IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 8, Issue 1 (Sep. - Oct. 2013), PP 18-22
www.iosrjournals.org

Numerical Experiment of Systems of Non-Linear Volterra's Integro-Differential Equations Using New Variational Homotopy Perturbation Method

O. E. Abolarin
Department of Industrial Mathematics Landmark University, Omu-Aran, Nigeria.

Abstract: *This research paper deals with the solution of systems of Non-Linear Volterra's Integro-Differential Equations using the New Variational Homotopy Perturbation Method. The New Method does not require discritization, linearization or any restrictive assumption of any form in providing analytical or approximate solutions to linear and nonlinear equation. Theses virtues make it to be reliable and its efficiency is demonstrated with numerical examples.*
Keywords: *systems of Non-Linear Volterra's Integro-Differential equations; Variational Iteration Method, Homotopy Perturbation Method, New Variational Homotopy Perturbation Method.*

I. Introduction

One of the interesting topics among researchers is solving integro-differential equations. In fact, integro-differential equations arise in many physical processes, such as glass-forming process [1], nanohydrodynamics [2], drop wise condensation [3], and wind ripple in the desert [4]. There are various numerical and analytical methods to solve such problems, for example, the Homotopy perturbation method [5], the Adomian decomposition method [6], but each method limits to a special class of integro-differential equations. J.H. He used the variational iteration method for solving some integro-differential equations [7]. This Chinese mathematician chooses [7] initial approximate solution in the form of exact solution with unknown constants. M. Ghasemi et al solved the nonlinear Volterra's integro-differential equations [8] by using homotopy perturbation method. In [9], the variational iteration method was applied to solve the system of linear integro-differential equations. Also, J. Biazar et al solved systems of integro-differential equations by He's homotopy perturbation method [10]. S. Abbasbandy and E. Shivanian solved system of nonlinear volterra's integro-differential equations using Variational Iteration Method[11].The aim of this paper is to extend the analysis of the variational Homotopy Perturbation method to solve the system of nonlinear Volterra's integro-differential equations, as demonstrated by M. Matinfar et al [12], O. A. Taiwo and O. E. Abolarin [13].

II. New Variational Homotopy Perturbation Method

To demonstrate the new variational homotopy perturbation method, we consider the first **order** integro-differential equation given by

$$du/dx = f(x) + \int_0^x \psi(t,u(t),u'(t))dt \quad (1)$$

where $f(x)$ is the source term and $u(x)$ is the unknown which is to be determined via the new variational Homotopy perturbation method.

The correction functional according to variational iteration method can be constructed as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)[(u_n)_s - f(s) - \int_0^s \psi(t,\bar{u}(t)\bar{u}'(t))dt]ds \quad (2)$$

where $\bar{u}_n$ is considered as restricted variations, which means $\bar{u}_n = 0$. To find the optimal $\lambda(s)$, we proceed as follows:

$$\delta u_{n+1(x)} = \delta u_n(x) + \delta\int_0^x \lambda(s)[(u_n)_s - f(s) - \int_0^s \psi(t,\bar{u}(t)\bar{u}'(t))dt]ds \quad (3)$$

And consequently,

$$\delta u_{n+1(x)} = \delta u_n(x) + \delta\int_0^x \lambda(s)(u_n)_s\, ds \quad (4)$$

which results in,

$$\delta u_{n+1(x)} = \delta u_n(x) + \lambda(s)\delta u_n(x) - \int_0^x \delta u_n(x)\lambda'(s)ds \qquad (5)$$

The stationary conditions can be obtained as follows:

$$\lambda'(s) = 0 \text{ and } 1+\lambda(s)\big|_{s=t} = 0 \qquad (6)$$

The lagrange multipliers, therefore, can be identified as

$$\lambda(s) = -1$$

And the iteration formula is given as

$$u_{n+1}(x) = u_n(x) - \int_0^x [(u_n)_s - f(s) - \int_0^s \psi(t, u(t)u'(t))dt]ds \qquad (7)$$

Now, we implement the new variational Homotopy perturbation method on equ. (7) to obtain

$$\sum_{n=0}^{\infty} p^n u_n = u_0 - p\int_0^x [\sum_{n=0}^{\infty}(p^n u_n)_s - f(s) - \int_0^s \sum_{n=0}^{\infty} p^n \psi_n dt]ds \qquad (8)$$

the expansion of equ. (8), gives:

$$u_0 + pu_1 + p^2 u_2 + ... = u_0(x) -$$
$$p\int_0^x [(u_0 + pu_1 + ..)_s - f(s) - \int_0^s (\psi_0 + p\psi_1 + ..)dt]ds \qquad (9)$$

The comparison of the coefficients of like powers of p gives solutions of various orders of the form:

$$p^0 : u_0 = u_0(x)$$
$$p^1 : u_1 = -\int_0^x [\frac{\partial u_0}{\partial s} - f(s) - \int_0^s \psi_0 dt]ds$$
$$p^2 : u_2 = -\int_0^x [\frac{\partial u_1}{\partial s} - \int_0^s \psi_1 dt] \qquad (10)$$
$$.$$
$$p^n : u_n = -\int_0^x [\frac{\partial u_{n-1}}{\partial s} - \int_0^s \psi_{n-1} dt]ds$$

3. **Numerical Examples**

Example 1: consider the system of nonlinear integro-differential equations as follow [11]:

$$u''(x) = 1 - \frac{1}{3}x^3 - \frac{1}{2}v'^2(x) + \frac{1}{2}\int_0^x (u^2(t) + v^2(t))dt \qquad (11)$$

$$v''(x) = -1 + x^2 - xu(x) + \frac{1}{4}\int_0^x (u^2(t) - v^2(t))dt$$

$$u(0) = 1, u'(0) = 2, v(0) = -1, v'(0) = 0,$$

with the exact solutions

$$u(x) = x + e^x, v(x) = x - e^x.$$

We construct a correction functional according to the variational iteration method as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x (s-x)[u_n''(s) - 1 + \frac{1}{3}s^3 + \frac{1}{2}v'^2{}_n(s) - \frac{1}{2}\int_0^s (u^2{}_n(t) + v^2{}_n(t))dt]ds \quad (12)$$

$$v_{n+1}(x) = v_n(x) + \int_0^x (s-x)[v_n''(s) + 1 - s^2 + su_n(s) - \frac{1}{4}\int_0^s (u^2{}_n(t) - v^2{}_n(t))dt]ds \quad (13)$$

Applying Homotopy Perturbation Method to equations (12) and (13), we have

$$\begin{aligned} u_0 + pu_1 + .. = u_0 + p\int_0^x (s-x)[(u_0 + pu_1 + ..)'' - 1 + \frac{1}{3}s^3 + \frac{1}{2}(v_0 + pv_1 + ..)'^2 \\ - \frac{1}{2}\int_0^s ((u_0 + pu_1 + ..)^2 + (v_0 + pv_1 + ..)^2)dt]ds \end{aligned} \quad (14)$$

$$\begin{aligned} v_0 + pv_1 + .. = v_0 + p\int_0^x (s-x)[(v_0 + pv_1 + ..)'' + 1 - s^2 + s(u_0 + pu_1 + ..) \\ - \frac{1}{4}\int_0^s ((u_0 + pu_1 + ..)^2 - (v_0 + pv_1 + ..)^2))dt]ds \end{aligned} \quad (15)$$

Choosing initial approximations $u_0(x) = 2x + 1$ and $v_0(x) = 1$ that satisfies our initial conditions, then comparing the coefficient of like powers of p , we obtain

p^0 : $u_0(x) = 2x + 1$

$v_0(x) = 1$

$$\begin{aligned} p^1: \quad & u_1(x) = 1 + 2x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{12}x^4 + \frac{1}{60}x^5 \\ & v_1(x) = -1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{60}x^5 \end{aligned} \quad (16)$$

.

.

It is clear that this iteration converges to the exact solution and the same as obtained by variational iteration method [11].

Example 2: Consider the following nonlinear system of two integro-differential Equations

$$\begin{aligned} u'(x) &= 1 - \frac{1}{2}v'^2(x) + \int_0^s ((x-t)v(t) + u(t)v(t))dt \\ v'(x) &= 2x + \int_0^s ((x-t)u(t) - v^2(t) + u^2(t))dt \end{aligned} \quad (17)$$

$u(0) = 0,\ v(0) = 0$

with the exact solutions

$u(x) = \sinh(x),\ v(x) = \cosh(x).$

We construct a correction functional according to the variational iteration method as follows:

$$u_{n+1}(x) = u_n(x) - \int_0^x [u_n'(s) - 1 + \frac{1}{2}v'^2{}_n(s) - \int_0^s ((s-t)v_n(t) + u_n(t)v_n(t))dt]ds \quad (18)$$

$$v_{n+1}(x) = v_n(x) - \int_0^x [v_n'(s) - 2s - \int_0^s ((s-t)u_n(t) - v^2{}_n(t)u^2{}_n(t))dt]ds \quad (19)$$

Applying Homotopy perturbation method to equations (18) and (19), we have

$$(u_0 + pu_1 + ..) = u_0 - \int_0^x [(u_0 + pu_1 + ..)' - 1 + \frac{1}{2}(v_0 + pv_1 + ..)'^2$$
$$- \int_0^s ((s-t)(v_0 + pv_1 + ..) + (u_0 + pu_1 + ..)(v_0 + pv_1 + ..))dt]ds \quad (20)$$

$$v_0 + pv_1 + .. = v_0 - \int_0^x [(v_0 + pv_1 + ..)' - 2s -$$
$$\int_0^s ((s-t)u_n(t) - (v_0 + pv_1 + ..)^2 u^2{}_n(t))dt]ds \quad (21)$$

Choosing initial approximations $u_0(x) = 0$ and $v_0(x) = 1$ that satisfies our initial conditions, then comparing the coefficient of like powers of p , we obtain

$$p^0: \quad u_0(x) = 0$$
$$v_0(x) = 1$$
$$p^1: \quad u_1 = x + \frac{1}{6}x^3$$
$$v_1 = 1 + \frac{1}{2}x^2 \quad (22)$$
$$p^2: \quad u_1 = x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{504}x^7$$
$$v_1 = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{240}x^6 + \frac{1}{2016}x^8$$

.

.

Also, as the same of example 1, it is clear that this iteration converges to the exact solution and the same as obtained by variational iteration method [11].

III. Conclusion

In this paper, New Variational Homotopy Perturbation Method has been successfully applied to find the solutions of system of Non-Linear Volterra's Integro-Differential Equations and the results obtained compared favourably with the two convectional variational iteration and Homotopy Perturbation Method. It can be concluded that the NVHPM is very strong and efficient technique for finding approximation solutions for wide classes of problems. It is worth mentioning that the Method is computational cost friendly.

References:

[1] H. Wang, H. M. Fu, H. F. Zhang, et al., A practical thermodynamic method to calculate the best glass-forming composition for bulk metallic glasses, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 171-178.

[2] L. Xu, J. H. He, Y. Liu, Electrospun nanoporous spheres with Chinese drug, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 199-202.

[3] F. Z. Sun, M. Gao, S .H. Lei, et al., The fractal dimension of the fractal model of dropwise condensation and its experimental study, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 211-222.

[4] T. L. Bo, L. Xie, X. J. Zheng, Numerical approach to wind ripple in desert, Int. J. Nonlinear Sci. Numer. Simul. 8 (2) (2007) 223-228.

[5] M. El-Shahed, Application of He's homotopy perturbation method to Volterra's integro-differential equation, Int. J. Nonlinear Sci. Numer. Simul. 6 (2) (2005) 163-168.

[6] S. M. El-Sayed, D. Kaya, S. Zarea, The decomposition method applied to solve high-order linear Volterra–Fredholm integro-differential equations, Int. J. Nonlinear Sci. Numer. Simul. 5 (2) (2004) 105-112.

[7] S. Q. Wang, J. H. He, Variational iteration method for solving integro-differential equations, Phys. Lett. A 367 (2007) 188-191.

[8] J. Biazar, Solution of systems of integral-differential equations by Adomian decomposition method, Appl. Math. Comput. 168 (2005) 1232–1238.

[9] J, Saberi-Nadjafi, M. Tamamgar, The variational iteration method: A highly promising method for solving the system of integro-differential equations, Comput. Math. Appl., (Article in press).
[10] J. Biazar , H. Ghazvini, M. Eslami, He's homotopy perturbation method for systems of integro-differential equations, Chaos, Solitons \& Fractals (Article in press).
[11] S. Abbasbandy and E. Shivanian ''Application of the Variational Iteration Method for System of Nonlinear Volterra's Integro-Differential Equations'' Mathematical and Computational Applications, Vol. 14, No. 2, pp. 147-158, 2009.
[12] M. Matinfar et al ''Variational Homotopy Perturbation Method for the Fisher's Equation'' International Journal of Nonlinear Science Vol.9(2010) No.3,pp.374-378
[13] O. A. Taiwo and O. E. Abolarin ''Application of New Variational Homotopy Perturbation Method (NVHPM) for solving Fredholm Integral Equations of the Second Kind'' Pioneer Journal of Mathematics and Mathematical Sciences Vol. 3, No.2, 2011. Pp 237-246. http://www.pspchv.com/content 1 PJMMS Volume 3 2.html