Some forms of N-closed Maps in supra Topological spaces

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Abstract: In this paper, we introduce the concept of N-closed maps and we obtain the basic properties and their relationships with other forms of N-closed maps in supra topological spaces.

Keywords: supra N-closed map, almost supra N-closed map, strongly supra N-closed map.

I. Introduction:


In this paper, we introduce the concept of supra N-closed maps and study its basic properties. Also we introduce the concept of almost supra N-closed maps and strongly supra N-closed maps and investigate their properties in supra topological spaces.

II. Preliminaries:

Definition 2.1[4]
A subfamily μ of X is said to be supra topology on X if
i) X, φ ∈ μ
ii) If Aᵢ ∈ μ ∀ i ∈ j then $\bigcup Aᵢ \in μ$. (X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ⁺.

Definition 2.2[4]
The supra closure of a set A is denoted by $clμ(A)$, and is defined as supra closure $\{B : B \text{ is supra closed and } A \subseteq B\}$.

The supra interior of a set A is denoted by $intμ(A)$, and is defined as supra interior $\{B : B \text{ is supra open and } A \supseteq B\}$.

Definition 2.3[4]
Let (X, τ) be a topological space and μ be a supra topology on X. We call μ a supra topology associated with τ, if τ ⊆ μ.

Definition 2.4[3]
Let (X, μ) be a supra topological space. A set A of X is called supra semi-open set, if $A \subseteq clμ(intμ(A))$. The complement of supra semi-open set is supra semi-closed set.

Definition 2.5[1]
Let (X, μ) be a supra topological space. A set A of X is called supra α-open set, if $A \subseteq intμ(clμ(intμ(A)))$. The complement of supra α-open set is supra α-closed set.

Definition 2.6[5]
Let (X, μ) be a supra topological space. A set A of X is called supra Ω closed set, if $scclμ(Ωclμ(A)) \subseteq intμ(U)$ whenever $A \subseteq U$, U is supra open set. The complement of the supra Ω closed set is supra Ω open set.

Definition 2.7[5]
The supra Ω closure of a set A is denoted by $Ωclμ(A)$, and is defined as supra Ω closed and $A \subseteq B$.

The supra Ω interior of a set A is denoted by $Ωintμ(A)$, and is defined as supra Ω open and $A \supseteq B$.

Definition 2.8[6]
Let (X, μ) be a supra topological space. A set A of X is called supra regular open if $A = intμ(clμ(A))$ and supra regular closed if $A = clμ(intμ(A))$. 
**Definition 2.9[7]**

Let \((X, \mu)\) be a supra topological space. A set \(A\) of \(X\) is called supra \(N\)-closed set if \(\Omega \cap^{\mu} (A) \subseteq U\), whenever \(A \subseteq U\), \(U\) is supra \(\alpha\) open set. The complement of supra \(N\)-closed set is supra \(N\)-open set.

**Definition 2.10[7]**

The supra \(N\) closure of a set \(A\) is denoted by \(\text{Ncl}^{\mu}(A)\), and defined as

\[
\text{Ncl}^{\mu}(A) = \bigcap \{B : B \text{ is supra } N \text{ closed and } A \subseteq B\}.
\]

The supra \(N\) interior of a set \(A\) is denoted by \(\text{Nint}^{\mu}(A)\), and defined as

\[
\text{Nint}^{\mu}(A) = \bigcup \{B : B \text{ is supra } N \text{ open and } A \supseteq B\}.
\]

**Definition 2.11[7]**

Let \((X, \tau)\) and \((Y, \sigma)\) be two topological spaces and \(\mu\) be an associated supra topology with \(\tau\). A function \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called supra \(N\)-continuous function if \(f^{-1}(V)\) is supra \(N\)-closed in \((X, \tau)\) for every supra closed set \(V\) of \((Y, \sigma)\).

**Notations:** Throughout this paper \(O^{\mu}(\tau)\) represents supra open set of \((X, \tau)\) and \(N^{\mu} O(\tau)\) represents supra \(N\)-open set of \((X, \tau)\).

### III. Supra \(N\)-Closed Maps

**Definition 3.1**

A map \(f: (X, \tau) \rightarrow (Y, \sigma)\) is called supra \(N\)-closed map (resp. supra \(N\)-open) if for every supra closed (resp. supra open) \(F\) of \(X\), \(f(F)\) is supra \(N\)-closed (resp. supra \(N\)-open) in \(Y\).

**Theorem 3.2**

Every supra closed map is supra \(N\)-closed map.

**Proof**

Let \(f: (X, \tau) \rightarrow (Y, \sigma)\) be supra closed map. Let \(V\) be supra closed set in \(X\). Since \(f\) is supra closed map then \(f(V)\) is supra closed set in \(Y\). We know that every supra closed set is supra \(N\)-closed, then \(f(V)\) is supra \(N\)-closed in \(Y\). Therefore \(f\) is supra \(N\)-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 3.3**

Let \(X = \{a, b, c\}\) and \(\tau = \{X, \emptyset, \{a\}, \{b, c\}\}\), \(\sigma = \{\{a\}, \emptyset\}\). \(f(X, \tau) \rightarrow (Y, \sigma)\) be the function defined by \(f(a) = b\), \(f(b) = c\), \(f(c) = a\). Here \(f\) is supra \(N\)-closed map but not supra closed map, since \(V = \{b, c\}\) is closed in \(X\) but \(f([b, c]) = \{a, c\}\) is supra \(N\)-closed set but not supra closed in \(Y\).

**Theorem 3.4**

A map \(f(X, \tau) \rightarrow (Y, \sigma)\) is supra \(N\)-closed iff \(f(\text{cl}^{\mu}(V)) = \text{Ncl}^{\mu}(f(V))\)

**Proof**

Suppose \(f\) is supra \(N\)-closed map. Let \(V\) be supra closed set in \((X, \tau)\). Since \(V\) is supra closed, \(\text{cl}^{\mu}(V) = V\). \(f(V)\) is supra \(N\)-closed in \((Y, \sigma)\). Since \(f\) is supra \(N\)-closed map, then \(f(\text{cl}^{\mu}(V)) = f(V)\). Since \(f(V)\) is supra \(N\)-closed, we have \(\text{Ncl}^{\mu}(f(V)) = f(V)\). Hence \(f(\text{cl}^{\mu}(V)) = \text{Ncl}^{\mu}(f(V))\).

Conversely, suppose \(f(\text{cl}^{\mu}(V)) = \text{Ncl}^{\mu}(f(V))\). Let \(V\) be supra closed set in \((X, \tau)\), then \(\text{cl}^{\mu}(V) = V\). Since \(f\) is a mapping, \(f(V)\) is in \((Y, \sigma)\) and we have \(f(\text{cl}^{\mu}(V)) = f(V)\). Since \(f(\text{cl}^{\mu}(V)) = \text{Ncl}^{\mu}(f(V))\), we have \(f(V) = \text{Ncl}^{\mu}(f(V))\), implies \(f(V)\) is supra \(N\)-closed in \((Y, \sigma)\). Therefore \(f\) is supra \(N\)-closed map.

**Theorem 3.5**

A map \(f(X, \tau) \rightarrow (Y, \sigma)\) is supra \(N\)-open iff \(f(\text{int}^{\mu}(V)) = \text{Nint}^{\mu}(f(V))\)

**Proof**

Suppose \(f\) is supra \(N\)-open map. Let \(V\) be supra open set in \((X, \tau)\). Since \(V\) is supra open, \(\text{int}^{\mu}(V) = V\), \(f(V)\) is supra \(N\)-open in \((Y, \sigma)\). Since \(f\) is supra \(N\)-open map, Therefore \(f(\text{int}^{\mu}(V)) = f(V)\). Since \(f(V)\) is supra \(N\)-open, we have \(\text{Nint}^{\mu}(f(V)) = f(V)\). Hence \(f(\text{int}^{\mu}(V)) = \text{Nint}^{\mu}(f(V))\).

Conversely, suppose \(f(\text{int}^{\mu}(V)) = \text{Nint}^{\mu}(f(V))\). Let \(V\) be a supra open set in \((X, \tau)\), then \(\text{int}^{\mu}(V) = V\). Since \(f\) is a mapping, \(f(V)\) is in \((Y, \sigma)\) and we have \(f(\text{int}^{\mu}(V)) = f(V)\). Since \(f(\text{int}^{\mu}(V)) = \text{Nint}^{\mu}(f(V))\), we have \(f(V) = \text{Nint}^{\mu}(f(V))\), implies \(f(V)\) is supra \(N\)-open in \((Y, \sigma)\). Therefore \(f\) is supra \(N\)-open map.

**Remark 3.6**

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If $f: (X, \tau) \to (Y, \sigma)$ is supra $N$-closed map and $g: (Y, \sigma) \to (Z, \upsilon)$ is supra $N$-closed map then its composite need not be supra $N$-closed map in general and this is shown by the following example.

**Example 3.7**
Let $X = Y = Z = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$, $\sigma = \{Y, \phi, \{a\}\}$. $\upsilon = \{Z, \phi, \{a\}, \{b, ab, bhc\}\}$.
$f: (X, \tau) \to (Y, \sigma)$ be the function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. $g: (Y, \sigma) \to (Z, \upsilon)$ be the function defined by $g(a) = b$, $g(b) = c$, $g(c) = a$. Here $f$ and $g$ is supra $N$-closed map, but its composition is not $N$-closed map, since $g \circ f \{b, c\} = \{a, b\}$ is not $N$-closed in $(Z, \upsilon)$.

**Theorem 3.8**
If $f: (X, \tau) \to (Y, \sigma)$ is supra closed map and $g: (Y, \sigma) \to (Z, \upsilon)$ is supra $N$-closed map then the composition $g \circ f$ is supra $N$-closed map.

**Proof**
Let $V$ be supra closed set in $X$. Since $f$ is supra closed map, $f(V)$ is supra closed set in $Y$. Since $g$ is supra $N$-closed map, $g(f(V))$ is supra $N$-closed in $Z$. This implies $g \circ f$ is supra $N$-closed map.

**IV. Almost supra $N$-closed map and strongly supra $N$-closed map.**

**Definition 4.1**
A map $f: (X, \tau) \to (Y, \sigma)$ is said to be almost supra $N$-closed map if for every supra regular closed set $F$ of $X$, $f(F)$ is supra $N$-closed in $Y$.

**Definition 4.2**
A map $f: (X, \tau) \to (Y, \sigma)$ is said to be strongly supra $N$-closed map if for every supra $N$-closed set $F$ of $X$, $f(F)$ is supra $N$-closed in $Y$.

**Theorem 4.3**
Every strongly supra $N$-closed map is supra $N$-closed map.

**Proof**
Let $V$ be supra closed set in $X$. Since every supra closed set is supra $N$-closed set, then $V$ is supra $N$-closed in $X$. Since $f$ is strongly supra $N$-closed map, $f(V)$ is supra $N$-closed set in $Y$. Therefore $f$ is supra $N$-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.4**
Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, ab, bhc\}\}$. $f: (X, \tau) \to (Y, \sigma)$ be the function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Here $f$ is supra $N$-closed map but not strongly supra $N$-closed map, since $V = \{a, b\}$ is supra $N$-closed set in $X$, but $f(\{a, b\}) = \{b, c\}$ is not a supra $N$-closed set in $Y$.

**Theorem 4.5**
Every supra $N$-closed map is almost supra $N$-closed map.

**Proof**
Let $V$ be a supra regular closed set in $X$. We know that every supra regular closed set is supra closed set. Therefore $V$ is supra closed set in $X$. Since $f$ is supra $N$-closed map, $f(V)$ is supra $N$-closed set in $Y$. Therefore $f$ is almost supra $N$-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.6**
Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, ab, bhc\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, ab, bhc\}\}$. $f: (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Here $f$ is almost supra $N$-closed map but it is not supra $N$-closed map, since $V = \{a, c\}$ is supra closed set in $X$ but $f(\{a, c\}) = \{a, c\}$ is not supra $N$-closed set in $Y$.

**Theorem 4.7**
Every strongly supra $N$-closed map is almost supra $N$-closed map.

**Proof**
Let $V$ be supra regular closed set in $X$. We know that every supra regular closed set is supra closed set and every supra closed set is supra $N$-closed set. Therefore $V$ is supra $N$-closed set in $X$. Since $f$ is strongly supra $N$-closed map, $f(V)$ is supra $N$-closed set in $Y$. Therefore $f$ is almost supra $N$-closed map.

The converse of the above theorem need not be true. It is shown by the following example.

**Example 4.8**
Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{c, \{a\}\}\}$, $\sigma = \{Y, \phi, \{a\}, \{b, ab, bhc\}\}$. $f: (X, \tau) \to (Y, \sigma)$ be the function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Here $f$ is almost supra $N$-closed map but it is not strongly supra $N$-closed map, since $V = \{a\}$ is supra $N$-closed in $X$ but $f(\{a\}) = \{b\}$ is not supra $N$-closed set in $Y$.

**Theorem 4.9**
If \( f: (X, \tau) \to (Y, \sigma) \) is strongly supra N-closed map and \( g: (Y, \sigma) \to (Z, \upsilon) \) is strongly supra N-closed map then its composition \( g f \) is strongly supra N-closed map.

**Proof**

Let \( V \) be supra N-closed set in \( X \). Since \( f \) is strongly supra N-closed, then \( f(V) \) is supra N-closed in \( Y \). Since \( g \) is strongly supra N-closed, then \( g(f(V)) \) is supra N-closed in \( Z \). Therefore \( g f(V) \) is supra N-closed.

**Theorem 4.10**

If \( f: (X, \tau) \to (Y, \sigma) \) is almost supra N-closed map and \( g: (Y, \sigma) \to (Z, \upsilon) \) is strongly supra N-closed map then its composite \( g f \) is almost supra N-closed map.

**Proof**

Let \( V \) be supra regular closed set in \( X \). Since \( f \) is almost supra N-closed, then \( f(V) \) is supra N-closed set in \( Y \). Since \( g \) is strongly supra N-closed, then \( g(f(V)) \) is supra N-closed in \( Z \). Therefore \( g f(V) \) is almost supra N-closed.

**Theorem 4.11**

Let \( f: (X, \tau) \to (Y, \sigma) \) and \( g: (Y, \sigma) \to (Z, \upsilon) \) be two mappings such that their composition \( g f: (X, \tau) \to (Z, \upsilon) \) be a supra N-closed mapping then the following statements are true:

(i) If \( f \) is supra continuous and surjective then \( g \) is supra N-closed map

(ii) If \( g \) is supra N-irresolute and injective then \( f \) is supra N-closed map.

**Proof**

i) Let \( V \) be a supra closed set of \( (Y, \sigma) \). Since \( f \) is supra continuous \( f^{-1}(V) \) is supra closed set in \( (X, \tau) \). Since \( g \) is supra N-closed map, we have \( (gf)(f^{-1}(V)) \) is supra N-closed in \( (Z, \upsilon) \). Therefore \( g(V) \) is supra N-closed in \( (Z, \upsilon) \), since \( f \) is surjective. Hence \( g \) is supra N-closed map.

ii) Let \( V \) be supra closed set of \( (X, \tau) \). Since \( g \) is supra N-closed, we have \( g f(V) \) is supra N-closed in \( (Y, \sigma) \). Since \( g \) is injective and supra N-irresolute \( g^{-1}(g f(V)) \) is supra N-closed in \( (Y, \sigma) \). Therefore \( f(V) \) is supra N-closed in \( (X, \tau) \). Hence \( f \) is supra N-closed map.

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**V. Applications**

A supra topological space \( (X, \tau) \) is \( T^{\mu}_N - space \) if every supra N-closed set is supra closed in \( (X, \tau) \).

**Theorem 5.2**

Let \( (X, \tau) \) be a supra topological space then

(i) \( O^{\mu}(\tau) \subseteq N^{\mu}O(\tau) \)

(ii) A space \( (X, \tau) \) is \( T^{\mu}_N - space \) iff \( O^{\mu}(\tau) = N^{\mu}O(\tau) \)

**Proof**

(i) Let \( A \) be supra open set, then \( X-A \) is supra closed set. We know that every closed set is N-closed. Therefore \( X-A \) is N-closed, implies \( A \) is N-open. Hence \( O^{\mu}(\tau) \subseteq N^{\mu}O(\tau) \)

(ii) Let \( (X, \tau) \) be \( T^{\mu}_N - space \). Let \( A \in N^{\mu}O(\tau) \), then \( X-A \) is N-closed, by hypothesis \( X-A \) is closed and therefore \( A \in O^{\mu}(\tau) \). Hence we have \( O^{\mu}(\tau) = N^{\mu}O(\tau) \). Conversely the proof is obvious.

**Theorem 5.3**

If \( (X, \tau) \) is \( T^{\mu}_N - space \), then every singleton set of \( (X, \tau) \) is either supra \( \alpha \)-closed set or supra open set.

**Proof**

Suppose that for some \( x \in X \), the set \( \{x\} \) is not supra \( \alpha \)-closed set of \( (X, \tau) \), then \( \{x\} \) is not N-closed set in \( (X, \tau) \), since we know that every \( \alpha \)-closed set is N-closed set. So trivially \( \{x\} \) is N-closed set. From the hypothesis \( \{x\} \) is supra closed set in \( (X, \tau) \). Therefore \( \{x\} \) is supra open set.

**Theorem 5.4**
Let \( f: (X, \tau) \to (Y, \sigma) \) be supra \( N \)-closed map and \( g: (Y, \sigma) \to (Z, \upsilon) \) be supra \( N \)-closed map then their composition \( gof: (X, \tau) \to (Z, \upsilon) \) is a supra \( N \)-closed map if \((Y, \sigma)\) is \( T_N^\mu \)-space.

**Proof**

Let \( V \) be a supra closed set in \( X \). Since \( f \) is supra \( N \)-closed map, then \( f(V) \) is supra \( N \)-closed set in \( Y \). Since \( Y \) is \( T_N^\mu \)-space, \( f(V) \) is supra closed set in \( Y \). Since \( g \) is supra \( N \)-closed map, we have \( g(f(V)) \) is supra \( N \)-closed in \( Z \). Hence \( gof \) is a \( N \)-closed map.

**Reference**


