IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728,p-ISSN: 2319-765X, Volume 7, Issue 4 (Jul. - Aug. 2013), PP 11-13
www.iosrjournals.org

Integral Solutions to the Biquadratic Equation with Four Unknowns $(x+y+z+w)^2 = xyzw+1$

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Abstract: *The main thrust of this paper is to study the biquadratic equation with four unknowns* $(x+y+z+w)^2 = xyzw+1$. *We present six different infinite families of positive integral solutions to this equation.*

I. Introduction

Integral solutions to equations in three or more variables are given in various parametric forms [1,2,3]. In [4], the theory of general pell's equation is used to generate four infinite families of positive integral solutions to the equation $(x+y+z)^2 = xyz$. In[5] other choices of integral solutions to the above equation are presented . In[6], the Diophantine equation $(x+y+z+t)^2 = xyzt$ is considered and nine different infinite families of positive integral solutions are given.

In this paper, the biquadratic equation with four unknowns given by $(x+y+z+w)^2 = xyzw+1$ is studied for its non-zero distinct integral solution. In particular, six different infinite families of positive integral solutions are obtained.

II. Method of Analysis

The biquadratic equation with four unknowns to be solved is $(x+y+z+w)^2 = xyzw+1$ (1)
Introduction of the linear transformations

$$x = u+v+a, \quad y = u-v+a, \quad z = b, \quad w = c. \tag{2}$$

where a,b,c,are positive integers
in (1) leads to the form

$$(bc-4)u^2 - bcv^2 = (2a+b+c)^2 - a^2bc - 1 \tag{3}$$

In which $bc > 4$ and $2(2a+b+c)^2 = abc$ (4)

There are six triples (a,b,c) satisfying the above conditions (4) namely (5,2,3), (7,1,6), (12,1,5), (1,10,3), (3,14,1), (4,1,9).

The following table contains the general pell's equation (3) corresponding to the above triples (a,b,c), their pell's resolvent, both equations with their fundamental solutions.

(a,b,c)	General pell's equation(3) and its fundamental solution	Pell's resolvent and its fundamental solution
(5,2,3)	$u^2 - 3v^2 = 37$, (7,2)	$r^2 - 3s^2 = 1$, (2,1)
(7,1,6)	$u^2 - 3v^2 = 73$, (10,3)	$r^2 - 3s^2 = 1$, (2,1)
(12,1,5)	$u^2 - 5v^2 = 179$, (28,11)	$r^2 - 5s^2 = 1$, (9,4)
(1,10,3)	$13u^2 - 15v^2 = 97$, (7,6)	$r^2 - 195s^2 = 1$, (14,1)
(3,14,1)	$5u^2 - 7v^2 = 157$, (10,7)	$r^2 - 35s^2 = 1$, (6,1)
(4,1,9)	$5u^2 - 9v^2 = 179$, (50,37)	$r^2 - 45s^2 = 1$, (161,24)

In view of (2), the following are the six families of positive integer solutions to (1)

$$(i)x_{n+1}=\frac{9f}{2}+\frac{13\sqrt{3}}{6}g+5$$

$$y_{n+1}=\frac{5f}{2}-\frac{\sqrt{3}g}{6}+5, z=2, w=3$$

where $f=[(2+\sqrt{3})^{n+1}+(2-\sqrt{3})^{n+1}]$

$$g=[(2+\sqrt{3})^{n+1}-(2-\sqrt{3})^{n+1}]$$

$$(ii)x_{n+1}=\frac{13f}{2}+\frac{19}{2\sqrt{3}}g+7$$

$$y_{n+1}=\frac{7f}{2}-\frac{g}{2\sqrt{3}}+7, z=1, w=6$$

$$(iii)x_{n+1}=\frac{49f_1}{2}+\frac{83\sqrt{5}}{10}g_1+12$$

$$y_{n+1}=\frac{17f_1}{2}-\frac{27\sqrt{5}g_1}{10}+12, z=1, w=5$$

where $f_1=[(9+4\sqrt{5})^{n+1}+(9-4\sqrt{5})^{n+1}]$

$$g_1=[(9+4\sqrt{5})^{n+1}-(9-4\sqrt{5})^{n+1}]$$

$$(iv)x_{n+1}=\frac{13f_2}{2}+\frac{181}{2\sqrt{195}}g_2+1$$

$$y_{n+1}=\frac{-5f_2}{2}-\frac{g_2}{2\sqrt{195}}+1, z=10, w=3$$

where $f_2=[(14+\sqrt{195})^{n+1}+(14-\sqrt{195})^{n+1}]$

$$g_2=[(14+\sqrt{195})^{n+1}-(14-\sqrt{195})^{n+1}]$$

$$(v)x_{n+1}=\frac{17f_3}{2}+\frac{99}{2\sqrt{35}}g_3+3$$

$$y_{n+1}=\frac{3f_3}{2}-\frac{g_3}{2\sqrt{35}}+3, z=14, w=1$$

where $f_3=[(6+\sqrt{35})^{n+1}+(6-\sqrt{35})^{n+1}]$

$$g_3=[(6+\sqrt{35})^{n+1}-(6-\sqrt{35})^{n+1}]$$

$$(vi)x_{n+1}=\frac{87f_4}{2}+\frac{583}{2\sqrt{45}}g_4+4$$

$$y_{n+1}=\frac{-13f_4}{2}-\frac{83g_4}{2\sqrt{45}}+4, z=1, w=9$$

where $f_4=[(161+24\sqrt{45})^{n+1}+(161-24\sqrt{45})^{n+1}]$

$$g_4=[(161+24\sqrt{45})^{n+1}-(161-24\sqrt{45})^{n+1}]$$

III Conclusion

To conclude, one may search for other patterns of solutions.

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