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On The Surd Transcendental Equation With Five Unknowns $\sqrt[4]{x^2+y^2}+\sqrt[2]{z^2+w^2}=(k^2+1)^{2n}R^5$

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Abstract: *The transcendental equation with five unknowns represented by the diophantine equation* $\sqrt[4]{x^2+y^2}+\sqrt[2]{z^2+w^2}=(k^2+1)^{2n}R^5$ *is analyzed for its patterns of non-zero distinct integral solutions.*
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NOTATIONS

$t_{m,n}$: *Polygonal number of rank* n *with size* m

P_n^m : *Pyramidal number of rank* n *with size* m

Cp_n^m : *Centered Pyramidal number of rank* n *with size* m

I. Introduction

Diophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equations $[1,2,3]$.It seems that much work has not been done to obtain integral solutions of transcendental equations.In this context one may refer $[4-16]$. This communication analyses a transcendental equation with five unknowns given by $\sqrt[4]{x^2+y^2}+\sqrt[2]{z^2+w^2}=(k^2+1)^{2n}R^5$. Infinitely many non-zero integer quintuples (x, y, X, Y, z, w) satisfying the above equation are obtained.

II. Method Of Analysis

The diophantine equation representing a transcendental equation with five unknowns is

$$\sqrt[4]{x^2+y^2}+\sqrt[2]{z^2+w^2}=(k^2+1)^{2n}R^5 \qquad (1)$$

To start with,the substitution of the transformations

$$\left.\begin{aligned} x &= 4pq(p^2-q^2) \\ y &= 4p^2q^2-(p^2-q^2)^2 \end{aligned}\right\} \qquad (2a)$$

$$\left.\begin{aligned} z &= 2pq \\ w &= p^2-q^2 \end{aligned}\right\} \qquad (2b)$$

in(1) leads to

$$2(p^2+q^2)=(k^2+1)^{2n}R^5 \qquad (3)$$

Assume $R=R(A,B)=A^2+B^2, A,B>0$ (4)

and write 2 as $2=(1+i)(1-i)$ (5)

Substituting (5),(4) in (3),and employing the method of factorization,define

$$(1+i)(p+iq)=(\alpha+i\beta)(A+iB)^5 \qquad (6)$$

where ,
$$\alpha = \frac{1}{2}\left((k+i)^{2n} + (k-i)^{2n}\right)$$
$$\beta = \frac{1}{2i}\left((k+i)^{2n} - (k-i)^{2n}\right)$$

Equating real and imaginary parts in (6),we get

$$\left.\begin{aligned} p-q &= \alpha f(A,B) - \beta g(A,B) \\ p+q &= \beta f(A,B) + \alpha g(A,B) \end{aligned}\right\} \qquad (7)$$

where
$$f(A,B) = (A^5 - 10A^3B^2 + 5AB^4)$$
$$g(A,B) = (5A^4B - 10A^2B^3 + B^5)$$

Solving the system of equations (7),we get

$$\left.\begin{aligned} p &= \frac{(\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)}{2} \\ q &= \frac{(\alpha+\beta)g(A,B) - (\alpha-\beta)g(A,B)}{2} \end{aligned}\right\} \qquad (7a)$$

It is to be noted that p and q are integers only when A and B are of the same pairty.

Replacing A by $2A$, B by $2B$ in(7a) and (4) ,we have

$$\left.\begin{aligned} p &= 2^4\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right) \\ q &= 2^4\left((\alpha+\beta)g(A,B) - (\alpha-\beta)g(A,B)\right) \end{aligned}\right\} \qquad (7b)$$

$$R = 4(A^2 + B^2) \qquad (7c)$$

Substituting (7b) in (2a) and (2b), the values of (x, y, z, w) are represented by

$$x(A,B) = 2^{16}\left\{\begin{aligned} &4\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)\left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right) - \\ &\left(\left[\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)\right]^2 - \left[\left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right)\right]^2\right) \end{aligned}\right\}$$

$$y(A,B) = 2^{16}\left\{\begin{aligned} &4\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)^2\left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right)^2 - \\ &\left(\left[\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)\right]^2 - \left[\left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right)\right]^2\right)^2 \end{aligned}\right\}$$

$$z(A,B) = 2^8\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)\left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right)$$

$$w(A,B) = 2^8\left((\alpha+\beta)f(A,B) + (\alpha-\beta)g(A,B)\right)^2 - \left((\alpha+\beta)g(A,B) - (\alpha-\beta)f(A,B)\right)^2$$

The above equations and (7c) represents the non-zero integer solutions to (1).

In a similar manner , replacing A by 2A+1, B by 2B+1 one obtains the corresponding set of non-zero integer solutions to (1).

It is worth to observe that one may employ different set of transformations for z and w leading to a different solution pattern which is illustrated as follows.

in (1), we get

$$z^2 + w^2 = s^2 \qquad (8)$$

(8) can be rewritten as

$$z^2 + w^2 = s^2 = s^2 * 1 \qquad (9)$$

Assume $\quad s = s(p,q) = p^2 + q^2, p,q > 0 \qquad (10)$

and write 1 as

$$\left.1=\frac{(m^2-n^2+i2mn)(m^2-n^2-i2mn)}{(m^2+n^2)^2}\right\}\text{...............................} \qquad (11)$$

Substituting (11),(10) in (9),and employing the method of factorization,define

$$\left.\begin{array}{l} z=\dfrac{\left((m^2-n^2)(p^2-q^2)-4pqmn\right)}{(m^2+n^2)} \\ w=\dfrac{\left(2mn(p^2-q^2)+2pq(m^2-n^2)\right)}{(m^2+n^2)} \end{array}\right\}\text{...............................} \qquad (12)$$

As our thurst is an finding integer solution,replacing p by $(m^2+n^2)P$, q by $(m^2+n^2)Q$ in (2a),(12),we have

$$\left.\begin{array}{l} x=4(m^2+n^2)^4PQ(P^2-Q^2) \\ y=(m^2+n^2)^4\left[4P^2Q^2-(P^2-Q^2)^2\right] \\ z=(m^2+n^2)\left[(m^2-n^2)(P^2-Q^2)-4PQmn\right] \\ w=(m^2+n^2)\left[2mn(P^2-Q^2)+2PQ(m^2-n^2)\right] \end{array}\right\}\text{..........} \qquad (13)$$

where

$$\mathrm{P}=2^4(\mathrm{m}^2+\mathrm{n}^2)^8\left[\left((\mathrm{m}^2-\mathrm{n}^2)-2\mathrm{mn}\right)\left(\beta\mathrm{f}(\mathrm{A},\mathrm{B})+\alpha\mathrm{g}(\mathrm{A},\mathrm{B})\right)-\left((\mathrm{m}^2-\mathrm{n}^2)+2\mathrm{mn}\right)\left(\alpha\mathrm{f}(\mathrm{A},\mathrm{B})-\beta\mathrm{g}(\mathrm{A},\mathrm{B})\right)\right]$$

$$\mathrm{Q}=2^4(\mathrm{m}^2+\mathrm{n}^2)^8\left[\left((\mathrm{m}^2-\mathrm{n}^2)-2\mathrm{mn}\right)\left(\alpha\mathrm{f}(\mathrm{A},\mathrm{B})-\beta\mathrm{g}(\mathrm{A},\mathrm{B})\right)-\left((\mathrm{m}^2-\mathrm{n}^2)+2\mathrm{mn}\right)\left(\beta\mathrm{f}(\mathrm{A},\mathrm{B})+\alpha\mathrm{g}(\mathrm{A},\mathrm{B})\right)\right]$$

Note that (13) and (4) represent the integral solutions to (1),provided A and B are of the same pairty.

2.1 : Properties:

1. (z,w,y) satisfies the hyperbolic paraboloid $z^2-w^2=y$
2. $w(q+1,q)z(q+1,q)=12P_q^4$
3. $x=2wz$
4. $w^2(q+1,q)=8t_{3,q}+1$
5. $z\big(q(q+1),q\big)=4P_q^5$
6. $z(p,1)w(p,1)=12P_{p-1}^3$
7. $m[z(p,1)w(p,1)]+6z(p,1)=12Cp_p^m, m\geq 3$
8. $2z(p,1)+(m-2)z(p,p-1)=4t_{m,p}, m>2$
9. $(y+w^2)z$ is a cubical integer
10. $z^2y=z^4-(x-zw)^2$
11. $x^2=4w^2(y+w^2)$
12. $2w-y+1$ is a difference of two squares.

III. Conclusion:

To conclude,one may search for other pattern of solutions and their corresponding properties.

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