \frac{dr(W)}{n(n-1)}[g(Y,Z) - \eta(Y)\eta(Z)]$$

Putting $z = \xi$ in (3.3) and using , we get

(3.4) $(D_w S)(Y,\xi) - \frac{dr(W)}{n}\eta(Y) - \eta\big(((D_W R)(e_i,Y)\xi\big)\eta(e_i) = 0$

After simplification ,we get $\eta\big(((D_W R)(e_i,Y)\xi\big)\eta(e_i) = 0$

Then from (3.4), we have

(3.5) $(D_w S)(Y,\xi) = \frac{1}{n}dr(W)\eta(Y)$

Putting $Y = \xi$, we get $dr(w) = 0$ and this implies r is constant . So from (3.5) we have $(D_w S)(Y,\xi) = 0$ and this implies that $S(Y,W) = -(n-1)g(Y,W)$, Hence we state the following

Theorem (3.1): If a Para Saskian manifold is globally $\emptyset -$ concircularly symmetric , then manifold is Einstein manifold.

Further it is also well known that if the Ricci tensor S of the manifold is of the form $S(X,Y) = \lambda g(X,Y)$, where λ is constant and X,Y ∈ $X_n M$, then the manifold is Einstein manifold.

Now let us suppose that $S(X,Y) = \lambda g(X,Y)$,that is manifold is Einstein manifold then from (1.3) , we have

$$(D_W C)(X,Y)Z = (D_W R)(X,Y)Z$$

Apply $\emptyset^2$ both side , we get

$\emptyset^2(D_W C)(X,Y)Z = \emptyset^2(D_W R)(X,Y)Z$, Hence we state following

Theorem (3.2): A globally $\emptyset -$concircularly symmetric para Saskian manifold is globally $\emptyset -$symmetric.

Since a globally $\emptyset -$ symmetric para Saskian manifold is always globally $\emptyset -$ concircularly symmetric manifold then by theorem (3.2) we have

Theorem (3,3): On a para Saskian manifold globally $\emptyset -$ symmetric and globally $\emptyset -$ concircularly symmetric are equivalent.

IV. 3- dimensional Locally $\emptyset$ −concircularly symmetric para Sasakian manifold:

In a 3 dimensional para Sasakian manifold concircular curvature tensor is given by

(4.1) $C(X,Y)Z = \left(\frac{r+4}{2}\right)[g(Y,Z)X - g(X,Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X -$
$\eta X \eta Z Y - r6 g Y, Z X - g X, Z Y$

Taking covariant differentiation of (4.1), we get

(4.2)
$$(D_W C)(X,Y)Z = \frac{dr(w)}{2}[g(Y,Z)X - g(X,Z)Y] - \frac{dr(w)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)(D_W\eta)(X)\xi + g(Y,Z)\eta(X)(D_W\xi) - g(X,Z)(D_W\eta)(Y)\xi - g(X,Z)\eta(Y)(D_W\xi) + (D_W\eta)(Y)\eta(Z)X + (D_W\eta)(Z)\eta(Y)X - (D_W\eta)(X)\eta(Z)X - (D_W\eta)(Z)\eta(X)Y] - \frac{dr(w)}{6}[[g(Y,Z)X - g(X,Z)Y]]$$

Taking X, Y, Z horizontal vector field and using (2.5) and (2.6) , we get

(4.3)
$$(D_W C)(X,Y)Z = \frac{dr(w)}{3}[g(Y,Z)X - g(X,Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)]\xi$$

From (4.3) , it follows that

$$\emptyset^2(D_W C)(X,Y)Z = \frac{dr(w)}{3}[g(Y,Z)X - g(X,Z)Y]$$

Hence we state the following

Theorem (4.1): A 3-dimensional Para Sasakian manifold is locally $\emptyset -$ concircularly symmetric if and only if scalar curvature r is constant.

In 1977[4] has proved that

Corollarly (4.2): A 3-dimensional Para sasakian manifold is locally $\emptyset -$ symmetric if and only if scalr curvature r is constant.

Using corollary (4.2) we state the following

Theorem (4.3): A 3-dimensional Para Sasakian manifold is locally $\emptyset -$ concircularly symmetric if and only if it is locally $\emptyset -$ symmetric.

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