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On Semi-$\mathcal{I}_s$-Open Sets and Semi-$\mathcal{I}_s$-Continuous Functions

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Abstract: *We study the concepts of semi-$\mathcal{I}_s$-open sets and semi-$\mathcal{I}_s$-continuous functions introduced in [13] and some properties of the functions. Also we introduce notion of semi-$\mathcal{I}_s$- open and semi-$\mathcal{I}_s$-closed functions.*
Keywords: *semi-$\mathcal{I}_s$-open set, semi-$\mathcal{I}_s$-continuous function.*

I. Introduction

Ideal in topological space have been considered since 1930 by Kuratowski[9] and Vaidyanathaswamy[14]. After several decades, in 1990, Jankovic and Hamlett[6] investigated the topological ideals which is the generalization of general topology. Where as in 2010, Khan and Noiri[7] introduced and studied the concept of semi-local functions. The notion of semi-open sets and semi-continuity was first introduced and investigated by Levine [10] in 1963. Finally in 2005, Hatir and Noiri [4] introduced the notion of semi-$\mathcal{I}$-open sets and semi-$\mathcal{I}$-continuity in ideal topological spaces. Recently we introduced semi-$\mathcal{I}_s$-open sets and semi-$\mathcal{I}_s$-continuity to obtain decomposition of continuity.

In this paper, we obtain several characterizations of semi-$\mathcal{I}_s$-open sets and semi-$\mathcal{I}_s$-continuous functions. Also we introduce new functions semi-$\mathcal{I}_s$-open and semi-$\mathcal{I}_s$-closed functions

II. Preliminaries

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X, τ), cl(A) and int(A) denote the closure and interior of A in (X, τ) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is an nonempty collection of subsets of X which satisfies: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$ (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

If (X, τ) is a topological space and $\mathcal{I}$ is an ideal on X, then (X, τ, $\mathcal{I}$) is called an ideal topological space or an ideal space.

Let P(X) be the power set of X. Then the operator $(\ )^*$: P(X) → P(X) called a local function [9] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I}, \tau) = \{ x \in X / U \cap A \notin \mathcal{I}$ for every open set U containing x}. We simply write A^* instead of $A^*(\mathcal{I}, \tau)$ in case there is no confusion. For every ideal topological space (X, τ, $\mathcal{I}$) there exists topology τ^* finer than τ, generated by $\beta(\mathcal{I}, \tau) = \{ U \setminus J: U \in \tau$ and $J \in \mathcal{I} \}$but in general $\beta(\mathcal{I}, \tau)$ is not always a topology. Additionally $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ. Throughout this paper X denotes the ideal topological space (X, τ, $\mathcal{I}$) and also $int^*(A)$ denotes the interior of A with respect to τ^*.

Definition 2.1. Let (X, τ) be a topological space. A subset A of X is said to be semi-open [10] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by SO(X) (resp. SC(X)). The semi-closure of A in (X, τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by scl(A).

Definition 2.2. For $A \subseteq X$, $A_*(\mathcal{I}, \tau) = \{ x \in X / U \cap A \notin \mathcal{I}$ for every $U \in SO(X)\}$ is called the semi-local function[7] of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{ U \in SO(X) / x \in U\}$. We simply write A_* instead of $A_*(\mathcal{I}, \tau)$ in this case there is no ambiguity.

It is given in [2] that $\tau^{*s}(\mathcal{I})$ is a topology on X, generated by the sub basis $\{ U - E : U \in SO(X)$ and $E \in \mathcal{I} \}$ or equivalently $\tau^{*s}(\mathcal{I}) = \{U \subseteq X: cl^{*s}(X - U) = X - U \}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathcal{I})$ is defined as follows: for $A \subseteq X$, $cl^{*s}(A) = A \cup A_*$ and int^{*s} denotes the interior of the set A in (X, τ^{*s}, $\mathcal{I}$). It is known that $\tau \subseteq \tau^*(\mathcal{I}) \subseteq \tau^{*s}(\mathcal{I})$. A subset A of (X, τ, $\mathcal{I}$) is called semi-∗-perfect[8] if $A = A_*$. $A \subseteq$ (X, τ, $\mathcal{I}$) is called ∗-semi dense in-itself [8] (resp. Semi-∗-closed [8]) if $A \subset A_*$ (resp. $A_* \subseteq A$).

Lemma 2.3. [7] Let (X, τ, $\mathcal{I}$) be an ideal topological space and A, B subsets of X. Then for the semi-local function the following properties hold:

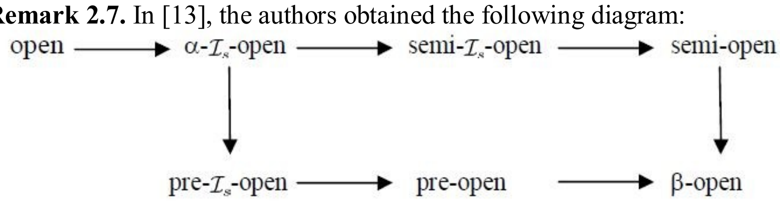
1. If $A \subseteq B$ then $A_* \subseteq B_*$.
2. If $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$
3. $A_* = scl(A_*) \subseteq scl(A)$ and A_* is semi-closed in X.
4. $(A_*)_* \subseteq A_*$
5. $(A \cup B)_* = A_* \cup B_*$
6. If $\mathcal{I} = \{\phi\}$, then $A_* = scl(A)$.

Definition 2.4. A subset A of a topological space X is said to be
1. α-open [12] if $A \subseteq int(cl(int(A)))$,
2. pre-open [11] if $A \subseteq int(cl(A))$,
3. β-open[1] if $A \subseteq cl(int(cl(A)))$.

Definition 2.5. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be
1. α-$\mathcal{I}$-open[4] if $A \subseteq int(cl^*(int(A)))$,
2. semi-$\mathcal{I}$-open [4] if $A \subseteq cl^*(intA))$,
3. pre-$\mathcal{I}$-open [3] if $A \subseteq int(cl^*(A))$.

Definition2.6. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be
1. α-$\mathcal{I}_s$-open[13] if $A \subseteq int(cl^{*s}(int(A)))$,
2. semi-$\mathcal{I}_s$-open [13] if $A \subseteq cl^{*s}(intA))$,
3. pre-$\mathcal{I}_s$-open [13] if $A \subseteq int(cl^{*s}(A))$.

Remark 2.7. In [13], the authors obtained the following diagram:



By SISO(X,τ), we denote the family of all semi-$\mathcal{I}_s$-open sets of a space $(X, \tau, \mathcal{I})$.

III. Semi-$\mathcal{I}_s$-open sets

Theorem3.1. A subset A of a space $(X, \tau, \mathcal{I})$ is semi-$\mathcal{I}_s$-open if and only if $cl^{*s}(A) = cl^{*s}(int(A))$.

Proof. Let A be semi-$\mathcal{I}_s$-open, we have $A \subseteq cl^{*s}(int(A))$. Then $cl^{*s}(A) \subseteq cl^{*s}(int(A))$. Obviously $cl^{*s}(int(A)) \subseteq cl^{*s}(A)$. Hence $cl^{*s}(A) = cl^{*s}(int(A))$. The converse is obvious.

Theorem3.2. A subset A of a space $(X, \tau, \mathcal{I})$ is semi-$\mathcal{I}_s$-open if and only if there exists $U \in \tau$ such that $U \subseteq A \subseteq cl^{*s}(U)$.

Proof. Let A be semi-$\mathcal{I}_s$-open, we have $A \subseteq cl^{*s}(int(A))$. Take $int(A) = U$. Then we have $U \subseteq A \subseteq cl^{*s}(U)$. Conversely, let $U \subseteq A \subseteq cl^{*s}(U)$ for some $U \in \tau$. Since $U \subseteq A$ we have $U \subseteq int(A)$ and hence $cl^{*s}(U) \subseteq cl^{*s}(int(A))$. Thus we obtain $A \subseteq cl^{*s}(int(A))$.

Theorem3.3. If A is semi-$\mathcal{I}_s$-open set in a space $(X, \tau, \mathcal{I})$ and $A \subseteq B \subseteq cl^{*s}(A)$, then B is semi-$\mathcal{I}_s$-open in $(X, \tau, \mathcal{I})$.

Proof. Since A is semi-$\mathcal{I}_s$-open, there exists an open set U such that $U \subseteq A \subseteq cl^{*s}(U)$. Then we have $U \subseteq A \subseteq B \subseteq cl^{*s}(A) \subseteq cl^{*s}(cl^{*s}(U)) = cl^{*s}(U)$ and hence $U \subseteq B \subseteq cl^{*s}(A)$. By Theorem 3.2, we obtain B is semi-$\mathcal{I}_s$-open.

Theorem3.4. Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B subsets of X.
1. If $U_\alpha \in SISO(X, \tau)$ for each $\alpha \in \Delta$, then $\cup\{U_\alpha : \alpha \in \Delta\} \in SISO(X, \tau)$,
2. If $A \in SISO(X, \tau)$ and $B \in \tau$, then $A \cap B \in SISO(X, \tau)$.

Proof.
1. Since $U_\alpha \in SISO(X, \tau)$, we have $U_\alpha \subseteq cl^{*s}(int(U_\alpha))$ for each $\alpha \in \Delta$. Thus by using Lemma 2.3, we obtain

$$\bigcup_{\alpha\in\Delta} U_\alpha \subseteq \bigcup_{\alpha\in\Delta} cl^{*s}(int(U_\alpha)) \subseteq \bigcup_{\alpha\in\Delta}\{(int(U_\alpha))_* \cup (int(U_\alpha))\} \subseteq \left(\bigcup_{\alpha\in\Delta}(int(U_\alpha))\right)_* \cup int\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)$$

$$\subseteq \left(\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)\right)_* \cup \mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right) = \mathrm{cl}^{*s}\left(\mathrm{int}\left(\bigcup_{\alpha\in\Delta} U_\alpha\right)\right).$$

This shows that $\bigcup_{\alpha\in\Delta} U_\alpha \in \mathrm{SISO}(X,\tau)$.

2. Let A ∈ SISO(X, τ) and B ∈ τ. Then A ⊆ cl^{*s}(int(A)) and by using Lemma 2.3, we have A ∩ B ⊆ cl^{*s}(int(A)) ∩ B = ((int(A))$_*$ ∪ (int(A))) ∩ B = ((int(A))$_*$ ∩ B) ∪ (int(A) ∩ B) ⊆ (int(A) ∩ B)$_*$ ∪ int(A ∩ B) = (int(A ∩ B))$_*$ ∪ int(A ∩ B) = cl^{*s}(int(A ∩ B)). This shows that A ∩ B ∈ SISO(X, τ).

Definition3.5. A subset F of a space (X, τ, $\mathcal{I}$) is said to be semi-$\mathcal{I}_s$-closed if its complement is semi-$\mathcal{I}_s$-open.

Remark3.6. For a subset A of a space (X, τ, $\mathcal{I}$), we have X− int(cl^{*s}(A)) ≠ cl^{*s}(int(X − A)) as shown from the following example.

Example3.7. Let X = {a, b, c, d}, τ = {ϕ, X, {a}, {b}, {a, b}} and $\mathcal{I}$ = {ϕ, {b}, {c}, {b, c}}. Then we put A = {b}, we have cl^{*s}(int(X − A)) = cl^{*s}({a}) = {a} and X − int(cl^{*s}(A)) = X − int({b}) = {a, c, d}.

Theorem 3.8. If a subset A of a space (X, τ, $\mathcal{I}$) is semi-$\mathcal{I}_s$-closed, then int(cl^{*s}(A)) ⊆ A.
Proof. Since A is semi-$\mathcal{I}_s$-closed, X − A ∈ SISO(X, τ). Since τ^{*s}(I) is finer than τ, we have X − A ⊆ cl^{*s}(int(X − A)) ⊆ cl(int(X − A)) = X − int(cl(A)) ⊆ X − int(cl^{*s}(A)). Therefore we obtain int(cl^{*s}(A)) ⊆ A.

Corollary3.9. Let A be a subset of a space (X, τ, $\mathcal{I}$) such that X − int(cl^{*s}(A)) = cl^{*s}(int(X − A)). Then A is semi-$\mathcal{I}_s$-closed if and only if int(cl^{*s}(A)) ⊆ A.
Proof. This is an immediate consequence of Theorem 3.8.

Theorem3.10. [8] Let (X, τ, $\mathcal{I}$) be an ideal space and A ⊆ Y ⊆ X, where Y is α-open in X. Then $A_*(\mathcal{I}_Y, \tau|_Y) = A_*(\mathcal{I}, \tau) \cap Y$.

Theorem3.11. Let (X, τ, $\mathcal{I}$) be an ideal topological space. If Y ∈ τ and W ∈ SISO(X), then Y ∩ W ∈ SISO(Y, $\tau|_Y$, $\mathcal{I}_Y$).
Proof. Since Y is open, we have int_Y(A) = int(A) for any subset A of Y. Now Y ∩ W ⊆ Y ∩ cl^{*s}(int(W)) = Y ∩ ((int(W))$_*$ ∪ int(W)) = [(Y ∩ (int(W))$_*$) ∪ (Y ∩ int(W))] ∩ Y = [Y ∩ (Y ∩ (int(W)))$_*$] ∪ [(Y ∩ int(W)) ∩ Y] = Y ∩ [int_Y (Y ∩W)]$_*$ ∪ (Y ∩ [int_Y (Y ∩ W)]) = [int_Y (Y ∩W)]$_*$ ($\mathcal{I}_Y$, $\tau|_Y$) ∪ [int_Y (Y ∩ W)] = cl_Y^{*s}(int_Y(Y ∩ W)). This shows that Y ∩ W ∈ SISO(Y, $\tau|_Y$, $\mathcal{I}_Y$).

IV. Semi-$\mathcal{I}_s$-continuous functions

Definition4.1. A function *f* : (X, τ, $\mathcal{I}$) → (Y, σ) is said to be semi-$\mathcal{I}_s$-continuous [13] (resp. semi-$\mathcal{I}$-continuous [4], semi-continuous [10]) if f^{-1}(V) is semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}$-open, semi-open) in (X, τ, $\mathcal{I}$) for each open set V of (Y, σ).

Definition4.2. A function *f* : (X, τ, $\mathcal{I}$) → (Y, σ, $\mathcal{J}$) is said to be $\mathcal{I}_s$-irresolute (resp. $\mathcal{I}$-irresolute [5]) if f^{-1} (V) is semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}$-open) in (X, τ, $\mathcal{I}$) for each semi-$\mathcal{J}_s$-open set(resp. semi-$\mathcal{J}$-open set) V of (Y, σ, $\mathcal{J}$).

Remark4.3. It is obvious that continuity implies semi-$\mathcal{I}_s$-continuity, semi-$\mathcal{I}_s$-continuity implies semi-$\mathcal{I}$-continuity and semi-$\mathcal{I}$-continuity implies semi-continuity.

Theorem 4.4. For a function *f* : (X, τ, $\mathcal{I}$) → (Y, σ), the following are equivalent:
1. *f* is semi-$\mathcal{I}_s$-continuous,
2. for each x ∈ X and each V ∈ σ containing *f* (x), there exists W ∈ SISO(X, τ) containing x such that *f* (W) ⊆ V,
3. the inverse image of each closed set in Y is semi-$\mathcal{I}_s$-closed.

Proof. (1) ⇒ (2). Let x ∈ X and V be any open set of Y containing *f* (x). Set W = f^{-1}(V), then by Definition 4.1, W is a semi-$\mathcal{I}_s$-open set containing x and *f* (W) ⊆ V.
(2) ⇒ (3). Let F be a closed set of Y. Set V = Y − F, then V is open in Y. Let x ∈ f^{-1}(V), by (2), there exists a semi-$\mathcal{I}_s$-open set W of X containing x such that *f* (W) ⊆ V. Thus, we obtain x ∈ W ⊆ cl^{*s}(int(W)) ⊆

$cl^{*s}(int(f^{-1}(V)))$ and hence $f^{-1}(V) \subseteq cl^{*s}(int(f^{-1}(V)))$. This shows that $f^{-1}(V)$ is semi-$\mathcal{I}_s$-open in X. Hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(V)$ is semi-$\mathcal{I}_s$-closed in X.
(3) $\Rightarrow$ (1). Let V be a open set of Y. Set F = Y − V, then F is closed in Y. $f^{-1}(V) = X - f^{-1}(Y - V) = X - f^{-1}(F)$. By (3) f^{-1} (V) is semi-$\mathcal{I}_s$-open in X.

Theorem 4.5. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, be semi-$\mathcal{I}_s$-continuous and U $\in \tau$. Then the restriction $f|_U : (U, \tau|_U, \mathcal{I}_U) \to (Y, \sigma)$ is semi-$\mathcal{I}_s$-continuous.
Proof. Let V be any open set of (Y, σ). Since f is semi-$\mathcal{I}_s$-continuous, $f^{-1}(V) \in SISO(X, \tau)$ and by Theorem 3.11, $(f|_U)^{-1}(V) = f^{-1}(V) \cap U \in SISO(U, \tau|_U)$. This shows that $f|_U: (U, \tau|_U, \mathcal{I}_U) \to (Y, \sigma)$ is semi-$\mathcal{I}_s$-continuous.

Theorem 4.6. For function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$, the following hold.
1. $g \circ f$ is semi-$\mathcal{I}_s$-continuous if f is semi-$\mathcal{I}_s$-continuous and g is continuous.
2. $g \circ f$ is semi-$\mathcal{I}_s$-continuous if f is $\mathcal{I}_s$-irresolute and g is semi-$\mathcal{I}_s$-continuous.
Proof. It is Obvious.

Theorem 4.7. A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is semi-$\mathcal{I}_s$-continuous if and only if the graph function $g : X \to X \times Y$, defined by $g(x) = (x, f(X))$ for each $x \in X$, is semi-$\mathcal{I}_s$-continuous.
Proof. Necessity. Suppose that f is semi-$\mathcal{I}_s$-continuous. Let $x \in X$ and W be any open set of X × Y containing g(x). Then there exists a basic open set U × V such that $g(x) = (x, f(x)) \in U \times V \subseteq W$. Since f is semi-$\mathcal{I}_s$-continuous, then there exists a semi-$\mathcal{I}_s$-open set $U_\circ$ of X containing x such that $f(U_\circ) \subseteq V$. By Theorem 3.4 $U_\circ \cap U \in SISO(X, \tau)$ and $g(U_\circ \cap U) \subseteq U \times V \subseteq W$. This shows that g is semi-$\mathcal{I}_s$-continuous.
Sufficiency: Suppose that g is semi-$\mathcal{I}_s$-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then X × V is open in X × Y and by semi-$\mathcal{I}_s$-continuity of g, there exists $U \in SISO(X, \tau)$ containing x such that $g(U) \subseteq X \times V$. Therefore we obtain $f(U) \subseteq V$. This shows that f is semi-$\mathcal{I}_s$-continuous.

Theorem 4.8. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ be semi-$\mathcal{I}_s$-continuous and $f^{-1}(V_*) \subseteq (f^{-1}(V))_*$ for each $V \in \sigma$. Then f is $\mathcal{I}_s$-irresolute.
Proof. Let B be any semi-$\mathcal{J}$-open set of (Y, σ, $\mathcal{J}$). By Theorem 3.2, there exists $V \in \sigma$ such that $V \subseteq B \subseteq cl^{*s}(V)$. Therefore, we have $f^{-1}(V) \subseteq f^{-1}(B) \subseteq f^{-1}(cl^{*s}(V)) \subseteq cl^{*s}(f^{-1}(V))$. Since f is semi-$\mathcal{I}_s$-continuous and $V \in \sigma$, $f^{-1}(V) \in SISO(X, \tau)$ and hence by Theorem 3.3, $f^{-1}(B)$ is semi-$\mathcal{I}_s$-open in (X, τ, $\mathcal{I}$). This shows that f is $\mathcal{I}_s$-irresolute.

V. Semi-$\mathcal{I}_s$-open and semi-$\mathcal{I}_s$-closed functions

Definition 5.1. A function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ is called semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}_s$-closed) if for each $U \in \tau$ (resp. U is closed) $f(U) \in SISO(Y, \sigma, \mathcal{J})$ (resp. $f(U)$ is semi-$\mathcal{I}_s$-closed set).

Definition 5.2. [5] A function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ is called semi-$\mathcal{I}$-open (resp. semi-$\mathcal{I}$-closed) if for each $U \in \tau$ (resp. U is closed) $f(U)$ is semi-$\mathcal{I}$-open (resp. $f(U)$ is semi-$\mathcal{I}$-closed) set in (Y, σ, $\mathcal{J}$).

Remark 5.3. 1. Every semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}_s$-closed) function is semi-open (resp. semi-closed) and the converses are false in general.
2. Every semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}_s$-closed) function is semi-$\mathcal{I}$-open (resp. semi-$\mathcal{I}$-closed) and the converses are false in general.
3. Every open function is semi-$\mathcal{I}_s$-open but the converse is not true in general.

Example 5.4. Let X = {a, b, c, d}, τ = {ϕ, X, {a, b}}, σ = {ϕ, X, {a}, {b}, {a, b}} and $\mathcal{J}$ = {ϕ, {b}, {c}, {b, c}}. Define a function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ as follows $f(a) = b, f(b) = c, f(c) = f(d) = a$. Then f is semi-open, but it is not semi-$\mathcal{I}_s$-open.

Example 5.5. Let X = {a, b, c, d}, τ = {ϕ, X, {a, b}}, σ = {ϕ, X, {a}, {b}, {a, b}} and $\mathcal{J}$ = {ϕ, {b}, {c}, {b, c}}. Define a function $f : (X, \tau) \to (X, \sigma, \mathcal{J})$ as follows $f(a) = a, f(b) = c, f(c) = f(d) = d$. Then f is semi-$\mathcal{I}$-open, but it is not semi-$\mathcal{I}_s$-open.

Example 5.6. Let X = {a, b, c, d}, τ = {ϕ, X, {c}, {a, b, d}}, σ = {ϕ, X, {c}, {a, b}, {a, b, c}} and $\mathcal{J}$= {ϕ, {a}}. The identity function f : (X, τ) → (Y, σ, $\mathcal{J}$) is semi-$\mathcal{I}_s$-open, but it is not open.

Example 5.7. Let X = {a, b, c, d}, τ = {ϕ, X, {a, b}}, σ = {ϕ, X, {a}, {b}, {a, b}} and $\mathcal{J}$ = {ϕ, {b}, {c}, {b, c}}. The identity function f : (X, τ) → (Y, σ, $\mathcal{J}$) is semi-closed, but it is not semi-$\mathcal{I}_s$-closed.

Theorem 5.8. A function f : (X, τ) → (Y, σ, $\mathcal{J}$) is semi-$\mathcal{I}_s$-open if and only if for each x ∈ X and each neighbourhood U of x, there exists V ∈ SISO(Y, σ) containing f(x) such that V ⊆ f(U).
Proof. Suppose that f is a semi-$\mathcal{I}_s$-open function. For each x ∈ X and each neighbourhood U of x, there exists U_o∈ τ such that x ∈U_o ⊆ U. Since f is semi-$\mathcal{I}_s$-open, V = f(U_o) ∈ SISO(Y, σ) and f(x) ∈ V ⊆ f(U).
Conversely, let U be an open set of (X, τ) . For each x ∈ U, there exists V_x ∈ SISO(Y, σ) such that f(x) ∈ V_x ⊆ f(U). Therefore we obtain f(U) = ∪{V_x : x ∈ U} and hence by Theorem 3.4, f(U) ∈ SISO(Y, σ). This shows that f is semi-$\mathcal{I}_s$-open.

Theorem 5.9. Let f : (X, τ) → (Y, σ, $\mathcal{J}$) be a semi-$\mathcal{I}_s$-open (resp. semi-$\mathcal{I}_s$-closed) function. If W is any subset of Y and F is a closed (resp. open) set of X containing f^{-1}(W), then there exists a semi-$\mathcal{I}_s$-closed (resp. semi-$\mathcal{I}_s$-open) subset H of Y containing W such that f^{-1}(H) ⊆ F.
Proof. Suppose that f is a semi-$\mathcal{I}_s$-open function. Let W be any subset of Y and F a closed subset of X containing f^{-1}(W). Then X − F is open and since f is semi-$\mathcal{I}_s$-open, f (X − F) is semi-$\mathcal{I}_s$-open. Hence H = Y − f(X − F) is semi-$\mathcal{I}_s$-closed. It follows from f^{-1}(W) ⊆ F that W ⊆ H. Moreover we obtain f^{-1}(H) ⊆ F. For a semi-$\mathcal{I}_s$-closed function can be proved similarly.

Theorem 5.10. For any bijective function f : (X, τ) → (Y, σ, $\mathcal{J}$), the following are equivalent:
1. f^{-1} : (X, σ, $\mathcal{J}$) → (X, τ) is semi-$\mathcal{I}_s$-continuous,
2. f is semi-$\mathcal{I}_s$-open,
3. f is semi-$\mathcal{I}_s$-closed,
Proof. Obvious.

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