Deformation of a Uniform Half-Space with Rigid Boundary Due to a Strike-Slip Line Source

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Abstract: *Closed-form analytical expressions for the displacement and stresses at an arbitrary point caused by strike-slip line source buried in a homogeneous, isotropic, perfectly elastic half-space with rigid boundary are obtained. These expressions are used, further, to find the expressions for the displacement and stresses caused by vertical as well as horizontal strike-slip line source. The variation of the displacement and stress fields due to vertical strike-slip line source and horizontal strike-slip line source with distance from the fault and depth from the fault is studied numerically.*
Keywords – Half-space, rigid boundary, static deformation, strike-slip faulting

I.   Introduction

The theory of dislocation was first introduced to the field of seismology by Steketee (1958a, b). A dislocation is considered to be a defect in an elastic (or viscoelastic) medium which is represented by a discontinuity in the displacement field. Static dislocation models are used to analyze the static deformation of the medium caused by earthquake faults. These faults are slip planes across which discontinuous motion occurs in the Earth. If the slip on the fault is parallel to the fault strike, the fault is known as strike-slip fault. Steketee (1958a, b) obtained the displacement and stress field of a strike-slip fault with uniform slip. He gave the method for construction of the Green's functions and calculated one set of Green's functions corresponding to a vertical strike-slip fault. Following Steketee, Chinnery (1961) derived closed- form analytic expressions for the displacement field in the vicinity of a vertical, rectangular, strike-slip fault and obtained contour maps for two representative cases: Tango and North Idu earthquakes and the San Andreas Fault. Press (1965) computed residual displacements, strains and tilts at intermediate and large distances from major earthquakes due to vertical, rectangular, strike-slip and dip-slip faults in a half-space. The static deformation of various Earth models caused by two-dimensional sources has been studied by many investigators. Maruyama (1966) obtained the displacement and stress fields corresponding to long strike-slip and dip-slip faults of arbitrary dip. Rybicki (1971) investigated the effect of a vertical and a horizontal discontinuity in a half-space on the elastic residual field of a long strike-slip fault using the method of images. Sato (1971) and Sato and Yamashita (1975) derived the expressions for the static surface deformations due to two-dimensional strike slip and dip-slip faults located along the dipping boundary between the two different media. McHugh and Johnston (1977) and Rybicki and Kasahara (1977) studied the static deformation of a laterally inhomogeneous medium by a two- dimensional strike-slip fault. Rybicki (1978) derived the formulae for the displacement, strain and stress fields of a two-dimensional strike-slip fault in a laterally inhomogeneous half-space. Mahrer and Nur (1979a) developed a model for a very long strike-slip fault in a non- homogeneous half-space with a downward varying shear modulus. Mahrer and Nur (1979b) considered a static two-dimensional model of a long strike-slip fault in a crust with horizontally varying rigidity and calculated the surface displacement and strain fields for a number of slip regimes and crustal rigidity profiles. Wang and Wu (1983), following Steketee's method of integration, obtained a closed-form analytical solution for the displacement and stress field due to a "trapezodial" type of non-uniform slip along a strike-slip fault in an elastic half-space.

Singh (1985) considered the problem of the static deformation of a multilayered half-space by a long strike-slip dislocation. The representation of sources corresponding to antiplane strain has been considered. Singh and Garg (1986) obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Sharma and Garg (1991) have obtained the deformation field at any point of a layered half-space due to very long strike-slip dislocations. Madan and Garg (1997) obtained the analytic expressions for the displacement and stresses at any point of an orthotropic horizontal elastic layer coupling in different ways to a base due to a very long inclined strike-slip fault situated in a layer. Kumar *et al.* (2002) studied static deformation of two monoclinic elastic half-spaces in welded contact due to a long inclined strike-slip fault situated in one of the half-spaces analytically and numerically. Chug *et al.* (2010) obtained closed-form analytical expressions for the deformation at any point of a two-phase medium consisting of a

homogeneous orthotropic half-space in welded contact with a homogeneous isotropic elastic half-space caused by non-uniform slip along a very long vertical strike-slip dislocation, situated in the orthotropic elastic half-space. Singh *et al*. (2011) obtained analytical expressions for stresses at an arbitrary point of homogeneous, isotropic, perfectly elastic half-space with rigid boundary caused by a long tensile fault of finite width. Malik *et al*. (2012) obtained analytical expressions for stresses and the displacements at an arbitrary point of homogeneous, isotropic, perfectly elastic half-space with rigid boundary caused by a long dip-slip fault.

Beginning with the expression for the displacement in an infinite medium given by Singh (1985), we have obtained the integral expressions for the displacement and stresses due to a long strike-slip fault in a homogeneous, isotropic, perfectly elastic half-space by applying the boundary conditions of rigid boundary at the surface of the half-space. The integrals were then evaluated analytically, obtaining closed-form expressions for the displacement and the stresses at any point of the half-space caused by a long strike-slip fault. The expressions for the displacement and stresses for a vertical and a horizontal strike-slip dislocation follow immediately.

II.   Theory

In the Cartesian co-ordinates system $(x, y, z) \equiv (x_1, x_2, x_3)$, the equations of equilibrium are

$$p_{ij,j} + f_i = 0 \qquad (i = 1,2,3) \tag{1.1a, b, c}$$

where p_{ij} is the stress tensor, f_i is the body force per unit volume and

$$p_{ij,j} \equiv \frac{\partial}{\partial x_j} p_{ij}$$

Summation over the repeated suffix is understood. The stress-strain relations for an isotropic medium are

$$p_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij} \tag{1.2}$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ $i \neq j$), λ and μ are the lame constants and e_{ij} is the strain tensor. If (u_1, u_2, u_3) denote the components of the displacement vector, the strain-displacement relations are

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{1.3}$$

Using equations (1.2) and (1.3), we obtain the stress-displacement relations

$$p_{ij} = \lambda \delta_{ij} u_{k,k} + \mu(u_{i,j} + u_{j,i}) \tag{1.4}$$

We shall be considering a two-dimensional approximation in which the displacement components and consequently, the stresses are independent of a single co-ordinate, say x_1, so that $\partial/\partial x_1 \equiv 0$. Under this assumption. equations (1.1) - (1.4) get decoupled into two mutually independent sets- one representing the antiplane strain case ($u_2 = u_3 = 0$) and the other the plane strain case ($u_1 = 0$).

III.   Antiplane Strain

For antiplane case, we have

$$u_1 = u_1(y, z), \qquad u_2 = u_3 = 0$$

The non-zero strain and stress components are

$$e_{12} = \frac{1}{2}\frac{\partial u_1}{\partial y}, \qquad e_{13} = \frac{1}{2}\frac{\partial u_1}{\partial z} \tag{1.5}$$

$$p_{12} = \mu\frac{\partial u_1}{\partial y}, \qquad p_{13} = \mu\frac{\partial u_1}{\partial z} \tag{1.6}$$

For zero body forces, equilibrium equations (1.1b) and (1.1c) are identically satisfied and (1.1a) gives, using equation (1.6),

$$\nabla^2 u_1 = 0 \tag{1.7}$$

where ∇^2 is the two-dimensional Laplacian differential operator.

A solution of equation (1.7) corresponding a line source at (0, 0, h) in an infinite medium is (Singh, 1985):

$$u_1^{(0)} = \int_0^\infty \left[A_0 \sin ky + B_0 \cos ky \right] e^{-k|z-h|} dk \tag{1.8}$$

where the source coefficients A_0 and B_0 are independent of k. Singh (1985) has obtained these coefficients for the single couple (12) and (13). For the single couple (12)

$$A_0 = \frac{F_{12}}{2\pi\mu}, \qquad B_0 = 0 \tag{1.9a}$$

while for the single couple (13)

$$A_0 = 0, \qquad B_0 = \pm\frac{F_{13}}{2\pi\mu} \tag{1.9b}$$

where the upper sign is for z > h and the lower sign for z < h.
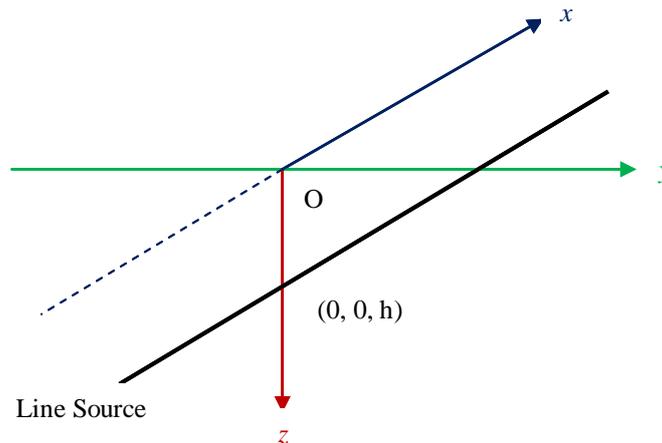
Fig.1 A uniform half-space ($z \geq 0$) with a line source acting at the point (0, 0, h)

For a line source parallel to the *x*-axis acting at the point (0, 0, h) of the half space $z \geq 0$ (Figure 1), a suitable solution of equation (1.7) is of the form

$$u_1 = u_1^{(0)} + \int_0^\infty \left[A \sin ky + B \cos ky \right] e^{-kz} dk \tag{1.10}$$

where $u_1^{(0)}$ is given by the equation (1.8) and A and B are unknown functions of k to be determined from the boundary conditions. From equations (1.6) and (1.10), the stresses are found to be

$$p_{12} = \mu\int_0^\infty \left(A_0 e^{-k|z-h|} + Ae^{-kz} \right)\cos ky \; kdk - \mu\int_0^\infty \left(B_0 e^{-k|z-h|} + Be^{-kz} \right)\sin ky \; kdk \tag{1.11}$$

$$p_{13} = \mu\int_0^\infty \left(\mp A_0 e^{-k|z-h|} - Ae^{-kz} \right)\sin ky \; kdk + \mu\int_0^\infty \left(\mp B_0 e^{-k|z-h|} - Be^{-kz} \right)\cos ky \; kdk \tag{1.12}$$

where the upper sign is for z > h and the lower sign for z < h.

We assume that the surface of the half-space $z \geq 0$ is with rigid boundary. Therefore, the boundary condition is

$$u_1 = 0 \text{ at } z = 0 \tag{1.13}$$

It is noticed from equations (1.9a, b) that A_0 and B_0 have different values for z $\gtrless$ h. Let A^- and B^- be respectively, the values of A_0 and B_0 for z < h. From equation (1.10) and (1.13), we get

$$A = -A^- e^{-kh}$$
$$B = -B^- e^{-kh} \tag{1.14}$$

Putting the values of A and B in equations (1.10) – (1.12), we get the following expressions for the displacement and stresses at any point of the half-space.

$$u_1 = \int_0^\infty \left[A_0 \sin ky + B_0 \cos ky \right] e^{-k|z-h|} dk - \int_0^\infty \left[A^- \sin ky + B^- \cos ky \right] e^{-k(z+h)} dk \tag{1.15}$$

$$p_{12} = \mu \int_0^\infty \left[A_0 \cos ky - B_0 \sin ky \right] e^{-k|z-h|} k \, dk - \mu \int_0^\infty \left[A^- \cos ky - B^- \sin ky \right] e^{-k(z+h)} k \, dk \tag{1.16}$$

$$p_{13} = \mu \int_0^\infty \left(A_0 \sin ky + B_0 \cos ky \right) e^{-k|z-h|} k \, dk + \mu \int_0^\infty \left(A^- \sin ky + B^- \cos ky \right) e^{-k(z+h)} k \, dk \tag{1.17}$$

Evaluating the integrals by using standard integral transform tables (Appendix I), we obtain

$$u_1 = \frac{A_0 y}{R^2} + \frac{B_0 (h-z)}{R^2} - \frac{A^- y}{S^2} - \frac{B^- (h+z)}{S^2} \tag{1.18}$$

$$p_{12} = \mu \left[\frac{A_0}{R^2} \left\{ \frac{2(h-z)^2}{R^2} - 1 \right\} - \frac{2B_0 y(h-z)}{R^4} - \frac{A^-}{S^2} \left\{ \frac{2(h+z)^2}{S^2} - 1 \right\} + \frac{2B^- y(h+z)}{S^4} \right] \tag{1.19}$$

$$p_{13} = \mu \left[\frac{2A_0 y(h-z)}{R^4} + \frac{B_0}{R^2} \left\{ \frac{2(h-z)^2}{R^2} - 1 \right\} + \frac{2A^- y(h+z)}{S^4} + \frac{B^-}{S^2} \left\{ \frac{2(h+z)^2}{S^2} - 1 \right\} \right] \tag{1.20}$$

IV.   Strike-Slip Dislocation

Let u_i^j denote the x_j-component of the displacement at $P(x_2, x_3)$ caused by a unit concentrated force acting at $Q(y_2, y_3)$ in the x_i - direction. Then, for antiplane strain, the displacement field due to a long displacement dislocation of arbitrary orientation is given by (Maruyama, 1966)

$$u_1 = \int_L \Delta u_1 u_{1k}^1 n_k ds \tag{1.21}$$

where $u_{1k}^1 = \mu \dfrac{\partial}{\partial y_k} u_1^1$ \qquad (1.22)

From equation (1.22), we note that u_{1k}^1 is the displacement at $P(x_2, x_3)$ in the x_1 - direction caused by a source, specified by [1k], we observe that

$$\left.\begin{array}{l} [12] = \mu(12) \\ [13] = \mu(13) \end{array}\right\} \tag{1.23}$$

Equation (1.23) shows that the sources [12] and [13] represent single couples with arms in the x_2 - and x_3 - directions, respectively.

Writing $\Delta u_1 = b$, $n_2 = -\sin \delta$, $n_3 = \cos \delta$, where δ is the dip of the fault, equation (1.21) becomes

$$u_1 = \int_L b(u_{13}^1 \cos \delta - u_{12}^1 \sin \delta) ds \tag{1.24}$$

Thus the source [12] corresponds to a vertical strike-slip fault ($\delta = 90^0$) and the source [13] corresponds to a horizontal strike-slip fault ($\delta = 0^0$). From equations (1.23) and (1.24), we have

$$u_1 = \int_L \mu b(u_{(13)}^1 \cos \delta - u_{(12)}^1 \sin \delta) ds \tag{1.25a}$$

In case of a line dislocation, we can do away with the integration in equation (1.25a), thus obtaining

$$u_1 = \mu b ds(u_{(13)}^1 \cos \delta - u_{(12)}^1 \sin \delta) \tag{1.25b}$$

Therefore, the field due to a long strike-slip fault of arbitrary dip can be expressed in terms of the fields due to a vertical strike-slip fault and a horizontal strike- slip fault.

V.   Vertical Strike-Slip

From equation (1.25b), the single couple (12) of moment F_{12} is equivalent to a vertical strike-slip line source such that

$$F_{12} = \mu bds \tag{1.26}$$

where b is the slip. Therefore, from equation (1.9a), the source coefficients for a vertical strike-slip source are given by

$$A_0 = A^- = \frac{bds}{2\pi}, \qquad B_0 = B^- = 0 \tag{1.27}$$

On putting these values of the source coefficients from equation (1.27) into equations (1.18-1.20), the results for the displacement and the stresses for a vertical strike-slip are found to be

$$u_1 = \frac{bdsy}{2\pi}\left[\frac{1}{R^2} - \frac{1}{S^2}\right] \tag{1.28}$$

$$p_{12} = \frac{\mu bds}{2\pi}\left[\frac{2(h-z)^2}{R^4} - \frac{1}{R^2} - \frac{2(h+z)^2}{S^4} + \frac{1}{S^2}\right] \tag{1.29}$$

$$p_{13} = \frac{\mu bdsy}{\pi}\left[\frac{(h-z)}{R^4} + \frac{(h+z)}{S^4}\right] \tag{1.30}$$

VI. Horizontal Strike-Slip

The single couple (13) of moment F_{13} is equivalent to a horizontal strike-slip line source such that

$$F_{13} = \mu bds \tag{1.31}$$

Therefore, from equation (1.9b), the source coefficients for a horizontal strike-slip source are given by

$$A_0 = A^- = 0, \qquad B_0 = \pm\frac{bds}{2\pi}, \qquad B^- = -\frac{bds}{2\pi} \tag{1.32}$$

On putting these values of the source coefficients from equation (1.32) into equations (1.18-1.20), the results for the displacement and the stresses for a horizontal strike-slip are found to be

$$u_1 = \frac{bds}{2\pi}\left[\frac{(z-h)}{R^2} + \frac{(z+h)}{S^2}\right] \tag{1.33}$$

$$p_{12} = \frac{\mu bdsy}{\pi}\left[\frac{(h-z)}{R^4} - \frac{(h+z)}{S^4}\right] \tag{1.34}$$

$$p_{13} = \frac{\mu bds}{2\pi}\left[\frac{1}{R^2} - \frac{2(h-z)^2}{R^4} + \frac{1}{S^2} - \frac{2(h+z)^2}{S^4}\right] \tag{1.35}$$

where $R^2 = y^2 + (z-h)^2$, $S^2 = y^2 + (z+h)^2$.

VII. Numerical Results

We study numerically displacement and the stress fields at any point of the uniform isotropic perfectly elastic half-space with rigid boundary caused by a vertical and horizontal strike-slip line source. We define the following dimensionless quantities

$$Y = \frac{y}{L}, \qquad Z = \frac{z}{L} \tag{1.36}$$

where L is the distance of the line source from the interface. Let the dimensionless displacement and stresses be denoted by U_i and P_{ij}. Then,

$$U_i = \frac{\pi L}{bds}u_i, \qquad P_{ij} = \frac{\pi L^2}{\mu bds}p_{ij} \tag{1.37}$$

where b is the magnitude of dislocation (slip) and ds is the width of the fault.

From equations (1.28-1.30), (1.36) and (1.37), we get the following expressions for the dimensionless displacement and stresses for a vertical strike-slip line source:

$$U_1 = \frac{Y}{2}\left[\frac{1}{A^2} - \frac{1}{B^2}\right] \tag{1.38}$$

$$P_{12} = \frac{1}{2}\left[\frac{2(1-Z)^2}{A^4} - \frac{1}{A^2} - \frac{2(1+Z)^2}{B^4} + \frac{1}{B^2}\right] \tag{1.39}$$

$$P_{13} = Y\left[\frac{(1-Z)}{A^4} + \frac{(1+Z)}{B^4}\right] \tag{1.40}$$

From equations (1.33-1.35), (1.36) and (1.37), we get the following expressions for the dimensionless displacement and stresses for a horizontal strike-slip line source:

$$U_1 = \frac{1}{2}\left[\frac{(Z-1)}{A^2} + \frac{(Z+1)}{B^2}\right] \tag{1.41}$$

$$P_{12} = Y\left[\frac{(1-Z)}{A^4} - \frac{(1+Z)}{B^4}\right] \tag{1.42}$$

$$P_{13} = \frac{1}{2}\left[\frac{1}{A^2} - \frac{2(1-Z)^2}{A^4} + \frac{1}{B^2} - \frac{2(1+Z)^2}{B^4}\right] \tag{1.43}$$

where $A^2 = Y^2 + (Z-1)^2$, $\qquad B^2 = Y^2 + (Z+1)^2$

VIII.   Discussion

Figures 2(a) - 2(c) show the variation of dimensionless vertical displacement U_1 and shear stresses P_{12} and P_{13} at the interface with the horizontal distance from the fault. Figure 2(a) shows the variation of displacement U_1 with distance from the fault at z = 2L, 2.5L and 3L respectively. Moreover, U_1 tends to zero as y approaches to infinity. Figure 2(b) shows the variation of the dimensionless shear stress P_{12} with the horizontal distance from the fault at z = 2L, 2.5L and 3L respectively. At y = 0, P_{12} attains its maximum value for z = 2L and minimum value at z = 3L. P_{12} approaches to zero as y approaches to infinity. Figure 2(c) shows the variation of the dimensionless shear stress P_{13} with y at z = 2L, 2.5L and 3L. It is observed that P_{13} is zero at y = 0 and also tends to zero as y approaches to infinity.

Figures 3(a) - 3(c) show the variation of dimensionless vertical displacement U_1 and shear stresses P_{12} and P_{13} at the interface with the depth at locations at y = 2L, 2.5L and 3L respectively. It is observed that for y = 3L, the variation of U_1 is smooth but for y = 2L, U_1 varies strongly in the range $0 < z < 2L$. Moreover it tends to zero as z approaches to infinity. Figure 3(b) shows the variation of the dimensionless shear stress P_{12} with the depth at y = 2L, 2.5L and 3L respectively. The variation of P_{12} for y =2L depends strongly on z whereas for y = 2.5L and y = 3L, the variation of stress component P_{12} is smooth. P_{12} tends to zero as z approaches to infinity. Figure 3(c) shows the variation of the dimensionless shear stress P_{13} with z at y = 2L, 2.5L and 3L. For y = 2L, P_{13} attains the maximum value. The variation is significant in the range o < z < 3L. P_{13} tends to zero as z approaches to infinity.

Figures 4(a) - 4(c) show the variation of dimensionless horizontal displacement U_1 and shear stresses P_{12} and P_{13} at the interface with the horizontal distance from the fault. Figure 4(a) shows the variation of the horizontal displacement U_1 with distance from the fault at z = 2L, 2.5L and 3L respectively. The value of U_1 is maximum at z = 2L and minimum at z = 3L and tends to zero as y approaches to infinity. Figures 4(b) and 4(c)

show the variation of the dimensionless shear stresses P_{12} and P_{13} respectively with the horizontal distance from the fault at z = 2L, 2.5L and 3L. P_{12} attains its minimum value for z = 2L and maximum value at z = 3L. P_{12} approaches to zero as y approaches to infinity. The stress P_{13} also attains maximum value at z = 2L and varies strongly in the range 0 < y < 2L. It also tends to zero as y approaches to infinity.

Figures 5(a) - 5(c) show the variation of dimensionless horizontal displacement U_1 and shear stresses P_{12} and P_{13} at the interface with the depth at locations at y = 2L, 2.5L and 3L respectively. The variation of U_1 is sharp for z = 2L while it is smooth for z = 3L. Moreover it tends to zero as z approaches to infinity. Figure 5(b) shows the variation of the dimensionless shear stress P_{12} with the depth at y = 2L, 2.5L and 3L respectively. The variation of P_{12} for y =2L depends strongly on z whereas for y = 2.5L and y = 3L, the variation of stress component P_{12} is smooth. P_{12} tends to zero as z approaches to infinity. Figure 5(c) shows the variation of the dimensionless shear stress P_{13} with z at y = 2L, 2.5L and 3L. The variation of the shear stress is noticeable in the range 2L< z < 3L for all values of y and tends to zero as z approaches to infinity.

VERTICAL STRIKE SLIP FAULT
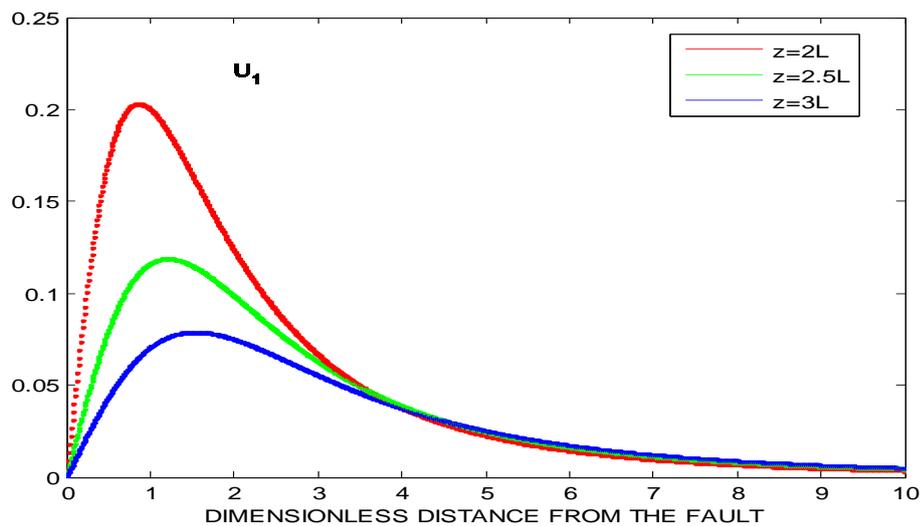
Fig. 2(a) Variation of dimensionless displacement U_1 with the distance from the fault
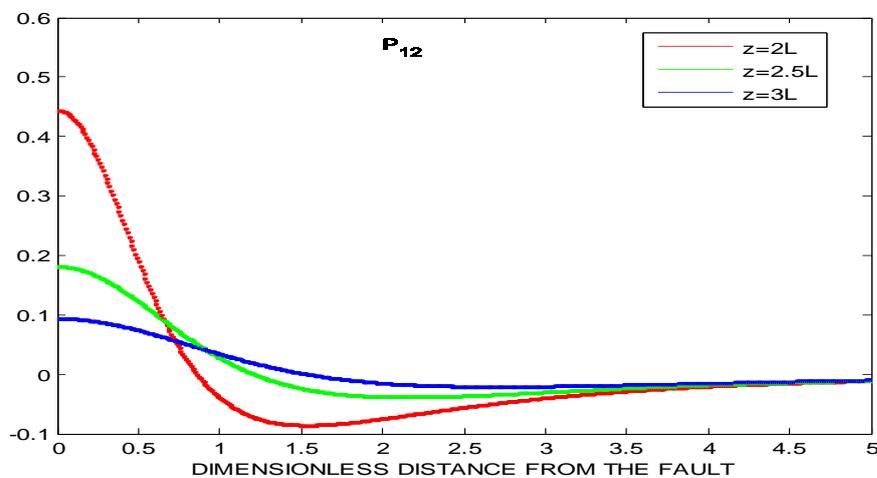
Fig. 2(b) Variation of dimensionless shear stress (P_{12}) with the distance from the fault
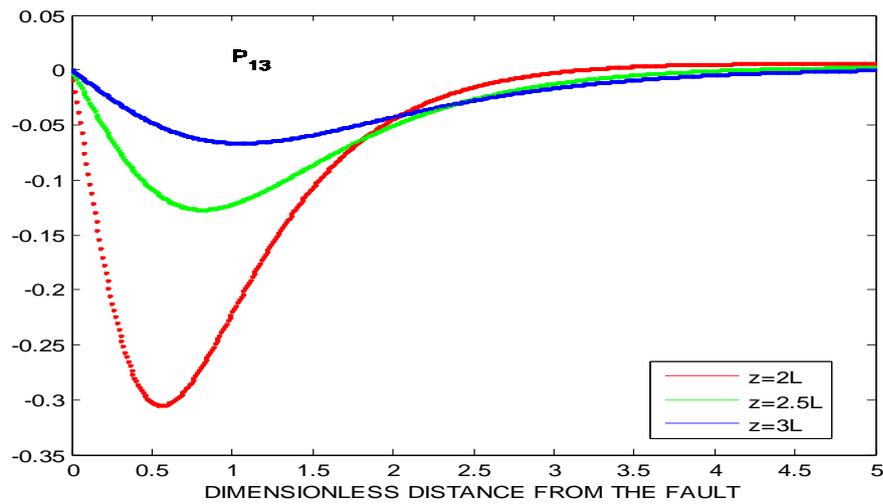
Fig. 2(c) Variation of dimensionless shear stress (P_{13}) with the distance from the fault
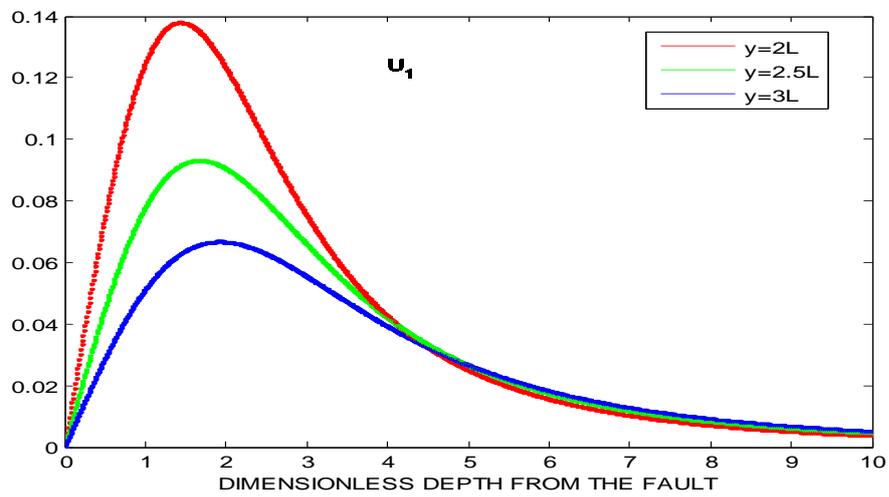
Fig. 3(a) Variation of dimensionless displacement U_1 with the depth from the fault
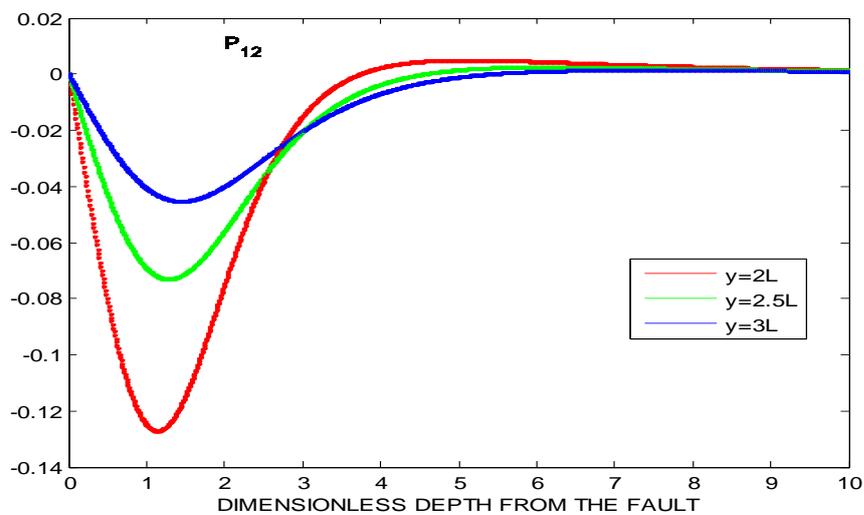
Fig. 3(b) Variation of dimensionless shear stress (P_{12}) with the depth from the fault
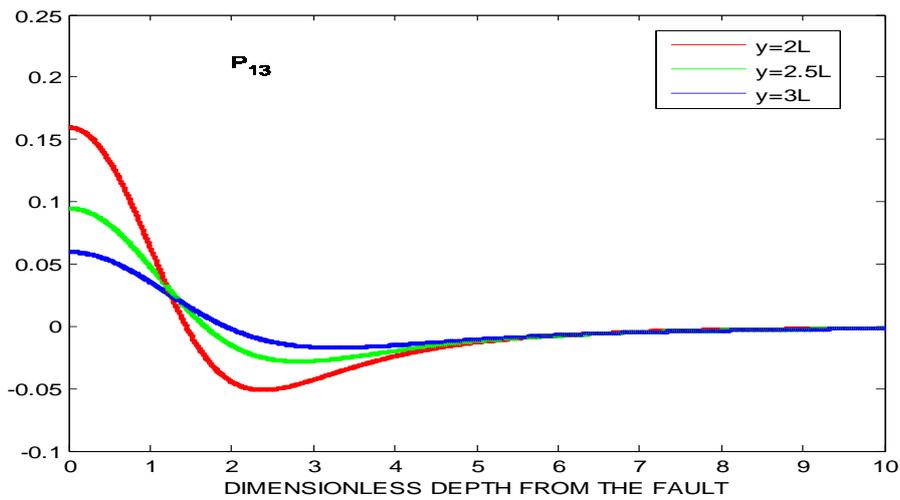
Fig. 3(c) Variation of dimensionless shear stress (P_{13}) with the depth from the fault

HORIZONTAL STRIKE-SLIP FAULT
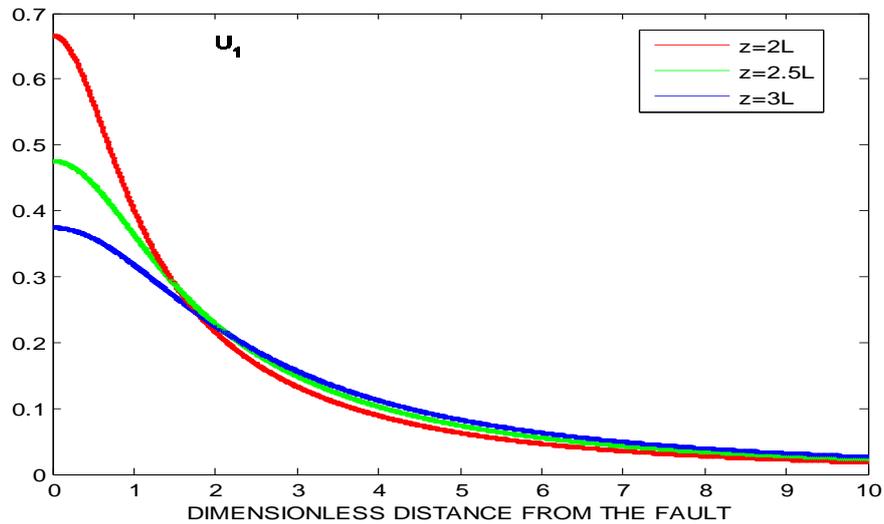
Fig. 4(a) Variation of dimensionless displacement U_1 with the distance from the fault
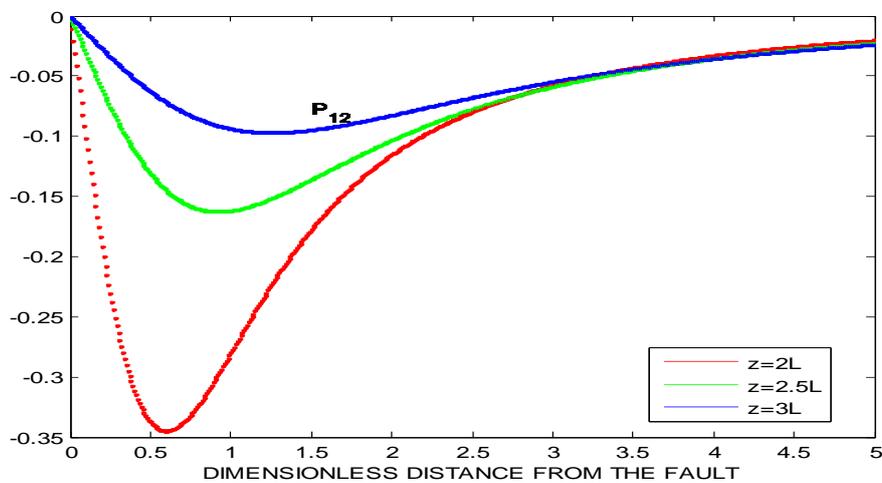
Fig. 4(b) Variation of dimensionless shear stress (P_{12}) with the distance from the fault
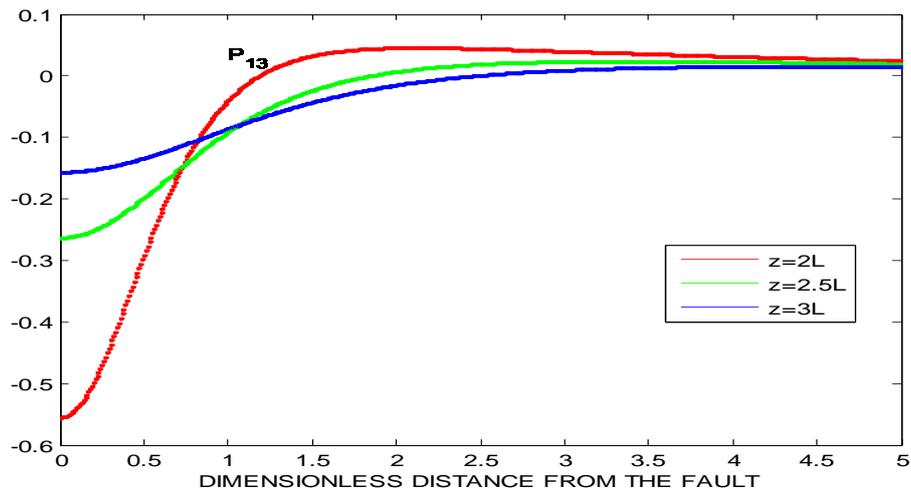
Fig. 4(c) Variation of dimensionless shear stress (P_{13}) with the distance from the fault
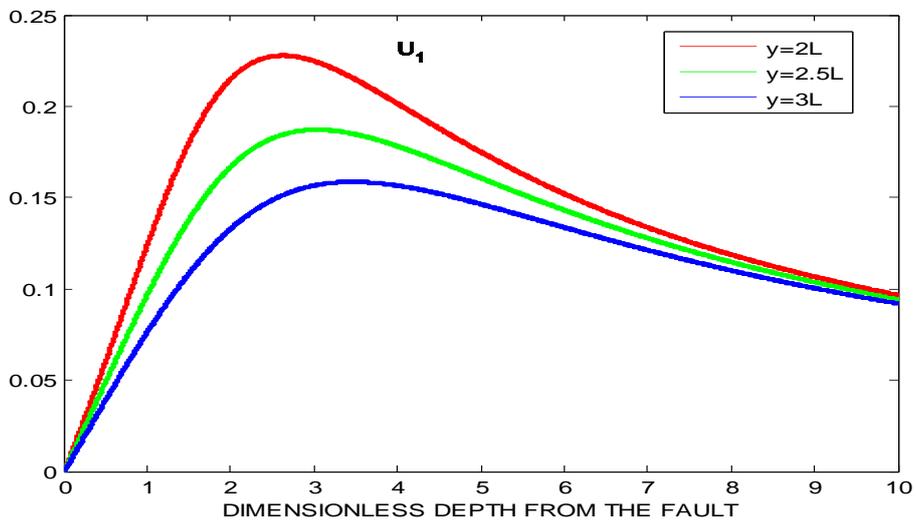
Fig. 5(a) Variation of dimensionless displacement U_1 with the depth from the fault
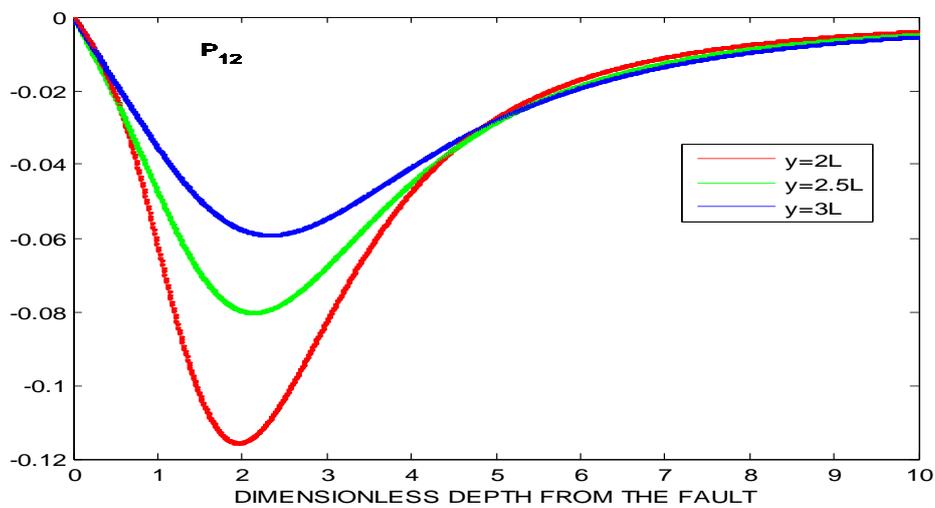
Fig. 5(b) Variation of dimensionless shear stress (P_{12}) with the depth from the fault
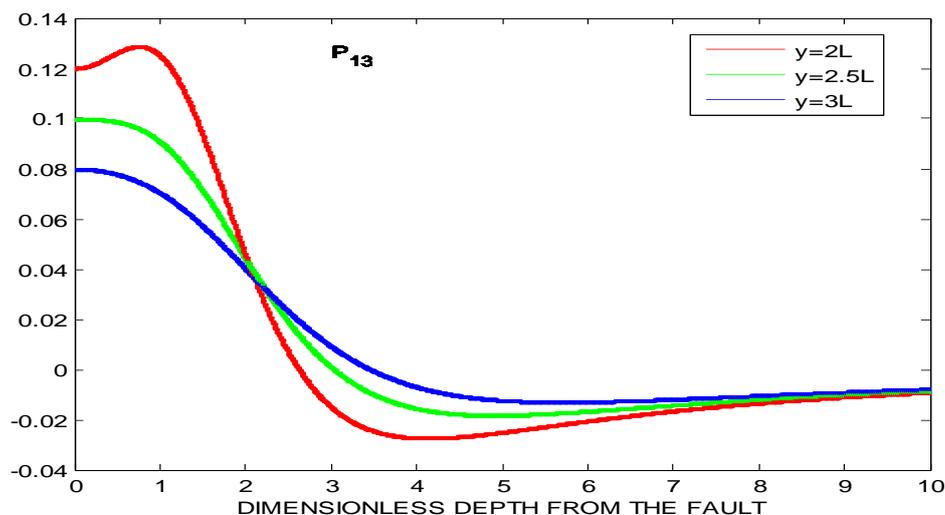
Fig. 5(c) Variation of dimensionless shear stress (P_{13}) with the depth from the fault

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Appendix I
$$\left[z > 0, \; y^2 + z^2 = R^2 \right]$$

i. $\displaystyle\int_0^\infty e^{-kz} \frac{\sin ky}{k} dk = \tan^{-1}\left(\frac{y}{z}\right)$ vi $\displaystyle\int_0^\infty e^{-kz} \cos ky k \, dk = \frac{1}{R^2}\left(\frac{2z^2}{R^2} - 1\right)$

ii. $\displaystyle\int_0^\infty e^{-kz} \frac{\cos ky}{k} dk = -\log R$ vii $\displaystyle\int_0^\infty e^{-kz} \sin ky k^2 \, dk = \frac{2y}{R^4}\left(\frac{4z^2}{R^2} - 1\right)$

iii. $\displaystyle\int_0^\infty e^{-kz} \sin ky \, dk = \left(\frac{y}{R^2}\right)$ viii $\displaystyle\int_0^\infty e^{-kz} \cos ky k^2 \, dk = \frac{2z}{R^4}\left(\frac{4z^2}{R^2} - 3\right)$

iv. $\displaystyle\int_0^\infty e^{-kz} \cos ky \, dk = \left(\frac{y}{R^2}\right)$ ix $\displaystyle\int_0^\infty e^{-kz} \sin ky k^3 \, dk = \frac{24yz}{R^6}\left(\frac{2z^2}{R^2} - 1\right)$

v. $\displaystyle\int_0^\infty e^{-kz} \sin ky k \, dk = \frac{2yz}{R^4}$ x $\displaystyle\int_0^\infty e^{-kz} \cos ky k^3 \, dk = \frac{6}{R^4}\left(\frac{8z^4}{R^4} - \frac{8z^2}{R^2} + 1\right)$