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Pythagorean Triangle and Special Pyramidal Numbers

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Abstract: *Patterns of Pythagorean triangle, where, in each of which either a leg or the hypotenuse is a pentagonal pyramidal number and Centered hexagonal pyramidal number, in turn are presented.*
Keywords: *Pythagorean triangles, pentagonal pyramidal,centered hexagonal pyramidal.*

I Introduction

The method of obtaining three non-zero integers α,β and γ under certain relations satisfying the equation $\alpha^2+\beta^2=\gamma^2$ has been a matter of interest to various mathematicians [1,2,3].In [4-12], special Pythagorean problems are studied.In this communication, we present yet another interesting Pythagorean problem.That is, we search for patterns of Pythagorean triangles where in each of which, either a leg or the hypotenuse is represented by a pentagonal pyramidal number and centered hexagonal pyramidal number,in turn.

II Notation

P_n^m - m-gonal pyramidal number of rank n
CP_n^m - centered m-gonal pyramidal number of rank n
$t_{m,n}$ - polygonal number of rank n.

III Method of Analysis

Let (*m,n,k*) represent a triple of non-zero distinct positive integers such that

$$m=(k+1)n$$

Let $P(\alpha,\beta,\gamma)$ be the Pythagorean triangle whose generators are *m,n.* Consider

$$\alpha=2mn\ ;\quad \beta=m^2-n^2\ ;\quad \gamma=m^2+n^2.$$

It is observed that, for suitable choices of n, either a leg or hypotenuse of the Pythagorean triangle P is represented by a pentagonal pyramidal number and centered hexagonal pyramidal number ,in turn.Different choices of n along with the corresponding sides of the Pythagorean triangle are illustrated below

Choice 3.1

Let $n=4k+3$.

The corresponding sides of the Pythagorean triangle are

$$\alpha=32k^3+80k^2+66k+18$$
$$\beta=16k^4+56k^3+57k^2+18k$$
$$\gamma=16k^4+56k^3+89k^2+66k+18$$

Note that $\alpha=P_n^5$

Choice 3.2

Let $n=2k^2+4k+3$

The corresponding sides of the Pythagorean triangle are

$$\alpha=8k^5+40k^4+88k^3+104k^2+66k+18$$
$$\beta=4k^6+24k^5+60k^4+80k^3+57k^2+18k$$
$$\gamma=4k^6+24k^5+68k^4+112k^3+113k^2+66k+18$$

Note that $\gamma=P_n^5$

Note

It is worth mentioning here that, for the following two choices of m,n given by (i)$n=4k, m=k(n+1)$ and (ii) $n=2k^3-3, m=kn$ the sides α and β represent P_n^5 respectively.

Choice 3.3

Let $n=2(k+1)$

The corresponding sides of the Pythagorean triangle are

$$\alpha=8k^3+24k^2+24k+8$$
$$\beta=4k^4+16k^3+20k^2+8k$$
$$\gamma=4k^4+16k^3+28k^2+24k+8$$

Note that $\alpha=CP_n^6$

Choice 3.4

Let $n = k(k+2)$

The corresponding sides of the Pythagorean triangle are

$$\alpha = 2k^5 + 10k^4 + 16k^3 + 8k^2$$
$$\beta = k^6 + 6k^5 + 12k^4 + 8k^3$$
$$\gamma = k^6 + 6k^5 + 14k^4 + 16k^3 + 8k^2$$

Note that $\beta = CP_n^6$

Choice 3.5

Let $n = k^2 + 2k + 2$

The corresponding sides of the Pythagorean triangle are

$$\alpha = 2k^5 + 10k^4 + 24k^3 + 32k^2 + 24k + 8$$
$$\beta = k^6 + 6k^5 + 16k^4 + 24k^3 + 20k^2 + 8k$$
$$\gamma = k^6 + 6k^5 + 18k^4 + 32k^3 + 36k^2 + 24k + 8$$

Note that $\gamma = CP_n^6$.

Properties

(1) $3(\gamma - \beta)$ is a Nasty Number.

(2) $\frac{\alpha\beta}{12p_k^3}$ is a biquadratic integer.

(3) $\frac{\alpha\beta}{P_k^5 + t_{3,k}}$ is a perfect square.

(4) $\frac{\gamma}{\beta} = \frac{CP_{k+1}^3}{P_k^5 + 2t_{3,k}}$

(5) α is a perfect square when $k = 2p^2 - 1$

(6) $6(\gamma - \alpha)$ is a Nasty number.

(7) $\frac{\gamma\alpha}{CP_{k+1}^3}$ is a biquadratic integer.

(8) $\frac{3\gamma}{\beta} = \frac{CP_{k+1}^3}{P_k^3}$

IV Conclusion

One may search for other patterns of Pythagorean triangles, where , in each of which either a leg or the hypotenuse is represented by other polygonal and pyramidal numbers.

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