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Application of Group Theory to a Local Game Called "Tsorry Checkerboard" (A Case of Klein Four- Group)

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Abstract: *Some popular recreational games have a group- theoretic foundation, and group are useful in modelling games that involve a series of discrete moves, with each move leading to a change in the board state. In this work we present to you our local game called "tsorry" (meaning they line up straight). Which we carefully examine and found that it has a board and checkers, each possible moves has an element of group, where the effect of performing a sequence of moves corresponds to the product of those elements. The group under consideration here is Klein four- group.*
Keyword: *Checkerboard, Group theory, Games, Klein four – group, Possible moves and Tsorry Checkerboard,*

I. Introduction

1.0 Introduction

Group theory is the branch of pure mathematics which emanated from abstract algebra.
Due to its abstract nature, it was seems to be an arts subject rather than a science subject. In fact it was consider pure abstract and not practical.

Even students of group theory after being introduce to the course seems not to believe as to whether the subject have any practical application in real life situations because of its abstract nature.

This problem prompt the researchers to study the different ways in which groups can be expressed concretely (its groups represention) both from theoretical and practical point of view, with intention of bringing its real life application in games for better understanding of the subject.

The modern concept of abstract group developed out of several fields of mathematics, [1]. The idea of group theory although developed from the concept of abstract algebra, yet can be applied in many other mathematical areas and other fields in sciences and as well as in games.

According to [2], said that group is a collection of reversible actions that can be carry out one after the other. [3] , said many groups are set of numbers, but the case of Rubik cube, the group is a set of permutation of the cubelets. [4], State in his paper that group theory can be applied to puzzles very effectively. A permutation representation can be used in many puzzles to define a group. He said he consider the 15- puzzle in terms of group theory.

We here now consider the Checker board game, this game is of different versions. Boards and checkers come in a wide variety ranging from squares drawn in dirt with rocks for pieces to expensive boards of rare woods and stone with pieces made from stone, plastic, word, metal, gems. Most of us probably started with a folding cardboard grid of red and black squares with thin plastic red and black pieces with grooves that allowed two pieces to interlock, [5].

The mechanics of this game varies from one version to another. To focus on the official American checkers rules**:**

The game is played on a "standard" 8 x 8 chess/checker board. In regulation play, the squares are green and buff rather than red and black squares which many are familiar with. The checkers are solid red and solid white in regulation play. The size of the board squares and checker pieces are stated in the official rules and regulations as well.

Checkers is a two player game and each side begins with 12 pieces or "men" on each side. Pieces are placed on the dark squares only and player takes pieces by dark squares as well. Single pieces can only move forward one square at a time. Jumps are mandatory, but if more than one jump is possible the moving player can choose as they see fit.

Once a player single piece reaches his opponents end row the piece becomes a "king" and another piece of the same colour is placed on top of that piece. A king can only move one space at a time like the singles pieces, but it now has the ability to move forward and backwards. Jumps follow the same rules as single pieces. Winning is accomplished by eliminating all the opponent pieces or by forcing the opponent into a situation where they cannot make any legal moves. Any other situation result in a draw, [6].

However here we are considering our own local game in Nigeria, which we use to play as young boys at Secondary School age. We call it tsorry in our native (Berom Language in Plateau State) as we use to call it, Meaning they line up straight.

II. Methodology

2.0 Method

A group is an algebraic structure consisting of a set together with an operation that combines any two of its elements to form a third element. To qualify as a group, the set and the operation must satisfy a few conditions called group axioms, namely closure, associativity, identity and invertibility. The idea of group is one which pervades the whole of mathematics both pure and applied, [7].

Definition A group G of transformations of a set X is equipped with the composition law GxG → G,(a, b) → ab, the identity element e =id, and the inversion operation $a \to a^{-1}$, which satisfy the following axioms, [8].

Axioms 1. Associativity: (ab) c = a (bc), a, b, cεG

Axioms 2. The unit axiom: ea = ae = a for any aεG

Axioms 3. The inverse axiom. $aa^{-1} = a^{-1}\ a = e$ for any aG

2.2 Klein four- group

Klein four- group is the unique (up to isomorphism) non cyclic group of order four. In this group every non- identity element has order two.

The multiplication table with non-identity elements a,b,c and identity element I can be described as follows (and this characterize the group) , [9].

- The product of identity element and any element is that element itself
- The product of any non- identity element with itself is the identity element
- The product of two distinct non-identity element is the third non- identity element.

2.3 Tsorry Checkerboard

We here introducing our own local game called tsorry checkerboard whose rules are as follows:

The game is played on a 2 x 2 checker board. This checker is a two player game and each side (player) begins with 1 piece per move and up to maximum of 3 pieces. The winning comes if all the three pieces with the same colours fall on the same line either vertically, horizontally or diagonally then the player with these colours win the game.

You can set up your opponent and force him into an uneven exchange of pieces. You are expected to control and force, that is control the board and force your opponent into making moves to your advantage and their ultimate demise. Capitalizing on the opponents' mistakes will be the only reason you win in most cases, since perfect play will almost always result in a long play and make it bored.

Board and checkers come in a wide variety ranging from squares drawn in the dirt with rocks for pieces to expensive boards. But our board is locally drawn on a cardboard or on a plane piece of wood, in fact sometimes it is drawn on the ground just to play for the fun it. For the checkers, mostly people locally use bottle tops.

III. Results

3.0 Tsorry Checker board

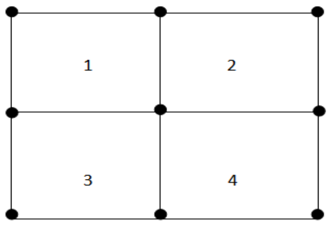

Fig. (a)

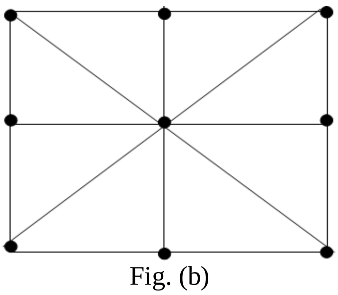

Fig. (b)

Our checker board has only four squares number 1, 2, 3 and 4, as in figure (a). There is a single checker on the board, and it has 4 possible moves. In figure (b) the dot showing the possible positions to which any player can place his checker as he contest to win this game.

V: move vertically that is move 1 to 3 or 4 to 2 or vice versa

H: move horizontally, that is move from 1 to 2 or from 3 to 4 or vice versa

D: move diagonally, that is move from 2 to 3 or from 1 to 4 or vice versa.

I: Stay put

We may consider an operation on the set of these moves, which consists of performing moves successively. For example, if we move horizontally and then vertically, we end up with the same result as if we had moved diagonally H ∗ V = D.

If we perform two horizontal moves in succession we end up where we started H ∗ H = I. And so on . Here the elements of this Klein four- group are V, H, D and I (Where I is the identity element)

G = (V, H, D, I) and ∗ is the operation, we have just described, write the table of G.

TABLE I showing the multiplication table of the elements of the Group

∗	I	V	H	D
I				
V				
H				
D				

By completing table I of G is granting associativity explain why (G, ∗) is a group.

Solution

TABLE II The complete Cayley table of the Klein four- group G(V, H, D ,I)

∗	I	V	H	D
I	I	V	H	D
V	V	I	D	H
H	H	D	I	V
D	D	H	V	I

Group Axioms

(i) Closure H ∗ D ∈ G

(ii) Associativity (V ∗ H) ∗ D = V ∗ (H ∗ D)

(iii) Identity I ∗ V = V, I ∗ D = D, I ∗ H = H

(iv) Inverse, every element is its own inverse V ∗ V = D ∗ D = H ∗ H = I

The above move of the checker has satisfies all the axiom of group theory. In deed this game has satisfied all the properties of Klein four- group.

IV. Conclusion

4.0 Conclusion

Group theory even though is from abstract mathematics yet can be seen as applied mathematics, since it has application in so many fields of studies ranging from science to non- sciences.

More research can be conducted by students of group theory to discover more fields that group theory is applicable , this will confirm the fact that group theory is an applied course, rather than viewing it otherwise.

We have successfully showed here that group theory is applicable to the game of tsorry checker board. The successive moves perform on this game obeys group axioms, With this we conclude that group theory has wide application in so many games even to those yet discover.

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