(τ₁, τ₂) – RGB Closed Sets in Bitopological Spaces

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Abstract: In this paper we introduce and study the concept of a new class of closed sets called (τ₁, τ₂) – regular generalized b-closed sets (briefly (τ₁, τ₂) – rgb-closed) in bitopological spaces. Further we define and study new neighborhood namely (τ₁, τ₂) – rgb- neighbourhood (briefly (τ₁, τ₂) – rgb-nhd) and discuss some of their properties in bitopological spaces. Also, we give some characterizations and applications of it.

I. Introduction

In 1963, Kelley J. C. [16] was first introduced the concept of bitopological spaces, where X is a nonempty set and τ₁, τ₂ are two topologies on X. 1970, M.K. Signal [28] introduced some more separation axioms that consider with bitopological spaces. 1977, V. Popo. [26] introduced some properties of bitopological semi separation.

In (1985), Fukutake [7] introduced and the studied the notions of generalized closed (g-closed) sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Sundaram, P. and Shiek John [29], El- Tantawy and Abu-Donia [6] introduced the concept of ω-closed sets and generalized semi-closed (gs-closed) sets in bitopological spaces respectively.


In §2 we recollect the basic definitions which are used in this paper.

In §3 we find basic properties and characteristics of (τ₁, τ₂) – rgb closed sets. Also we provide several properties of above concept and to investigate its relationships with certain types of closed sets with some new results and examples.

In §4 we provide several properties of characterizations of (τ₁, τ₂) – rgb-closed sets (τ₁, τ₂) – rgb-open sets and (τ₁, τ₂) – rgb – nhd of a point as well as some propositions and examples that are included throughout the section.

II. Introduction And Preliminaries

If A is a subset of a topological space X with a topology τ, with then the closure of A is denoted by τ -cl(A) or cl(A), the interior of A is denoted by τ -int(A) or int(A), semi-closure (resp. pre-closure) of A is denoted by τ - scl(A) or scl(A) (resp. τ - pcl(A) or pcl(A)), semi-interior of A is denoted by τ - sint(A) or sint(A) and the complement of A is denoted by A'.

Before entering into our work we recall the following definitions:

Definition 2.1. A subset A of a topological space (X, τ) is called:

1) an α-open set [18] if A ⊆ int(cl(int(A))).

2) a semi-open set [12] if A ⊆ cl(int(A)).

3) a pre-open set [13] if A ⊆ int(cl(A)).

4) a semi-pre-open set ([6]-open set)[5] if A ⊆ cl(int(cl(A))).

5) a regular open set [9] if A = int(cl(A)).

6) a b-open set [1] if A ⊆ int(cl(A)) ∪ cl(int(A)).
Remark 3.2: The family of all $\tau$ and $U$ is 1. Definition 2.4 The complements of the above mentioned sets are called their respective open sets.

8) a generalized semi closed set [25] (abbreviated gsclosed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$. 9) a semi generalized closed set [10] (abbreviated sgcl(A)) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $X$. 11) a strongly generalized closed set [27] (abbreviated $^{*}g$ closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is g-open in $X$. 12) a generalized ggbc closed set [30](abbreviated ggbc closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $X$. 13) a regular generalized bclosed set [23](abbreviated rgb- closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular in $X$. 2) intersection of all semi open sets [21] (abbreviated ag-open) if $acl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$. 3) an open set (21) $(X, \tau_{1}, \tau_{2})$ a topological space $(X, \tau_{1}, \tau_{2})$ is a topological space if $A \subseteq \tau_{1}$ - int $\tau_{1}$ - cl $\tau_{1}$ (A)]

Definition 2.3. A subset $A$ of a bitopological space $(X, \tau_{1}, \tau_{2})$ is called a 1. $(\tau_{1}, \tau_{2})$ -pre open [12] if $A \subseteq \tau_{1}$ - int $\tau_{1}$ - cl $\tau_{1}$ (A)]

Definition 2.4. A subset $A$ of a bitopological space $(X, \tau_{1}, \tau_{2})$ is called a 1. $\tau_{i}, \tau_{j}$ - g closed [7] if $\tau_{i} - cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \subseteq \tau_{i}$. 2. $(\tau_{1}, \tau_{2})$ - semi open [20] if $A \subseteq \tau_{1}$ - cl $\tau_{1}$ - int $\tau_{1}$ (A)]

III. $(\tau_{1}, \tau_{2})$ - RGB Closed Sets In Bitopological Spaces

In this section we introduce $(\tau_{1}, \tau_{2})$ - rgb-closed sets in bitopological spaces and study some of their properties.
Proposition 3.3: If A is $\tau_j$-closed subset of $(X, \tau_i, \tau_j)$ then A is $(\tau_i, \tau_j)$-rgb-closed set.

Proof. Let A be any $\tau_j$-closed set and U be any $\tau_i$-regular-open set containing A. Since $\tau_j$ - $\text{bcl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$, then $\tau_j - \text{bcl}(A) \subseteq U$. Hence A is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.4: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}\}$ and $\tau_j = \{X, \phi, \{\phi\}\}$, the set $\{b\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-b-closed.

Proposition 3.5: If A is $(\tau_i, \tau_j)$-b-closed subset of $(X, \tau_i, \tau_j)$ then A is $(\tau_i, \tau_j)$-rgb-closed set.

Proof. Let A be any $(\tau_i, \tau_j)$-b-closed set in $(X, \tau_i, \tau_j)$ such that $A \subseteq U$, where U is $\tau_i$-regular-open set. Since A is $(\tau_i, \tau_j)$-b-closed which implies that $\tau_j - \text{bcl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$, then $\tau_j - \text{bcl}(A) \subseteq U$. Hence A is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.6: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{a, c\}\}$ and $\tau_j = \{X, \phi, \{\phi\}\}$, the set $\{a, c\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-b-closed.

Proposition 3.7: If A is $\tau_j$-closed (resp. $\tau_j$-semi-closed) subset of $(X, \tau_i, \tau_j)$ then A is $(\tau_i, \tau_j)$-rgb-closed.

Proof. Let A be any $\tau_j$-closed set in $(X, \tau_i, \tau_j)$ such that $A \subseteq U$, where U is $\tau_j$-regular-open set. Since A is $\tau_j$-closed, then $\tau_j - \text{bcl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$, so $\tau_j - \text{bcl}(A) \subseteq U$. Therefore A is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.8: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\tau_j = \{X, \phi, \phi\}$, the set $\{c\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $\tau_i$-closed.

Remark 3.9: The concept of $(\tau_i, \tau_j)$-closed and $(\tau_i, \tau_j)$-rgb-closed sets are independent of each other as seen from the following examples.

Example 3.10: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{c\}\}$ and $\tau_j = \{X, \phi, \{b\}, \{c\}\}$, the set $\{b\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-closed.

Example 3.11: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}, \{c\}\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\tau_j = \{X, \phi, \{a\}\}$, the set $\{a, b\}$ is $(\tau_i, \tau_j)$-closed but not $(\tau_i, \tau_j)$-rgb-closed set.

Remark 3.12: The concept of $(\tau_i, \tau_j)$-semi-closed and $(\tau_i, \tau_j)$-rgb-closed sets are independent of each other as seen from the following examples.

Example 3.13: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{b\}, \{b, c\}\}$ and $\tau_j = \{X, \phi, \{a\}\}$. Then the set $\{c\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-semi-closed set.

Example 3.14: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}, \{c\}\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\tau_j = \{X, \phi, \{b\}, \{b, c\}\}$, the set $\{b\}$ is $(\tau_i, \tau_j)$-semi-closed but not $(\tau_i, \tau_j)$-rgb-closed set.

Remark 3.15: $(\tau_i, \tau_j)$-pre-closed and $(\tau_j, \tau_i)$-rgb-closed sets are independent of each other as seen from the following two examples.

Example 3.16: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}\}$ and $\tau_j = \{X, \phi, \{a\}, \{b\}\}$, the set $\{a, b\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-pre-closed.

Example 3.17: Let $X, \tau_i$ and $\tau_j$ be as in Example 3.14. The set $\{b, c\}$ is $(\tau_i, \tau_j)$-pre-closed but not $(\tau_i, \tau_j)$-rgb-closed.
Remark 3.18: \((\tau_i, \tau_j)\) - semi-pre-closed sets \((\beta\text{-closed sets})\) and \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed sets are independent of each other as seen from the following two examples.

Example 3.19: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{c\}\}\) and \(\tau_j = \{X, \varnothing, \{a, b\}, \{a, c\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{a, b\}\}\) the set \(\{a\}\) is \((\tau_i, \tau_j)\) - \(\beta\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Example 3.20: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{b\}, \{c\}\}\) the set \(\{a, c\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(\beta\) - closed.

Remark 3.21: The concept of \((\tau_i, \tau_j)\) - \(\text{rgb}^*\) - closed sets and \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed sets are independent of each other as seen from the following example.

Example 3.22: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{b\}\}\) . Then the set \(\{a\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(\text{rgb}^*\) - closed and \(\{b\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}^*\) - closed but not \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Proposition 3.23: If \(A\) is \((\tau_i, \tau_j)\) - \(g\) - closed subset of \((X, \tau_i, \tau_j)\) then \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Proof. Suppose that \(A\) is \((\tau_i, \tau_j)\) - \(g\) - closed set \(U\) be any \(\tau_i\) - regular - open set such that \(A \subseteq U\). Since \(A\) is \((\tau_i, \tau_j)\) - \(g\) - closed, then \(\tau_j \setminus \text{cl}(A) \subseteq \U\), we have \(\tau_j \setminus \text{bcl}(A) \subseteq \tau_j \setminus \text{cl}(A) \subseteq \U\). Hence \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.24: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) the set \(\{a\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(g\) - closed.

Proposition 3.25: If \(A\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed subset of \((X, \tau_i, \tau_j)\) then \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Proof. Let \(A\) be any \((\tau_i, \tau_j)\) - \(g^*\) - closed set and \(U\) be any \(\tau_i\) - regular - open set containing \(A\). Since \(A\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed set and \(\tau_j \setminus \text{cl}(A) \subseteq \U\), \(\tau_j \setminus \text{bcl}(A) \subseteq \tau_j \setminus \text{cl}(A) \subseteq \U\), so \(\tau_j \setminus \text{bcl}(A) \subseteq \U\). Therefore \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.26: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{b\}\}\) . Then the set \(\{a\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(g^*\) - closed.

Proposition 3.27: If \(A\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed subset of \((X, \tau_i, \tau_j)\) then \(A\) is \((\tau_i, \tau_j)\) - \(\text{gb}\) - closed.

Proof. Assume \(A\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed, \(A \subseteq \U\) and \(U\) is \(\tau_i\) - regular - open set. Since \(A\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed set, we have \(\tau_j \setminus \text{pcl}(A) \subseteq \U\) and \(\tau_j \setminus \text{pcl}(A) \subseteq \tau_j \setminus \text{bcl}(A) \subseteq \U\), \(\tau_j \setminus \text{bcl}(A) \subseteq \U\). Therefore \(A\) is \((\tau_i, \tau_j)\) - \(\text{gb}\) - closed.

The following example show that the converse of the above proposition is not true:

Example 3.28: Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}\}\) . Then the set \(\{a\}\) is \((\tau_i, \tau_j)\) - \(g^*\) - closed.

Proposition 3.29: If \(A\) is \((\tau_i, \tau_j)\) - \(\text{gb}\) - closed subset of \((X, \tau_i, \tau_j)\) then \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Proof. Let \(A\) be any \((\tau_i, \tau_j)\) - \(\text{gb}\) - closed set \((X, \tau_i, \tau_j)\) such that \(A \subseteq \U\), where \(U\) is \(\tau_i\) - regular - open set. Since \(A\) is \((\tau_i, \tau_j)\) - \(\text{gb}\) - closed set, which implise that \(\tau_j \setminus \text{bcl}(A) \subseteq \U\). Therefore \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.30 Let \(X = \{a, b, c\}\) and \(\tau_i = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}\) and \(\tau_j = \{X, \varnothing, \{a\}, \{b\}\}\) . The set \(\{a, b\}\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed but \(\text{not}(\tau_i, \tau_j)\) - \(\text{gb}\) - closed.

Proposition 3.31: If \(A\) is \((\tau_i, \tau_j)\) - \(\text{rb}\) - closed subset of \((X, \tau_i, \tau_j)\) then \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.

Proof. Let \(A\) be any \((\tau_i, \tau_j)\) - \(\text{rb}\) - closed set \((X, \tau_i, \tau_j)\) and \(U\) be any \(\tau_i\) - regular open set containing \(A\). Since \(A\) is \((\tau_i, \tau_j)\) - \(\text{rb}\) - closed set the \(\tau_i \setminus \text{cl}(A) \subseteq \U\) and \(\tau_i \setminus \text{bcl}(A) \subseteq \tau_i \setminus \text{cl}(A) \subseteq \U\). Hence \(A\) is \((\tau_i, \tau_j)\) - \(\text{rgb}\) - closed.
Proposition 3.43: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a, b\}, \{a, b\}\}$ and $\tau_j = \{X, \emptyset, \{a\}\}$, the set $\{a\}$ is (\(\tau_i, \tau_j\))-rgb-closed but not $(\tau_i, \tau_j)$-rw-closed.

Proposition 3.33: If $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set and $U$ be any $\tau_i$-regular open set containing $A$. Since $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set, then $\tau_j - \text{bcl}(A) \subseteq \tau_j - \alpha \text{cl}(A) \subseteq U$. Therefore $\tau_j - \text{bcl}(A) \subseteq U$. Hence $A$ is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true as seen from the following example.

Example 3.34: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a, b\}, \{a\}\}$ and $\tau_j = \{X, \emptyset, \{a\}\}$, the set $\{a, \{a\}\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-$g_{\alpha}$-closed.

Similarly, we prove the following Proposition:

Proposition 3.35: If $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed but not conversely.

Example 3.36: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a, b\}, \emptyset\}$ and $\tau_j = \{X, \emptyset, \{a\}\}$, the set $\{a\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-$g_{\alpha}$-closed.

Proposition 3.37: If $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set in $(X, \tau_i, \tau_j)$ such that $A \subseteq U$, where $U$ is $\tau_i$-regular open set. Since $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set, $\tau_i - \text{bcl}(A) \subseteq U$. Hence $A$ is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.38: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a, b\}, \emptyset\}$ and $\tau_j = \{X, \emptyset, \{a\}\}$, the set $\{a\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-$g_{\alpha}$-closed.

Proposition 3.39: If $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set and $U$ be any $\tau_i$-regular open set containing $A$. Since $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed set, then $\tau_j - \text{cl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$. Therefore $A$ is $(\tau_i, \tau_j)$-rgb-closed.

The following example show that the converse of the above proposition is not true:

Example 3.40: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a, b\}, \{b\}\}$ and $\tau_j = \{X, \emptyset, \{a\}\}$, the set $\{a, \{b\}\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-$g_{\alpha}$-closed.

Similarly, we prove the following Proposition:

Proposition 3.41: If $A$ is $(\tau_i, \tau_j)$-$g_{\alpha}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true in general, as seen from the following example.

Example 3.42: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b\}\}$ and $\tau_j = \{X, \emptyset, \{a, b\}\}$, the set $\{a\}$ is $(\tau_i, \tau_j)$-rgb-closed but not $(\tau_i, \tau_j)$-$g_{\alpha}$-closed.

Proposition 3.43: If $A$ is $(\tau_i, \tau_j)$-rg-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)$-rgb-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)$-rg-closed set and $U$ be any $\tau_i$-regular open set containing $A$. Since $A$ is $(\tau_i, \tau_j)$-rg-closed set, then $\tau_j - \text{cl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$. Therefore $A$ is $(\tau_i, \tau_j)$-rgb-closed.

The converse of the above proposition need not be true as seen from the following example.
Example 3.44: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{b\}, \{a,c\}\}$ and $\tau_j = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$, the set $\{b\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed but not $(\tau_i, \tau_j)^-\text{rgb}$-closed.

Proposition 3.45: If $A$ is $(\tau_i, \tau_j)^-\text{sgb}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)^-\text{sgb}$-closed set in $(X, \tau_i, \tau_j)$ such that $A \subseteq U$, where $U$ is $\tau_i$-regular open set. Since $A$ is $(\tau_i, \tau_j)^-\text{sgb}$-closed set, $\tau_i - \text{bcl}(A) \subseteq U$. Hence $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

The following example show that the converse of the above proposition is not true:

Example 3.46: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{b\}, \{a,b\}\}$ and $\tau_j = \{X, \emptyset, \{c\}\}$, the set $\{a,b\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed but not $(\tau_i, \tau_j)^-\text{sgb}$-closed.

Proposition 3.47: If $A$ is $(\tau_i, \tau_j)^-\text{w}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

Proof. Let $A$ be any $(\tau_i, \tau_j)^-\text{w}$-closed set and $U$ be any $\tau_i$-regular open set containing $A$. Since $A$ is $(\tau_i, \tau_j)^-\text{w}$-closed set, then $\tau_j - \text{cl}(A) \subseteq U$, so $\tau_j - \text{bcl}(A) \subseteq \tau_j - \text{cl}(A) \subseteq U$. Therefore $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

The converse of the above proposition need not be true as seen from the following example:

Example 3.48: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and $\tau_j = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$, the set $\{a,b\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed but not $(\tau_i, \tau_j)^-\text{w}$-closed.

Similarly, we prove the following Proposition

Proposition 3.49: If $A$ is $(\tau_i, \tau_j)^-\text{w}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

The converse of the above proposition need not be true as seen from the following example:

Example 3.50: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$ and $\tau_j = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$, the set $\{a\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed but not $(\tau_i, \tau_j)^-\text{w}$-closed.

Proposition 3.51: If $A$ is $(\tau_i, \tau_j)^-\text{rgw}$-closed subset of $(X, \tau_i, \tau_j)$ then $A$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed.

The converse of the above proposition need not be true as seen from the following example:

Example 3.52: Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and $\tau_j = \{X, \emptyset, \{a\}, \{b\}, \{a,b\}\}$, the set $\{b\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed but not $(\tau_i, \tau_j)^-\text{rgw}$-closed.

IV. Characterizations And Properties Of $(\tau_i, \tau_j)^-\text{RGB-Closed Sets}$, $(\tau_i, \tau_j)^-\text{RGB -Open Sets}$ And $(\tau_i, \tau_j)^-\text{RGB - Neighborhoods}$

In this section we introduce some characterizations of $(\tau_i, \tau_j)^-\text{rgb}$-closed sets and $(\tau_i, \tau_j)^-\text{rgb}$-open sets, also we define and study new neighborhood namely $(\tau_i, \tau_j)^-\text{rgb}$-neighborhood (briefly $(\tau_i, \tau_j)^-\text{rgb}$-nhd) and discuss some of their properties.

Definition 4.1. A subset $A$ of bitopological space $(X, \tau_i, \tau_j)$ is called $(\tau_i, \tau_j)^-\text{rgb}$-open set if and only if its complement is $(\tau_i, \tau_j)^-\text{rgb}$-closed in $X$.

The family of all $(\tau_i, \tau_j)^-\text{rgb}$-open subsets of $X$ is denoted by $D^r$ RGBO $(\tau_i, \tau_j)$

Remark 4.2 Let $A$ and $B$ be two $(\tau_i, \tau_j)^-\text{rgb}$-closed sets in $(X, \tau_i, \tau_j)$
1) The union $A \cup B$ is not generally $(\tau_i, \tau_j)^-\text{rgb}$-closed set.
2) The intersection $A \cap B$ is not generally $(\tau_i, \tau_j)^-\text{rgb}$-closed set as seen from the following examples.

Example 4.3. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}\}$ and $\tau_j = \{X, \emptyset, \{a\}, \{b\}\}$, the subsets $\{a\}$ is $(\tau_i, \tau_j)^-\text{rgb}$-closed sets, but their union $\{a\} \cup \{b\} = \{a, b\}$ is not $(\tau_i, \tau_j)^-\text{rgb}$-closed set.

Example 4.4. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \emptyset, \{a\}, \{b\}\}$ and $\tau_j = \{X, \emptyset, \{b\}\}$, the subsets $\{a, c\}$ are $(\tau_i, \tau_j)^-\text{rgb}$-closed sets, but their intersection $\{a\} \cap \{b\} = \{b\}$ is not $(\tau_i, \tau_j)^-\text{rgb}$-closed set.
Remark 4.5 Let A and B be two $(\tau_i, \tau_j)$-rgb - open sets in $(X, \tau_i, \tau_j)$  
1)The union $A \cup B$ is not generally $(\tau_i, \tau_j)$-rgb - open set .
2) The intersection $A \cap B$ is not generally $(\tau_i, \tau_j)$-rgb - open set as seen from the following examples.

Example 4.6. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ and $\tau_j = \{X, \phi, \{b\}\}$,then the subsets $\{a\}, \{c\}$ is $(\tau_i, \tau_j)$-rgb - open sets but their union $\{a\} \cup \{c\} = \{a, c\}$ is not $(\tau_i, \tau_j)$-rgb - open set.

Example 4.7. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a,b\}, \{c\}\}$ and $\tau_j = \{X, \phi, \{a,b\}\}$,then the subsets $\{a,c\}, \{b,c\}$ is $(\tau_i, \tau_j)$-rgb - open sets but their intersection $\{a,c\} \cup \{b,c\} = \{c\}$ is not $(\tau_i, \tau_j)$-rgb - open set.

Proposition 4.8: If a set $G$ is $(\tau_i, \tau_j)$-rgb-closed set in $(X, \tau_i, \tau_j)$, then $\tau_j - cl(A)$ contains no non-empty $\tau_i$-regular -closed set.

Proof. Let $G$ be $(\tau_i, \tau_j)$-rgb-closed and $F$ be a $\tau_i$-regular -closed set such that $F \subseteq (\tau_j - cl(G))^\circ$. Since $G$ is $(\tau_i, \tau_j)$-rgb-closed, then $G \subseteq D^\circ$ RGB $(\tau_i, \tau_j)$ which impluse that $\tau_j - cl(G) \subseteq F^\circ$. Then $F \subseteq \tau_j - cl(G) \cap (\tau_j - cl(G))^\circ$. Therefore $F$ is empty.

The converse of the above theorem need not be true as seen from the following example.

Example 4.9. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$, $\tau_j = \{X, \phi, \{b\}\}$. If $G = \{b\}$, then $\tau_j - cl(G) - G = \{a,c\}$ does not any non-empty $\tau_i$-regular -closed set. But $G$ is a $(\tau_i, \tau_j)$-rgb-closed set.

Proposition 4.10: If A is $(\tau_i, \tau_j)$-rgb-closed set and $A \subseteq B \subseteq \tau_j - cl(A)$, then $B$ is $(\tau_i, \tau_j)$-rgb-closed set.

Proof. Let $B \subseteq U$ where $U$ is - regular open set. Since $A \subseteq B$, so $\tau_j - cl(A) \subseteq U$. But $\tau_j - cl(A)$ ,

We have $\tau_j - bc l(B) \subseteq \tau_j - (\tau_i - cl(A))$ then $\tau_j - bc l(B) \subseteq U$. Therefore $B$ is rgb-closed in $X$.

Proposition 4.11: Let $A \subseteq Y \subseteq X$ and if $A$ is $(\tau_i, \tau_j)$-rgb - closed in $X$ then $A$ is $(\tau_i, \tau_j)$-rgb -closed relative to $Y$.

Proof. Let $A \subseteq Y \cap G$ where $G$ is $\tau_i$ - regular open in $X$. Since $A$ is $(\tau_i, \tau_j)$-rgb-closed . Then $\tau_i - cl(A) \subseteq clG$. Then $Y \cap \tau_i - cl(A) \subseteq Y \cap G$. Thus $A$ is rgb -closed relative to $Y$.

Proposition 4.12: If $A$ is $(\tau_i, \tau_j)$-rgb-closed set, then $\tau_j - cl(\{x\}) \cap A \neq \phi$ for each $x \in \tau_j - cl(A)$.

Proof. If $\tau_j - cl(\{x\}) \cap A = \phi$ for each $x \in \tau_j - cl(A)$, then $A \subseteq (\tau_i - cl(\{x\}))^\circ$. Since $A$ is $(\tau_i, \tau_j)$-rgb-closed set, so $\tau_i - cl(A) \subseteq (\tau_j - cl(\{x\}))^\circ$ which impluse that $x \notin \tau_j - cl(A)$. This contradicts to the assumption.

Definition 4.13. Let $(X, \tau_i, \tau_j)$ be bitopological space, and let $g \subseteq X$. A subset $N$ of $X$ is said to be, $(\tau_i, \tau_j)$-rgb-neighborhood (briefly $(\tau_i, \tau_j)$-rgb-nhd) of a point $g$ if and only if there exists a $(\tau_i, \tau_j)$-rgb-open set $G$ such that $g \in G \subseteq N$.

The set of all $(\tau_i, \tau_j)$-rgb-nhd of a point $g$ is denoted by $(\tau_i, \tau_j)$-rgb-N(g).

Proposition 4.14: Every $\tau_i$-nhd of $g \in X$ is a $(\tau_i, \tau_j)$-rgb-nhd of $g \in X$.

Proof. Since $N$ is $\tau_i$-nhd of $g \in X$, then there exists $\tau_i$-open set $G$ such that $g \in G \subseteq N$. Since every $\tau_i$-open set is $(\tau_i, \tau_j)$-rgb-open set, $G = (\tau_i, \tau_j)$-rgb-open set. By Definition 4.13. $N = (\tau_i, \tau_j)$-rgb-nhd of $x$.

Remark 4.15: The converse of the above proposition need not be true as seen from the following example.

Example 4.16. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. $\tau_j = \{X, \phi, \{a\}, \{b,c\}\}$.

$D^\circ$ RGBO $(\tau_i, \tau_j) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$, the set $\{b,c\}$ is $(\tau_i, \tau_j)$-rgb-nhd of $c$ since there exists a $(\tau_i, \tau_j)$-rgb-open set $G = \{c\}$ such that $c \in \{c\} \subseteq \{b,c\}$. However $\{b,c\}$ is not $\tau_i$-nhd of $c$ since no $\tau_i$-open set $G$ such that $c \in G \subseteq \{b,c\}$.

Remark 4.17. The $(\tau_i, \tau_j)$-rgb-nhd of a point $g \in X$ need not be a $(\tau_i, \tau_j)$-rgb-open set in $X$ as seen from the following example.

Example 4.18. Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$. $\tau_j = \{X, \phi, \{b\}\}$.
D’ RGBO $(\tau_i, \tau_j) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}, the set \{a,c\} is \{(\tau_i, \tau_j) - rgb - nhd of c\}, since there exists a $(\tau_i, \tau_j) - rgb - open set G = \{c\}$ such that $c \in \{c\} \subseteq \{a,c\}$. However, \{a,c\} is not $(\tau_i, \tau_j) - rgb - open set$.

**Proposition 4.19:** If $N$ a subset of a bitopological space $(X, \tau_i, \tau_j)$ is $(\tau_i, \tau_j) - rgb - open set$, then $N$ is $(\tau_i, \tau_j) - rgb - nhd$ of each of its points.

**Proof.** Let $N$ be a $(\tau_i, \tau_j) - rgb - open set$. By Definition 4.13. $N$ is an $(\tau_i, \tau_j) - rgb - nhd$ of each of its points.

**Remark 4.20.** The $(\tau_i, \tau_j) - rgb - nhd$ of a point $g \in X$ need not be a $(\tau_i, \tau_j) - nhd$ of $x$ in $X$ as seen from the following example.

**Example 4.21.** Let $X = \{a, b, c\}$ and $\tau_i = \{X, \phi, \{a\}, \{b\}, \{a,c\}\}$, $\tau_j = \{X, \phi, \{a,b\}, \{b\}\}$.

D’ RGBO $(\tau_i, \tau_j) = \{X, \phi, \{a\}, \{c\}, \{a,b\}, \{b,c\}\}$, the set \{a,c\} is $(\tau_i, \tau_j) - rgb - nhd$ of $a$, since there exists a $(\tau_i, \tau_j) - rgb - open set G = \{a\}$ such that $a \in \{a\} \subseteq \{a,c\}$. Also, the set \{a,c\} is $(\tau_i, \tau_j) - rgb - nhd$ of $c$, since there exists a $(\tau_i, \tau_j) - rgb - open set G = \{c\}$ such that $c \in \{c\} \subseteq \{a,c\}$. However, \{a,c\} is not $(\tau_i, \tau_j) - rgb - open set$ in $X$.

**Proposition 4.22.** Let $(X, \tau_i, \tau_j)$ be a bitopological space:

1) $\forall g \in X, (\tau_i, \tau_j) - rgb - N(g) \neq \phi$

2) $\forall N \in (\tau_i, \tau_j) - rgb - N(g), \text{then } g \in N$.

3) If $N \in (\tau_i, \tau_j) - rgb - N(g), N \subseteq M$, then $M \in (\tau_i, \tau_j) - rgb - N(g)$.

4) If $N \in (\tau_i, \tau_j) - rgb - N(g), \text{then there exists } M \in (\tau_i, \tau_j) - rgb - N(g) \subseteq N \cup M$.

**Proof.** Since $X$ is an $(\tau_i, \tau_j) - rgb - open set$, it is $(\tau_i, \tau_j) - rgb - nhd$ of every $g \in X$. Hence there exists at least one $(\tau_i, \tau_j) - rgb - nhd G$ for every $g \in X$. Therefore $(\tau_i, \tau_j) - rgb - N(g) \neq \phi, \forall g \in X$.

2) If $N \in (\tau_i, \tau_j) - rgb - N(g), \text{then } N = (\tau_i, \tau_j) - rgb - nhd G$ of $g$. Thus by Definition 4.13 $g \in N$.

3) If $N \in (\tau_i, \tau_j) - rgb - N(g), \text{then there is an } (\tau_i, \tau_j) - rgb - open set A$ such that $g \in A \subseteq N$, since $N \subseteq M$, $g \in A \subseteq M$ and $M$ is an $(\tau_i, \tau_j) - rgb - nhd$ of $g$. Hence $M \in (\tau_i, \tau_j) - rgb - N(g)$. If $N \in (\tau_i, \tau_j) - rgb - N(g), \text{then there exists an } (\tau_i, \tau_j) - rgb - open set M$ such that $g \in M \subseteq N$. Since $M$ is an $(\tau_i, \tau_j) - rgb - open set$, then it is $(\tau_i, \tau_j) - rgb - nhd$ of each of its points. Therefore $M \in (\tau_i, \tau_j) - rgb - N(h) \forall h \in M$.

**References**


$\tau_i, \tau_j$ – RGB Closed Sets In Bitopological Spaces


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