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Penalty Function Selection Method Of Best-Fit Model Parameters Of Interacting Agricultural Crops In Oil Uncontaminated Utisol:Part 1

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Abstract: *The time series data of interacting legumes grown in oil uncontam- inated utisol have been collected. However, the dynamics of a best-fit math- ematical model that can be used to describe the interaction between cowpea and groundnut poses a challenging interdisciplinary approach. We propose to use the 1-norm penalty function method to select the best-fit model parame- ters from a list of other candidate logistic models. A mathematical analysis of this best-fit interspecific interaction model will be conducted. The novel results which we have achieved in this study will be presented and discussed.*
Key words: *and phrases. Best-fit parameters, agricultural data, 1-norm.*

I. Introduction

This simulation study is based on the current data which was collected by two experts in microbiology who are working in the Niger Delta Region of Nigeria ([1]). In their research report, the growth data of cowpea and groundnut over a growing season in weeks were provided in an uncontaminated utisol were obtained. However, their focus of research was not on numerical simulation analysis which uses the notion of the three popular mathematical norms to select the best-fit model parameters which characterize the interaction dynamics between cowpea and groundnut.

There is an extensive collection of literatures on themes which relate to plant- plant interactions, modelling biological interacting populations, equations of deter- minate growth and plant growth analysis to mention a few ([3]; [4]; [5]; [6]; [7]; [8]; [9]; [10]; [11]; [12]). All these useful citations deal more with clearly defined model formulations which indicate sufficient ecological insights. However, the application of which selection method is used to select the best-fit model parameters is rarely a computational approach. Because of the sophistication of measuring the agreement between provided data and simulated data over a time interval which is usually taken for granted in most parameter estimation analysis of biological interaction data, we will attempt to define the method of penalty function selection method otherwise called the cost function selection method in this paper. We would think that there may be some environmental perturbations and other features of the eco- logical instability which might change the best-fit behaviour between the given data and simulated data which we do not propose to model in this present simulation study.

Given these data on the growth of legumes ([1]), it is a challenging scientific prob- lem to construct a mathematical model which describes the dynamics of any two interacting legumes within an uncontaminated environmental setting. Attempting to develop these distinct nonlinear model equations will provide crop scientists with useful information such as the growth rates for two types of legumes, the intraspe- cific coefficients of legumes and their mass law action or interspecific coefficients as is popularly known in mathematical biology and molecular physics. The informa- tion about the doubling times of the two interacting legumes is an important insight which will assist further research in

the stabilization of the mathematical model of interacting legumes and other land equivalent ratio studies in livelihood analysis and possibly about the numerical simulation of mutualism due to increasing sea level rise from a competition interaction model between two legumes in severely affected communities in the Niger Delta Region of Nigeria.
For the purpose of our present study, we propose to utilize the penalty-function selection methods to select the best-fit model parameters from these uncontami- nated agricultural data. As far as we know, this novel innovative numerical sim- ulation technique has not been previously implemented computationally to tackle this interesting proposed scientific problem in the biological sciences.

II. Methodology

In this study, we will focus on the application of the three popular mathematical norms ([2]) to select the best-fit parameters which are expected to characterize the dynamics of the interaction between two legumes such as the cowpea and groundnut over the growth season in weeks. The main thrust of this method concerns how well to select the parameters which will provide the best-fit between the provided data and our simulated data. We will deduce this characteristic of these data if we can successfully find the local minimum from a sequence of the 1-norm, 2- norm and infinity-norm monotonic values. Without a detailed familiar simplication about the notions of mathematical norms, we will simply present our calculations subsequently which we have obtained in the context of selecting the best-fit model parameters from some uncontaminated data of cowpea and groundnut.

The core part of our method depends on an appropriate set up of a single initial value logistic model with only two parameters namely the intrinsic growth rate for the growing cowpea as well as the two similar parameters for the growing groundnut over a growing period in weeks. The data of [1] were collected every two weeks. Our proposed simulation technique is constructed using the logic that if the beginning of the first week starts on the first day, then the seond week will start on day 8. In our context of the length of the growing period, subsequently after every two weeks the growth data will be obtained by simulation on day 22, followed by day 36, then on day 50, on day 64 and on day 78. By this proven procedure, our simulated data and the provided data will have six data points. Therefore, we can now measure the error between the provided data and our simulated data provided that we can successfully find the local minimum from the penalty-functions sequence of monotonic precise values.

In the following sequence of results, our aim is to find the best-fit model param- eters using the three popular mathematical norms. The estimated daily growth rate for the cowpea data is 0.0225. This value is calculated by dividing the second data value of 1.81 by the first data value of 1.32 and taking the logarithm to basee. This estimated growth rate of 0.315695108679455 is called the weekly growth rate. Our expected daily growth rate of 0.022549650619961 can now be obtained by dividing this weekly growth rate by 14 since the data provided were collected every two weeks. For the purpose of this study, we prefer to consider only the estimated daily growth rate which is approximated to only 4 decimal places.

Examples	Calculation of the local minimum			
no	a	b	ss	1 − norm
1	0.0225	0.0132	1.71	3.0556
2	0.0225	0.0115	1.96	2.3841
3	0.0225	0.0102	2.21	1.7767
4	0.0225	0.0091	2.46	1.2506
5	0.0225	0.0083	2.71	1.1121
6	0.0225	0.0076	2.96	1.0363
7	0.0225	0.0070	3.21	0.9671
8	0.0225	0.0065	3.46	1.2757
9	0.0225	0.0061	3.71	1.6423
10	0.0225	0.0057	3.96	2.0341

Table 1. Calculation of the local minimum for the Cowpea data:
the notation ss stands for the steady state

Now, what do we learn from Table 1? It is very characteristic of this table that all the values of the 1-norm start first to decrease to a value, then begin to increase. This critical value in the behaviour of the monotonic sequence of values is called the local minimum for the 1-norm. Hence, the local minimum for the 1-norm is 0.9671.

Our next dauting task is to attempt to search for the smaller values of the local minimum than this local minimum which we have calculated. We decided to tackle this challenging numerical simulation problem by the tedious process of a further gridding around this value of the intraspecific coefficient b of the cowpea data where a smaller local minimum can be found. Without detailed explanations which we have done in the previous sections, we will henceforth only display the final outputs of our calculated results about the search for a smaller local minimum from the cowpea time series data.

From Table 2, we are yet to find a smaller local minimum of which the best- fit model parameters of these cowpea data between the provided data and our simulated data can be selected. So far, all the error values are bigger than the local minimum value of the 1-norm which is 0.9671.
Next, we will conduct a second set of further gridding to find out if we can select best-fit parameters from a smaller local minimum for the 1-norm.

From Table 3, all the error values for the 1-norm best-fit selection methods are bigger than the local minimum value of 0.9671 for the 1-norm.

From Table 4, we are yet to find a smaller local minimum for the 1-norm best-fit penalty function selection method. In this scenario, we will continue to search for the best-fit model parameters which correspond to the local minimum using the **1-norm method.**

To embark upon the systematic search for a smaller local minimum using the 1- norm penalty function method which is smaller than the first local minimum value of 0.9671, we will prefer to present the summary of our several repeated calculations which would clarify how we have obtained a smaller 1-norm local minimum for a chosen parameter space. When the thirteen values of the intraspecific coefficient

Examples	Calculation of the local minimum			
no	a	b	ss	1 − norm
1	0.0225	0.008523	2.64	1.1526
2	0.0225	0.008491	2.65	1.1469
3	0.0225	0.008459	2.66	1.1411
4	0.0225	0.008427	2.67	1.1354
5	0.0225	0.008396	2.68	1.1297
6	0.0225	0.008364	2.69	1.1239
7	0.0225	0.008333	2.70	1.1182
8	0.0225	0.008303	2.71	1.1126
9	0.0225	0.008272	2.72	1.1069
10	0.0225	0.008242	2.73	1.1012
11	0.0225	0.008212	2.74	1.0956
12	0.0225	0.008182	2.75	1.0899
13	0.0225	0.008152	2.76	1.0869

Table 2. Calculation of the local minimum for the Cowpea data:
first set of further gridding

Examples	Calculation of the local minimum			
no	a	b	ss	1 − norm
1	0.0225	0.008123	2.77	1.0846
2	0.0225	0.008094	2.78	1.0821
3	0.0225	0.008065	2.79	1.0797
4	0.0225	0.008036	2.80	1.0772
5	0.0225	0.008007	2.81	1.0747
6	0.0225	0.007979	2.82	1.0723
7	0.0225	0.007951	2.83	1.0698
8	0.0225	0.007923	2.84	1.0673
9	0.0225	0.007895	2.85	1.0648
10	0.0225	0.007867	2.86	1.0622
11	0.0225	0.007840	2.87	1.0597
12	0.0225	0.007813	2.88	1.0572
13	0.0225	0.007786	2.89	1.0546

Table 3. Calculation of the local minimum for the Cowpea data:

second set of further gridding
are 0.007426, 0.007401, 0.007377, 0.007353, 0.007330, 0.007305, 0.007282, 0.007258,
0.007235, 0.007212, 0.007189, 0.007166 and 0.007143, we have found the monotonic sequence of the 1-norm values to be 1.0178, 1.0151, 1.0124, 1.0097, 1.0071, 1.0043,
1.0016, 0.9988, 0.9961, 0.9934, 0.9906, 0.9879 and 0.9851. We observe from our calculations that none of these 1-norm values or error values between the model of [1] and our simualted data over a time interval is smaller than our target 1-norm value of 0.9671.
In order to overcome this challenging parameter estimation problem, we choose another parameter space with the expectation that we may obtain a smaller 1-norm local minimum. In this scenario, we will present our calculations in the table below.

Examples	Calculation of the local minimum			
no	a	b	ss	1 − norm
1	0.0225	0.007759	2.90	1.0520
2	0.0225	0.007732	2.91	1.0494
3	0.0225	0.007706	2.92	1.0469
4	0.0225	0.007680	2.93	1.0443
5	0.0225	0.007653	2.94	1.0416
6	0.0225	0.007627	2.95	1.0390
7	0.0225	0.007601	2.96	1.0364
8	0.0225	0.007576	2.97	1.0338
9	0.0225	0.007550	2.98	1.0311
10	0.0225	0.007525	2.99	1.0285
11	0.0225	0.007500	3.00	1.0258
12	0.0225	0.007475	3.01	1.0232
13	0.0225	0.007450	3.02	1.0205

Table 4. Calculation of the local minimum for the Cowpea data:
third set of further gridding

Examples	Calculation of the local minimum			
no	a	b	ss	1 − norm
1	0.0225	0.007120	3.16	0.9822
2	0.0225	0.007098	3.17	0.9795
3	0.0225	0.007076	3.18	0.9767
4	0.0225	0.007053	3.19	0.9738
5	0.0225	0.007009	3.20	0.9682
6	0.0225	0.006988	3.21	0.9655
7	0.0225	0.006966	3.22	0.9626
8	0.0225	0.006944	3.23	0.9597
9	0.0225	0.006923	3.24	0.9569
10	0.0225	0.006902	3.25	0.9541
11	0.0225	0.006881	3.26	0.9570
12	0.0225	0.006860	3.27	0.9653

Table 5. Calculation of the local minimum for the Cowpea data:
another set of further gridding

From Table 5, we can clearly observe that we have now found a smaller local minimum value of 0.9541 when compared to the first local minimum value of 0.9671 when the 1-norm penalty function selection method was implemented.

III. Discussion of Results

In this numerical simulation problem in the context of using the 1-norm to select the best-fit logistic model parameters between the provided model of [1] and our simulated data, we have found the following useful results which we have not seen elsewhere.

Based on the [1] cowpea data, our estimated local minimum value using the 1-norm is 0.9671. This first local minimum corresponds to the best-fit parameter values of a = 0.0225, b = 0.0070, ss = 3.21, T = 0 : 1 : 90 and the initial condition value of 1.50. The provided fourth nightly data are 1.32, 1.81, 1.91, 2.0, 2.54and 2.71. In this scenario, our corresponding simulated fourthly nightly data are represented as Y (8), Y (22), Y (36), Y (50), Y (64) and Y (78). Here, the data point Y (8) of our simulated data specifies the value at the beginning of the second week which compares with the provided data value of 1.32. The data point Y (22) of our simulated data specifies the value after two weeks which compares with the provided data value of 1.81. Other data points of our simulated data specify the values after every two weeks. Our algorithm which we have designed to calculate the difference between the provided data and our simulated data over a time interval of 90 days and also implemented to calculate the size of the errors between the provide data and our simulated data is based on this detailed idea.

A further gridding produces a smaller local minimum value using the 1-norm penalty function selection method which is 0.9541. Here, the corresponding best-fit model parameters are a = 0.0225, b = 0.006902, ss = 3.26, T = 0 : 1 : 90 and the initial condition value of 1.50 with the same provided data sets and simulated data over a time interval.

Therefore, our best-fit selected logistic candidate model using the 1-norm penalty function selection method for the growing cowpea over time in weeks is

$$\frac{dC(t)}{dt} = C(t)(0.0225 - 0.006902C(t)). \quad (3.1)$$

with the initial groundnut biomass of 1.50 grams.

So far, we have obtained a total of 2 deterministic initial value logistic models which fully approximate the dynamics of self-interaction for individual cowpea and groundnut time series data. For this present simulation analysis to provide further insights into the understanding of the interaction dynamics between the cowpea and groundnut legumes in an uncontaminated environment, it is worthwhile to specify the interspecific interaction component of our proposed self-interaction or intraspecific models. In this context, we will match the 1-norm selected logistic model for cowpea to the 1-norm selected logistic model for groundnut.

Therefore, the 1-norm penalty function selected interspecific interaction model between cowpea and groundnut under a realistic assumed value of the interspecific coefficient is

$$\frac{dC(t)}{dt} = C(t)(0.0225 - 0.006902C(t) - 0.0005G(t)). \quad (3.3)$$

with the initial cowpea biomass of 1.50 grams.

For the groundnut data ([1]), we will only present our first local minimum and a further gridding local minimum without going into the detailed presentation of our calculations. First the local minimum using the 1-norm penalty function selection method is 3.1842. The best-fit parameters are a = 0.0446 (which is called the daily growth rate for the groundnut data is similarly calculated by taking the logarithm to base e of the second data value of 2.82 divided by the first data value of 1.51 and dividing the obtained value by 14), b = 0.0127, ss = 3.50, T = 0 : 1 : 90 and the initial biomass of groundnut having the value of 1.50 grams. By implementing the principle of a further gridding and using the 1-norm penalty function selection method, the new local minimum for the groundnut data is 3.1307. In this scenario, the corresponding best-fit parameters are a = 0.00446, b = 0.0133, ss = 3.35, T = 0 : 1 : 90 and the initial groundnut biomass of 1.50 grams. Therefore, the best- fit selected logistic candidate model using the 1-norm penalty function selection method for the growing groundnut over time in weeks is

$$\frac{dC(t)}{dt} = C(t)(0.0225 - 0.006902C(t) - 0.0005G(t)). \quad (3.3)$$

$$\frac{dG(t)}{dt} = G(t)(0.0446 - 0.0133G(t) - 0.01C(t)). \quad (3.4)$$

where the initial biomasses are as specified earlier on in this study.

Without loss of generality, following the mathematical theories of steady-state solutions and stability, this mathematical analysis of the above Lotka-Volterrra system is reported as follows:

Examples	Calculation of the stability behaviour			
no	steady-state solutions	λ_1	λ_2	Each Type of Stability
1	(0, 0)	0.0225	0.0446	Unstable
2	(0, 3.3534)	-0.0446	0.0208	Unstable
3	(3.2599, 0)	-0.0225	0.0120	Unstable
4	(3.1908, 0.9543)	-0.0234	-0.113	Stable

Table 6. Numerical calculations of the steady-state solutions and their stability

It is very clear from this study that our best-fit mathematical model has only four steady-state solutions out of which three are unstable (and will require further stabilizations) while the only co-existence steady-state solution is said to be stable. The trivial steady-state solution implies that the two interacting populations of cowpea and groundnut will go into the ecological risk of extinction while the semi- trivial steady-state solutions will imply the theory of competitive exclusion, that is, one population will survive at its carrying capacity while the second population will be driven into extinction.

IV. Concluding Remarks and Further Research

In this challenging study, we have achieved the following: we have used the numerical technique of a 1-norm penalty function selection method to select one mathematical model which can describe the interaction between growing legumes of cowpea and groudnut in an uncontaminated utisol. This emprically determined deterministic model guarantees the existence of the carrying capacity value for both the cowpea and groundnut populations and the inhibiting effect of cowpea on the growth of groundnut as well as the inhibiting effect of groundnut on the growth of cowpea. The mathematical analysis of the best-fit candidate model shows that the system consisting of a co-existence steady-state solution will be stable using the popular theory of the sign method of testing for the stability of a steady-state solution. This steady-state solution satisfies the critical inequalities for the survival of the cowpea and groundnut legumes in competition for limited resources.

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