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Interval-Valued Fuzzy KUS-Ideals in KUS-Algebras

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Abstract: *In this paper the notion of interval-valued fuzzy KUS-ideals (briefly i-v fuzzy KUS-ideal) in KUS-algebras is introduced. Several theorems are stated and proved. The image and inverse image of i-v fuzzy KUS-ideals are defined and how the homomorphic images and inverse images of i-v fuzzy KUS-ideals become i-v fuzzy KUS-ideals in KUS-algebras is studied as well.*
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I. Introduction

W. A. Dudek and X. Zhang ([2],[3]) studied ideals and congruences of BCC-algebras. C. Prabpayak and U. Leerawat ([5],[6]) introduced a new algebraic structure which is called KU-algebras and investigated some related properties. The concept of a fuzzy set, was introduced by L.A. Zadeh [8]. O.G. Xi [7] applied the concept of fuzzy set to BCK-algebras and gave some of its properties. In [9], L.A. Zadeh made an extension of the concept of fuzzy set by an interval-valued fuzzy set (i.e., a fuzzy set with an interval-valued membership function). This interval-valued fuzzy set is referred to as an i-v fuzzy set. He constructed a method of approximate inference using his i-v fuzzy sets. In [1], R. Biswas defined interval-valued fuzzy subgroups and investigated some elementary properties. Recently S.M. Mostafa, and et al ([4]) introduced a new algebraic structure, called KUS-algebra, They have studied a few properties of these algebras, the notion of KUS-ideals on KUS-algebras was formulated and some of its properties are investigated. In this paper, using the notion of interval-valued fuzzy set by L.A. Zadeh, we introduce the concept of an interval-valued fuzzy KUS-ideals (briefly, i-v fuzzy KUS-ideals) of a KUS-algebra, and study some of their properties. Using an i-v level set of an i-v fuzzy set, we state a characterization of an i-v fuzzy KUS-ideals. We prove that every KUS-ideals of a KUS-algebra X can be realized as an i-v level KUS-ideals of an i-v fuzzy KUS ideals of X. In connection with the notion of homomorphism, we study how the images and inverse images of i-v fuzzy KUS-ideals become i-v fuzzy KUS-ideals.

II. The Structure of KUS-algebras:

In this section we include some elementary aspects that are necessary for this paper

Definition 2.1([4]). Let $(X; *,0)$ be an algebra with a single binary operation $(*)$. X is called a KUS-algebra if it satisfies the following identities:

(kus_1) : $(z*y) * (z*x) = y*x$,
(kus_2) : $0 * x = x$,
(kus_3) : $x * x = 0$,
(kus_4) : $x * (y*z) = y* (x*z)$,for any $x, y, z \in X$,

In what follows, let $(X; *,0)$ be denote a KUS-algebra unless otherwise specified.
For brevity we also call X a KUS-algebra. In X we can define a binary relation $(\leq)$ by: $x \leq y$ if and only if $y * x = 0$.

Lemma 2.2 ([4]). In any KUS-algebra $(X; *,0)$, the following properties hold: for all $x, y, z \in X$;

a) $x* y = 0$ and $y* x = 0$ imply $x = y$,
b) $y * [(y*z) *z] = 0$,
c) $(0*x) * (y*x) = y*0$,
d) $x \leq y$ implies that $y*z \leq x*z$ and $z*x \leq z*y$,
e) $x \leq y$ and $y \leq z$ imply $x \leq z$,
f) $x*y \leq z$ implies that $z*y \leq x$.

Definition 2.3 ([4]). A nonempty subset I of a KUS-algebra X is called a KUS-ideal of X if it satisfies: for all x , y, z $\in$ X,

$(Ikus_1)$ $(0 \in I)$,

($Ikus_2$) $(z * y) \in I$ and $(y * x) \in I$ imply $(z * x) \in I$.

Definition 2.4([8]). Let X be a nonempty set , a fuzzy subset μ in X is a function $\mu : X \to [0,1]$.

Definition 2.5([4]). Let X be a KUS-algebra and , a fuzzy subset μ in X is called a fuzzy KUS-sub-algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all x , y ∈ X.

Definition 2.6([4]). Let X be a KUS-algebra , a fuzzy subset μ in X is called a fuzzy KUS-ideal of X if it satisfies the following conditions: for all x , y, z ∈ X ,

($Fkus_1$) $\mu(0) \geq \mu(x)$,

($Fkus_2$) $\mu(z * x) \geq \min\{\mu(z * y), \mu(y * x)\}$.

Proposition 2.7([4]). The intersection of any finite sets of fuzzy KUS-ideals of KUS-algebra X is also a fuzzy KUS-ideal .

Definition 2.8([9]). Let X be a set and μ be a fuzzy subset of X , for t ∈ [0,1] , the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a level subset of μ .

Theorem 2.9([4]). A fuzzy subset μ of KUS-algebra X is a fuzzy KUS-ideal of X if and only if , for every t ∈ [0,1] , μ_t is either empty or a KUS-ideal of X .

Definition 2.10([6]) . Let (X ; ∗ ,0) and (Y; ∗`,0`) be nonempty sets . The mapping $f : (X; *, 0) \to (Y; *`, 0`)$ is called a homomorphism if it satisfies $f(x * y) = f(x) *` f(y)$ for all x , y ∈X. The set $\{x \in X \mid f(x) = 0'\}$ is called the Kernel of f and is denoted by Ker f .

Definition 2.11 ([6]). Let $f : (X; *, 0) \to (Y; *', 0')$ be a mapping from the set X to a set Y. If μ is a fuzzy subset of X, then the fuzzy subset β of Y defined by:

$$f(\mu)(y) = \begin{cases} \sup\{\mu(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 \quad \text{otherwise} \end{cases}$$

is said to be the image of μ under f **.**

Similarly if β is a fuzzy subset of Y , then the fuzzy subset $\mu = (\beta \circ f)$ in X (i.e the fuzzy subset defined by $\mu(x) = \beta(f(x))$ for all x ∈ X) is called the pre-image of β under f .

Theorem 2.12([4]). An into homomorphic pre-image of a fuzzy KUS-ideal is a fuzzy KUS-ideal .

Theorem 2.13([4]). An into homomorphic image of a fuzzy KUS-ideal is a fuzzy KUS-ideal .

III. Interval-valued fuzzy KUS-ideal of KUS-algebra

Remark 3.1([9]). An interval-valued fuzzy subset (briefly i-v fuzzy subset) A defined in the set X is given by $A = \{(x, [\mu_A^L(x), \mu_A^U(x)])\}$, for all x ∈ X . (briefly, it is denoted by $A = [\mu_A^L, \mu_A^U]$ where μ_A^L and μ_A^U are any two fuzzy subsets in X such that $\mu_A^L(x) \leq \mu_A^U(x)$ for all x ∈ X.

Let $\tilde{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$, for all x ∈ X and let D[0,1] be denotes the family of all closed sub-interval of [0,1]. It is clear that if $\mu_A^L(x) = \mu_A^U(x) = c$, where $0 \leq c \leq 1$, then $\tilde{\mu}_A(x) = [c, c]$ in D[0,1] , then $\tilde{\mu}_A(x) \in [0,1]$, for all x∈ X. Therefore the i-v fuzzy subset A is given by :

$A = \{(x, \tilde{\mu}_A(x))\}$, for all x∈ X where $\tilde{\mu}_A : X \to D[0,1]$.

Now we define the refined minimum (briefly r min) and order " ≤ " on elements $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2]$ of D[0, 1] as follows:

$r\min(D_1, D_2) = [\min\{a_1, a_2\}, \min\{b_1, b_2\}]$, $D_1 \leq D_2 \Leftrightarrow a_1 \leq a_2$ and $b_1 \leq b_2$. Similarly we can define (≥) and (=) .

In what follows, let X denote a KUS-algebra unless otherwise specified, we begin with the following definition.

Definition 3.2. An i-v fuzzy subset A in X is called an i-v fuzzy KUS-sub-algebra of X if $\tilde{\mu}_A(x * y) \geq r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$, for all x, y∈ X.

Example 3.3. Let X = {0, 1, 2, 3} in which the operation (as in example (∗) be define by the following table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then (X; $*$,0) is a KUS-algebra. Define a fuzzy subset $\mu: X \to [0,1]$ by

$\mu(x) = \begin{cases} 0.7 & \text{if } x = \{0,1\} \\ 0.3 & \text{otherwise} \end{cases}$. $I_1=\{0,1\}$ is a KUS-ideal of X. Routine calculation given that μ is a fuzzy KUS-ideal of X. Define $\tilde{\mu}_A$ (x) as follows:

$\tilde{\mu}_A(x) = \begin{cases} [0.3,0.9] & if\ x = \{0,1\} \\ [0.1,0.6] & otherwise \end{cases}$. It is easy to check that A is an i-v fuzzy KUS-sub-algebra.

Proposition 3.4. If A is an i-v fuzzy KUS-sub-algebra of X, then $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, for all $x \in X$.

Proof. For all $x \in X$, we have $\tilde{\mu}_A(0) = \tilde{\mu}_A(x * x) \geq r\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(x)\}$

$= r\min\{[\mu_A^L(x), \mu_A^U(x)], [\mu_A^L(x), \mu_A^U(x)]\} = r\min\{[\mu_A^L(x), \mu_A^U(x)]\} = \tilde{\mu}_A(x)$.⌂

Proposition 3.5. Let A be an i-v fuzzy KUS-sub-algebra of X, if there exist a sequence $\{X_n\}$ in X such that $\lim_{n\to\infty} \tilde{\mu}_A(x_n) = [1,1]$, then $\tilde{\mu}_A(0) = [1, 1]$.

Proof. By proposition (3.4), we have $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$, for all $x \in X$. Then $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x_n)$, for every positive integer n, Consider the inequality $[1,1] \geq \tilde{\mu}_A(0) \geq \lim_{n\to\infty} \tilde{\mu}_A(x_n) = [1,1]$. Hence $\tilde{\mu}_A(0) = [1,1]$.⌂

Definition 3.6. An i-v fuzzy subset $A = \{(x, \tilde{\mu}_A(x))\}$, $x \in X$ in KUS-algebra X is called an interval-valued fuzzy KUS-ideal (i-v fuzzy KUS-ideal, in short) if it satisfies the following conditions:

(A_1) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$,

(A_2) $\tilde{\mu}_A(z * x) \geq r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\}$, for all $x, y, z \in X$.

Example 3.7. Let X = {0, 1, 2, 3} as in example (3.3). Define $\tilde{\mu}_A$ (x) as follows:

$\tilde{\mu}_A(x) = \begin{cases} [0.3,0.9] & if\ x = \{0,1\} \\ [0.1,0.6] & otherwise \end{cases}$. It is easy to check that A is an i-v fuzzy KUS-ideal of X.

Theorem 3.8. An i-v fuzzy subset $A = [\mu_A^L, \mu_A^U]$ in X is an i-v fuzzy KUS-ideal of X if and only if μ_A^L and μ_A^U are fuzzy KUS-ideals of X.

Proof. If μ_A^L and μ_A^U are fuzzy KUS-ideals of X . For any $x, y, z \in X$. Observe $\tilde{\mu}_A(z * x) = [\mu_A^L(z * x), \mu_A^U(z * x)]$

$\geq [\min\{\mu_A^L(z * y), \mu_A^L(y * x)\}, \min\{\mu_A^U(z * y), \mu_A^U(y * x)\}]$

$= r\min\{[\mu_A^L(z * y), \mu_A^U(z * y)], [\mu_A^L(y * x), \mu_A^U(y * x)]\}$

$= r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)]$.

From what was mentioned above we can conclude that A is an i-v fuzzy KUS-ideal of X .

Conversely, suppose that A is an i-v fuzzy KUS-ideal of X . For all $x, y, z \in X$ we have $[\mu_A^L(z * x), \mu_A^U(z * x)] = \tilde{\mu}_A(z * x) \geq r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\}$

$= r\min\{[\mu_A^L(z * y), \mu_A^U(z * y)], [\mu_A^L(y * x), \mu_A^U(y * x)]\}$

$= [\min\{\mu_A^L(z*y), \mu_A^L(y*x)\}, \min\{\mu_A^U(z*y), \mu_A^U(y*x)\}]$. Therefore, $\mu_A^L(z*x) \geq \min\{\mu_A^L(z*y), \mu_A^L(y*x)\}$ and

$\mu_A^U(z*x) \geq \min\{\mu_A^U(z*y), \mu_A^U(y*x)\}$.

Hence, we get that μ_A^L and μ_A^U are fuzzy KUS-ideals of X. ⌂

Theorem 3.9. Let A_1 and A_2 be i-v fuzzy KUS-ideals of a KUS-algebra X. Then $A_1 \cap A_2$ is an i-v fuzzy KUS-ideal of X.

Proof. $\tilde{\mu}_{A_1\cap A_2}(0) = [\mu^L_{A_1\cap A2}(0), \mu^U_{A_1\cap A_2}(0)] \geq [\mu^L_{A_1\cap A2}(x), \mu^U_{A_1\cap A_2}(x)] = \tilde{\mu}_{A_1\cap A_2}(x)$.

Suppose $x, y, z \in X$ such that $(z*y) \in A_1 \cap A_2$ and $(y*x) \in A_1 \cap A_2$.
Since A_1 and A_2 are i-v fuzzy KUS-ideals of X, then by the theorem (3.8), we get

$\tilde{\mu}_{A_1\cap A_2}(z*x) = [\mu^L_{A_1\cap A2}(z*x), \mu^U_{A_1\cap A_2}(z*x)]$

$= [\min\{\mu^L_{A_1\cap A2}(z*y), \mu^L_{A_1\cap A2}(y*x)\}, \min\{\mu^U_{A_1\cap A_2}(z*y), \mu^U_{A_1\cap A_2}(y*x)\}]$

$= [\min\{\mu^L_{A_1\cap A2}(z*y), \mu^U_{A_1\cap A_2}(z*y)\}, \min\{\mu^L_{A_1\cap A2}(y*x), \mu^U_{A_1\cap A_2}(y*x)\}]$

$= r\min\{\tilde{\mu}_{A_1\cap A_2}(z*y), \tilde{\mu}_{A_1\cap A_2}(y*x)\}]$. ⌂

Corollary 3.10. Let $\{A_i \mid i\in\Lambda\}$ be a family of i-v fuzzy KUS-ideal of X. Then $\bigcap_{i\in\wedge} A_i$ is also an i-v fuzzy KUS-ideal of X.

Theorem 3.11. Let X be a KUS-algebra and A be an i-v fuzzy subset in X. Then A is an i-v fuzzy KUS-ideal of X if and only if the nonempty set

$\tilde{U}(A;[\delta_1,\delta_2]) := \{x \in X \mid \tilde{\mu}_A(x) \geq [\delta_1,\delta_2]\}$ is a KUS-ideal of X, for every

$[\delta_1, \delta_2] \in D[0, 1]$. We call $\tilde{U}(A; [\delta_1, \delta_2])$ the i-v level KUS-ideal of A.

Proof. Assume that A is an i-v fuzzy KUS-ideal of X and let $[\delta_1, \delta_2] \in D[0, 1]$ be such that $(z*y), (y*x) \in \tilde{U}(A; [\delta_1, \delta_2])$, then

$\tilde{\mu}_A(z*x) \geq r\min\{\tilde{\mu}_A(z*y), \tilde{\mu}_A(y*x)\} \geq r\min\{[\delta_1, \delta_2], [\delta_1, \delta_2]\} = [\delta_1, \delta_2]$ and so $(z*x) \in \tilde{U}(A; [\delta_1, \delta_2])$. Then $\tilde{U}(A; [\delta_1, \delta_2])$ the i-v level KUS-ideal of A.

Conversely, assume that $\tilde{U}(A; [\delta_1, \delta_2]) \neq \varnothing$ is a KUS-ideal of X, for every $[\delta_1, \delta_2] \in D[0, 1]$. In the contrary, suppose that there exist $x_0, y_0, z_0 \in X$, such that

$\tilde{\mu}_A(z_0*x_0) < r\min\{\tilde{\mu}_A(z_0*y_0), \tilde{\mu}_A(y_0*x_0)\}$.

Let $\tilde{\mu}_A(z_0*y_0) = [\gamma_1, \gamma_2]$, $\tilde{\mu}_A(y_0*x_0) = [\gamma_3, \gamma_4]$ and $\tilde{\mu}_A(z_0*x_0) = [\delta_1, \delta_2]$. If $[\delta_1, \delta_2] < r\min\{[\gamma_1, \gamma_2], [\gamma_3, \gamma_4]\} = \min\{\min\{\gamma_1, \gamma_2\}, \min\{\gamma_3, \gamma_4\}\}$.
So $\delta_1 < \min\{\gamma_1, \gamma_2\}$ and $\delta_2 < \min\{\gamma_3, \gamma_4\}$. Consider

$$[\lambda_1, \lambda_2] = \frac{1}{2}\{\tilde{\mu}_A(z_0*x_0) + r\min\{\tilde{\mu}_A(z_0*y_0)), \tilde{\mu}_A(y_0*x_0)\}\}$$

We find that

$$[\lambda_1, \lambda_2] = \frac{1}{2}\{[\delta_1, \delta_2] + r\min\{[\gamma_1, \gamma_2], [\gamma_3, \gamma_4]\}\}$$

$$= \frac{1}{2}\{(\delta_1 + \min\{\gamma_1, \gamma_3\}), (\delta_2 + \min\{\gamma_2, \gamma_4\})].$$

Therefore $\min\{\gamma_1, \gamma_3\} > \lambda_1 = \frac{1}{2}(\delta_1 + \min\{\gamma_1, \gamma_3\}) > \delta_1$,

$$\min\{\gamma_2, \gamma_4\} > \lambda_2 = \frac{1}{2}(\delta_2 + \min\{\gamma_2, \gamma_4\}) > \delta_2 .$$

Hence $[\min\{\gamma_1, \gamma_3\}, \min\{\gamma_2, \gamma_4\}] > [\lambda_1, \lambda_2] > [\delta_1, \delta_2] = \tilde{\mu}_A(z_0 * x_0)$,

so that, $(z_0 * x_0) \notin \tilde{U}(A; [\lambda_1, \lambda_2])$. which is a contradiction, since

$\tilde{\mu}_A(z_0 * y_0) = [\gamma_1, \gamma_2] \ge [\min\{\gamma_1, \gamma_3\}, \min\{\gamma_2, \gamma_4\}] > [\lambda_1, \lambda_2]$.

$\tilde{\mu}_A(y_0 * x_0) = [\gamma_3, \gamma_4] \ge [\min\{\gamma_1, \gamma_3\}, \min\{\gamma_2, \gamma_4\}] > [\lambda_1, \lambda_2]$, imply that

$(z_0 * y_0), (y_0 * x_0) \in \tilde{U}(A; [\lambda_1, \lambda_2])$. Then

$\tilde{\mu}_A(z * x) \ge r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\}$, for all $x, y, z \in X$. ⌂

Theorem 3.12. Every KUS-ideal of a KUS-algebra X can be realized as an i-v level KUS-ideal of an i-v fuzzy KUS-ideal of X.

Proof. Let Y be a KUS-ideal of X and let A be an i-v fuzzy subset on X defined by

$$\tilde{\mu}_A(x) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } x \in y \\ [0,0] & \text{otherwise} \end{cases}.$$

Where $\alpha_1, \alpha_2 \in [0, 1]$ with $\alpha_1 < \alpha_2$. It is clear that $\tilde{U}(A; [\alpha_1, \alpha_2]) = Y$. We show that A is an i-v fuzzy KUS-ideal of X. Let $x, y, z \in X$.

If $(z * y), (y * x) \in Y$, then $(z * x) \in Y$, and therefore

$\tilde{\mu}_A(z * x) = [\alpha_1, \alpha_2] = r\min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = r\min\{\tilde{\mu}_A(z * y)), \tilde{\mu}_A(y * x)\}$.

If $(z * y), (y * x) \notin Y$, then $\tilde{\mu}_A(z * y) = [0,0] = \tilde{\mu}_A(y * x)$ and so

$\tilde{\mu}_A(z * x) \ge [0,0] = r\min\{[0,0],[0,0]\} = r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\}$,

If $(z * y) \in Y$ and $(y * x) \notin Y$, then $\tilde{\mu}_A(z * y) = [\alpha_1, \alpha_2]$ and $\tilde{\mu}_A(y * x) = [0,0]$, then $\tilde{\mu}_A(z * x) \ge [0,0]$ $= r\min\{[\alpha_1, \alpha_2],[0,0]\} = r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\}$.

Similarly for the case $(z * y) \notin Y$ and $(y * x) \in Y$ we get

$\tilde{\mu}_A(z * x) \ge r\min\{\tilde{\mu}_A(z * y)), \tilde{\mu}_A(y * x)\}$.

Therefore A is an i-v fuzzy KUS-ideal of X, the proof is complete. ⌂

Proposition 3.13. Let X be a KUS-algebra, B be a fuzzy subset on X and let A be an i-v fuzzy subset on X defined by $\tilde{\mu}_A(x) = \begin{cases} [\alpha_1, \alpha_2] & \text{if } x \in y \\ [0,0] & \text{otherwise} \end{cases}$. Where $\alpha_1, \alpha_2 \in (0, 1]$ with $\alpha_1 < \alpha_2$. If A is an i-v fuzzy KUS-sub-algebra of X, then B is a fuzzy KUS- sub-algebra of X.

Proof. Clear .⌂

Theorem 3.14. If A is an i-v fuzzy KUS-ideal of X, then the set

$X_{\tilde{M}_A} := \{x \in X \mid \tilde{\mu}_A(x) = \tilde{\mu}_A(0)\}$ is a KUS-ideal of X.

Proof. Let $(z * y), (y * x) \in X_{\tilde{M}_A}$. Then $\tilde{\mu}_A(z * y) = \tilde{\mu}_A(0) = \tilde{\mu}_A(y * x)$, and so

$\tilde{\mu}_A(z * x) \ge r\min\{\tilde{\mu}_A(z * y), \tilde{\mu}_A(y * x)\} = r\min\{\tilde{\mu}_A(0), \tilde{\mu}_A(0)\} = \tilde{\mu}_A(0)$.

Combining this with condition (1) of definition (3.6), we get $\tilde{\mu}_A(z * x) = \tilde{\mu}_A(0)$, that is $(z * x) \in X_{\tilde{M}_A}$.

Hence $X_{\tilde{M}_A}$ is a KUS-ideal of X. ⌂

IV. Homomorphism of KUS-algebra

Definition 4.1 ([1]). Let $f : (X; *, 0) \to (Y; *', 0')$ be a mapping from set X into a set Y. let B be an i-v fuzzy subset in Y. Then the inverse image of B, denoted by $f^{-1}(B)$, is an i-v fuzzy subset in X with the membership function given by

$\mu_{f^{-1}(B)}$ (x) = $\tilde{\mu}_B$ (f (x)), for all x∈ X .

Proposition 4.2 ([1]). Let f be a mapping from set X into a set Y , let m = [m^L ,m^u], and n = [n^L ,n^u] be i-v fuzzy subsets in X and Y respectively. Then

(1) f^{-1} (n) = [f^{-1} (n^L), f^{-1} (n^u)] ,

(2) f (m) = [f (m^L), f (m^u)] .

Theorem 4.3. Let f be homomorphism from a KUS-algebra X into a KUS-algebra Y. If B is an i-v fuzzy KUS-ideal of Y, then the inverse image f^{-1} (B) of B is an i-v fuzzy KUS-ideal of X.

Proof. Since B= [μ_B^L , μ_B^u] is an i-v fuzzy KUS-ideal of Y, it follows that from theorem (3.8), that (μ_B^L) and (μ_B^u) are fuzzy KUS-ideals of Y. Using theorem (2.12), we know f^{-1} (μ_B^L) and f^{-1} (μ_B^u) are fuzzy KUS-ideals of X. Hence by proposition (4.2), we conclude that f^{-1} (B) = [f^{-1} (μ_B^L) , f^{-1} (μ_B^u)] is an i-v fuzzy KUS-ideal of X. ⌂

Definition 4.4 ([9]). Let f be a mapping from a set X into a set Y . let A be a an i-v fuzzy set in X. then the image of A, denoted by f (A), is the i-v fuzzy subset in Y with membership function denoted by :

$$\tilde{\mu}_{f(A)}(x)= \begin{cases} \sup_{z\in f^{-1}(y)} \tilde{\mu}_A(z) & \text{if } f^{-1}(y)\neq \phi, y\in Y \\ [0,0] & \text{otherwise} \end{cases},$$

where f^{-1}(y):={x∈ X | f (x) = y}.

Theorem 4.5. Let f be a homomorphism from a KUS-algebra X into a KUS-algebra Y . If A is an i-v fuzzy KUS-ideal of X, then f (A) of A is an i-v fuzzy KUS-ideal of Y.

Proof. Assume that A= [μ_A^L , μ_A^u] is an i-v fuzzy KUS-ideal of X. it follows that from theorem (3.8), that (μ_A^L) and (μ_A^u) are fuzzy KUS-ideals of X. Using theorem (2.13), that the images f (μ_A^L) and f (μ_A^u) are fuzzy KUS-ideal of Y. Hence by proposition (4.2), we conclude that f (A)= [f (μ_A^L), f (μ_A^u)] is an i-v fuzzy KUS-ideal of Y .⌂

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