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Heat and momentum transfer of isentropic fluid flow over a moving plate

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Abstract: *The paper investigates isentropic fluid flow over a moving plate with impact of heat and momentum transfer into consideration. The outcome equations are solved analytically using perturbation technique. Velocity, temperature and concentration profiles are achieved, as well as effect of Grashof and modified Grashof numbers on skin friction. Some of the important results seen are that increase in radiation parameter decreases the real temperature and increase in reaction parameter leads to decrease in concentration.*
Keywords: *Heat transfer, momentum transfer, isentropic.*

I. Introduction

Isentropic fluid is pronounced in nature and as a result attracted attention in the past few decades. It has many applications in Engineering and Science. One of such is the refrigerant which is a fluid used for heat transfer in a refrigeration system. Most refrigerants absorb heat during evaporation at low temperature and low pressure and reject heat during condensation at a higher temperature and higher pressure. Recently (Anco & Dar 2009) undertook a systematic study of local conservation laws. For the Euler equation governing isentropic compressible fluid flow in n>1 spatial dimensions, where the pressure of the fluid is a function of the fluid density. Moroso, Muthucumaraswamy and Senthic (2004) have studied heat and mass transfer effect on moving plate in the presence of thermal radiation. Hossain et al (1984) investigated the effects of mass transfer on the unsteady free-convection flow of an electrically conducting and viscous incompressible fluid past an infinite vertical plate subjected to variable suction in the presence of transverse magnetic field in which the flow is effected by the variable suction.

Noushima et al (2009) studied unsteady MHD memory flow and heat transfer over a moving continuous porous horizontal surface. Kia – long (2010) studied heat and mass transfer for viscous flow with radiation effect past a non-linear stretching sheet. Uwanta and Omokhuale (2012) also studied viscoelastic fluid flow in a fixed plane with heat and mass transfer.

Also Isreal – Cookey and Sigalo(2002) considered the problem of MHD free-convection flow past a semi-infinite heated porous vertical plate with time dependent suction and heat transfer in an optically thin environment. In all these studies, heat and momentum transfer of isentropic flow over a moving plate was neglected which the paper is able to capture.

II. Mathematical Formulation

We consider the flow of an electrically conducting fluid over a moving plate in the presence of applied transverse magnetic field and constant suction. The plate is assumed in x-axis taken along the upward direction and y-axis normal to the plate. We inject small magnetic field of small intensity Bo in the direction of y. however Reynold number is less than one as a result the induced magnetic field is ignored.

Continuity, momentum, concentration and energy are the governing equations

$$\frac{\partial \omega}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + \omega \frac{\partial u}{\partial y} = \frac{\partial u}{\partial t} + v \frac{\partial^2 u}{\partial y^2} - \frac{v}{k}(u - U) - \frac{c\mu^2 B^2{}_0}{\rho}(u - U) + g\beta_c (C - C_\infty) \qquad (2)$$

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial y} = \frac{\Delta}{v}\left(\frac{\partial^2}{\partial y^2} - k^2 y\right) C \qquad (3)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c \rho} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c \rho} \frac{\partial q_r}{\partial y} \qquad (4)$$

The manner of dynamic radiactive heat flux with respect to length and momentum transfer were put to the established equations. However, the mathematical derivation is an extension of Cogley et al . Integrating equation (1) $\omega = -\omega_O$ (5)

Where $\omega_O > 0$ and the negative sign indicates that the suction velocity is towards the plate.

Introducing the boundary conditions of the problem we have:

$$\begin{aligned} &y = O : U = \omega_O\left(1 + \varepsilon e'\omega' t'\right), T = T\omega, C = C\omega \\ &y = 1 : \quad U = 0, \ \ \theta = 0, \ C = 0 \end{aligned} \tag{6}$$

The non – dimensionless parameters are:

$$t = \frac{\omega_O{}^2 t}{4v}, \ y = \frac{\omega_O}{v} y, u = \frac{u}{u_.} U = \frac{U}{U_0},$$

$$\omega = \frac{4v}{\omega_O{}^2}\omega^1, \ \theta = \frac{T - T_\infty}{T_\omega - T_\infty}, \ \ C = \frac{C - C_\infty}{C_\omega - C_\infty}$$

$$F^2 = \frac{4\alpha^2}{\rho C_\rho K \omega_O^2}\left(T_\omega - T_\infty\right), \ p_r = \frac{\mu c_\rho}{k} \tag{7}$$

$$Gr = \frac{vg\beta\left(C_\omega - C_\infty\right)}{u_o \omega_O^2}, \chi = \frac{v^2}{\omega_O^2 k}$$

$$M = \sqrt{\frac{\mu_O B_O^2}{\rho \omega_O^2}} \quad Sc = \frac{V}{\Delta}, K_y^2 = k_y^2$$

Moreso, the term $\dfrac{\partial q_r}{\partial y}$ is the radiactive heat flux. Applying Vincenti et al approximation i.e.

$$\frac{\partial q_r}{\partial y} = 4\alpha^2\left(T - T_\infty\right) \tag{8}$$

Where $\alpha^2 = \int_O^\infty \delta\lambda\left(\dfrac{\partial B}{\partial T}\right)$ (9)

Putting the dimensionless variable in (2) (3) (4) and also using (8) and (9) we have

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{iwt}\right)\frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial u}{\partial t} + \frac{\partial^2}{\partial y^2}\left(X^2 + M^2\right)\left(u - U\right) - M^2 u + Gr\theta + Gc \tag{10}$$

$$\frac{1}{4} Gc\frac{\partial \theta}{\partial t} - \rho_r \left(1 + \varepsilon A e^{iwt}\right)\frac{\partial \theta}{\partial y} = \left(\frac{\partial \theta^2}{\partial y^2} - F^2\right)\theta \tag{11}$$

$$\frac{1}{4} Gc\frac{\partial c}{\partial t} - Gc \left(1 + \varepsilon A e^{iwt}\right)\frac{\partial \theta}{\partial y} = \left(\frac{\partial^2}{\partial y^2} - ky^2\right)C \tag{12}$$

With boundary conditions

$$\begin{aligned} &y = 0 : U = 1 + \varepsilon e^{iwt}, \qquad \theta = 1, \quad C = 1 \\ &y = 1 : U = 0, \ \ \theta = 0, \ C = 0 \ \ as\ y \to \infty \end{aligned} \tag{13}$$

Method of Solutions

Solving equations (10), (11) and (12) we assume a time dependent perturbation expansion of the form

$$U(y,t) = U_O(y) + \varepsilon e^{iwt} U_1(y) \qquad (14)$$

$$\theta(y,t) = \theta_O(y) + \varepsilon e^{iwt} \theta_1(y) \qquad (15)$$

$$C(y,t) = C_O(y) + \varepsilon e^{iwt} C_1(y) \qquad (16)$$

Substituting (14), (15), (16) into (10), (11) and (12) and neglecting the terms involving powers of ε^2, we have

$$U_o'' + U_o' - \left(X^2 + M^2\right)U_o = Gr\theta_o - G_c C_o - \left(X^2 + M^2\right) \qquad (17)$$

$$\theta_o'' + \rho_r \theta_o' - F^2 \theta_o = 0 \qquad (18)$$

$$C_o'' + S_c C_o' - K^2 y = 0 \qquad (19)$$

Subject to

$$\begin{aligned} &y = 0 : U_0 = 1, \quad \theta_0 = 1, \quad C_0 = 1 \\ &y = 1 : U_0 = 0, \ \theta_0 = 0, \ C_0 = 0 \ as\ y \to \infty \end{aligned} \qquad (20)$$

Again

$$U_1'' + U_1' - \left(X^2 + M^2 + \frac{iw}{4}\right)U_1 = -AU_o' - G_r C_1 - X^2 + M^2 - \frac{iw}{4} \qquad (21)$$

$$\theta_1'' + \rho_r \theta_1' - \left(\frac{iw\rho_r}{4} + F^2\right)\theta_1 = -A\rho_r \theta_o' \qquad (22)$$

$$C_1'' + ScC_1' - \left(K_y^2 + \frac{iwS_c}{4}\right)C_1 = -S_c AC_o' \qquad (23)$$

Subject to

$$\begin{aligned} &y = 0 : U_1 = 1, \quad \theta_1 = 0, \ C_1 = 0 \\ &y = 1 : U_1 = 0, \ \theta_1 = 0, \ C_1 = 0 \ as\ y \to \infty \end{aligned} \qquad (24)$$

By solving equation (17), (18) and (19) subject to (20). The solutions are

$$U_0 y = (\beta_1 + \beta_2)e^{m_5 y} - \beta_1 e^{m_1 y} + \beta_2 e^{m_3 y} + 1$$

Where

$$m_1 = -\frac{1}{2}\left(Sc + \sqrt{Sc_c^2 + 4k_y^2}\right)$$

$$m_3 = -\frac{1}{2}\left(\rho_r + \sqrt{\rho_r^{\ 2} + 4F^2}\right)$$

$$m_4 = -\frac{1}{2}\left[\rho_r + \sqrt{\rho_{r^2} + 4\left(F^2 + \frac{iw\rho_r}{4}\right)}\right]$$

$$m_5 = -\frac{1}{2}\left(1 + 4\sqrt{X^2 + M^2}\right)$$

$$m_6 = -\frac{1}{2}\left(1+\sqrt{1+(1+\alpha_0)}\right)$$

$$\beta_1 = \frac{G_c}{m_1^2 + m_1 - \left(X^2 + M^2\right)}$$

$$Sh = -\frac{\partial C}{\partial y}\bigg|_{y=0} \qquad \beta_2 = \frac{G_r}{m_3^2 + m_3 - \left(X^2 + M^2\right)}$$

$$\theta_o(y) = e^{m_3 y} \tag{25}$$

And

$$C_o(y) = e^{m_1 y} \tag{26}$$

Again, solving (21) to (23) with the boundary condition (24) and substituting the obtained solution into (14), we have the velocity, temperature and concentration fields expressed as

$$U(y,t) = (\beta_1 + \beta_2)e^{m_5 y} - \beta_1 e^{m_1 y} - \beta_2 e^{m_3 y} + 1 + \varepsilon e^{iwt}\left(D_1 + D_2 + D_3\right)e^{m_6 y} - D_1 e^{m_2 y} \tag{27}$$

$$\theta(y,t) = e^{m_1 y} + \varepsilon e^{iwt}\left(\varepsilon e^{m_4 y} - 1\right)L \tag{28}$$

$$C(y,t) = e^{m_1 y} + \varepsilon e^{iwt}\left(e^{m_2 y} - 1\right)H \tag{29}$$

Where $L = \dfrac{A\rho_r m_3}{m_3^2 + \rho_r m_3 - \left(F^2 + \dfrac{iw\rho r}{4}\right)}$

And $H = \dfrac{S_c A m_1}{m_1^2 + Scm_1 - \left(K_y^2 + \dfrac{iwsc}{4}\right)}$

However, skin friction is achieved when we differentiate (27) and evaluate at y=0

$$-\frac{\partial U}{\partial y}(y,t)\bigg|_{y=0} = m_5(\beta_1 + \beta_2) - \beta_1 m_3 - \beta_2 m_1 + m_6 \varepsilon e^{iwt}\left(D_1 + D_2 + D_3\right) - D_1 m_2 - D_2 M_4$$

Where $D_1 = \dfrac{G_c H m_2}{m_2^2 + m_2 - \alpha_0}$

$$D_2 = \frac{G_r L m_4}{m_4^2 + m_4 - \alpha_0}$$

$$D_3 = \frac{A(\beta_1 + \beta_2)m_5}{m_5^2 + m_5 - \alpha_0}$$

We obtain Nusselt number when we differentiate (28) and evaluate at y = 0

$$Nu = -\frac{\partial \theta}{\partial y}\bigg|_{y=0} = m_3 + \varepsilon e^{iwt} m_4 \tag{31}$$

Sherwood number is obtained when (29) is differentiated

$$= m_1 + \varepsilon e^{iwt} m_2 \qquad (32)$$

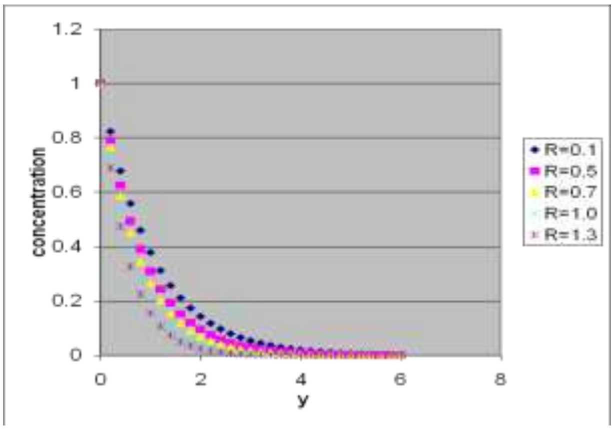

Fig.1Concentration profile for G_c=1 , A=0.01, t=0.5 and different values of reaction parameter

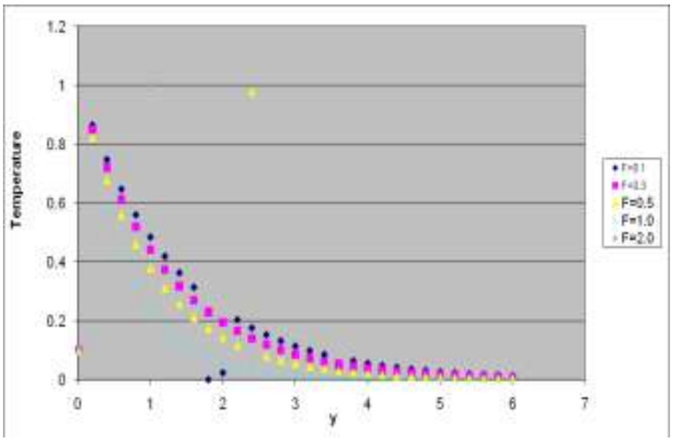

Fig.2Temperature profile for G_c=2, A=0.01, t=0.5 and different values of radiation parameter

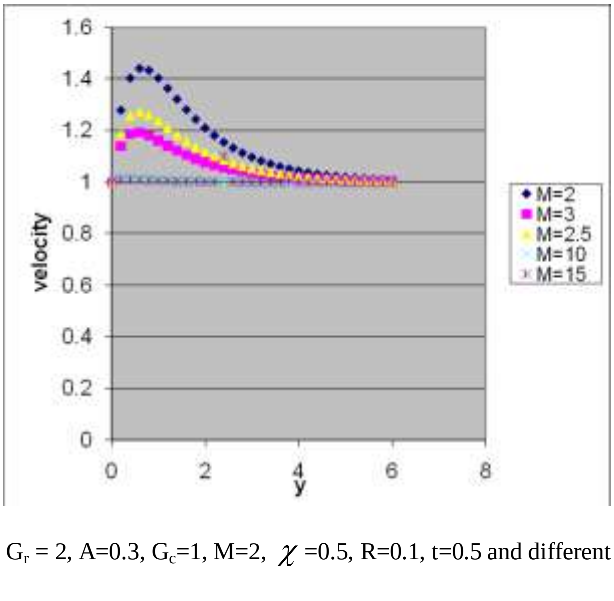

Fig.3 Velocity profiles for G_r = 2, A=0.3, G_c=1, M=2, χ =0.5, R=0.1, t=0.5 and different values of magnetic parameter

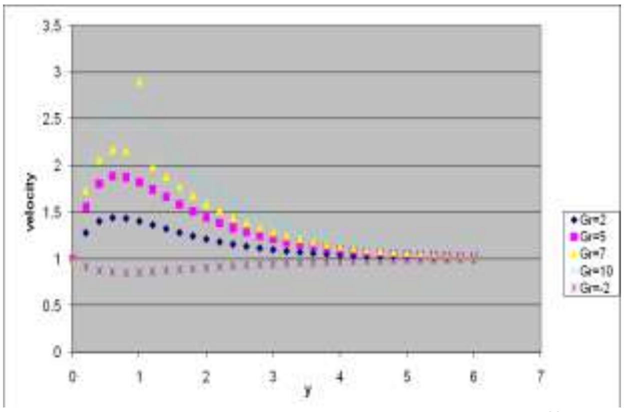

Fig.4Velocity profiles for A=0.3, G_c=1, M=2, χ =0.5, F=0.1, R=0.1, t=0.5 and different values of radiation parameter

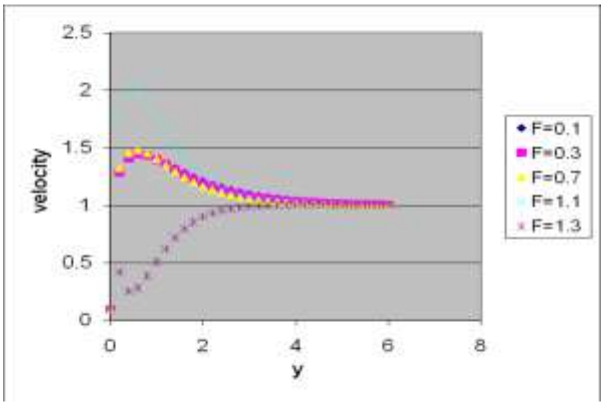

Fig.5Velocity profiles for G_r = 2, A=0.3, G_c=1, M=2, χ =0.5, R=0.1, t=0.5 and different values of radiation parameter

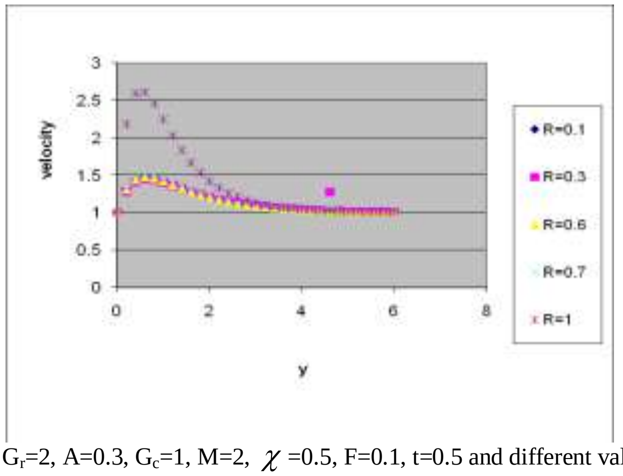

Fig.6Velocity profiles for G_r=2, A=0.3, G_c=1, M=2, χ =0.5, F=0.1, t=0.5 and different values of reaction parameter

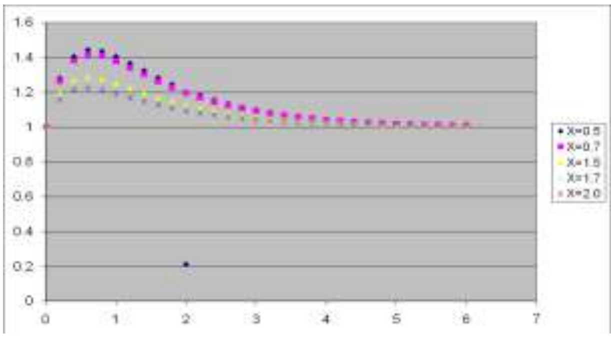

Fig.7 Velocity profiles for G_r=2, A=0.3, G_c=1, M=2, F=0.1, R=0.1, t=0.5 and different values of porosity parameter

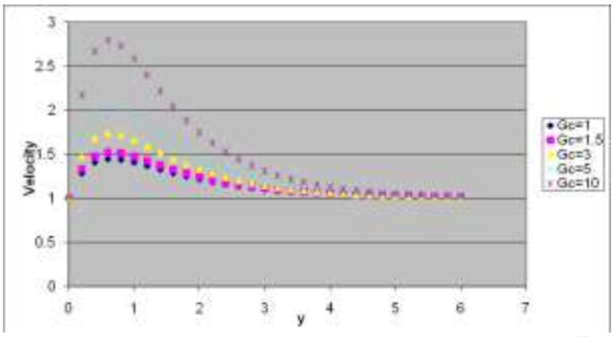

Fig. 8Velocity profiles for G_r=2, A=0.3, G_c=1, M=2, χ =0.5, F=0.1, R=0.1, t=0.5 and different values of modified Grashof parameter

However, we present the skin friction for A=0.3, G_c=1, M=2, X=0.5, F=0.1 and t=0.5 and various values of Grashof number. it is observed from velocity that when G_r=2, 5, 7, 10 and -2 when other parameters are kept constant that the corresponding skin frictions are -1.70238, -3.31424, -4.3881, -6.00067 and 0.446767. It indicates that increase in Grashof number result to decrease in skin frction.

Moreso, we present the skin friction for G_r=2, A=0.3, M=2, X=0.5, F=0.1, A=0.1 and t=0.5 and various values of modified Grashof number. It is seen that when G_c=1, 1.5, 3, 5 and 10 when other parameters are kept constants that the corresponding skin frictions are -1.70238, -2.95801, -4.21363 and -7.35268. It implies that increase in Grashof number result to decrease in skin friction.

III. Results and Discussion

Isentropic fluid flow over a moving plate was formulated and solved analytically. Computations with a software mathematica are adopted for different parameters like Gc, Gr, M, F, Sc, etc.

Figures 1 & 2 represent the concentration and temperature profiles respectively and figures 3 to 8 represent velocity profiles with different parameters.

In fig.1, it is observed that the real concentration profile decays exponentially across the boundary layer. Therefore increase in the reaction parameter led to a decrease in the real concentration. In fig.2 the temperature profile within the system for A = 0.01, ρ_r = 0.71, ε = 0.01 and t = 0.2 and different values of the frequency and radiation parameter F. We observed that increase in the radiation parameter led to a decrease in the real temperature.

The impact of velocity for different values of (M=2,3,2.5,10,15) is shown in fig.3, the impact of velocity for different values of (G_r = 2,5,7,10,2) is given in fig.4 and 5 represent the effect of velocity for (F = 0.1, 0.3, 0.7, 1.1, 1.3). The graphs show that velocity increases with the increase in M, Gr, and F.

The impact of velocity for different values of (R = 0.1, 0.3, 0.6, 0.7, 1) is shown in fig.6, the impact of velocity for different values of (X = 0.5, 0.7, 1.5, 1.7, 2.0) is indicated in fig.7 and 8 denote the impact of velocity for different values of (Gc = 1, 1.5, 3, 5, 10). The graphs show that velocity increases with increase in R, X and Gc. Moreso, effect of variations of Grashof and modified Grashof numbers on skin friction for various values of material parameter were presented. The result show that increase in Grashof and modified Grashof number decrease in skin friction.

IV. Summary and Conclusion

We therefore conclude that velocity increases with the increase in M, Gr, F, R, χ and G_c. Concentration decreased led to an increase in the reaction parameter while decrease in temperature led to increase in radiation parameter. Also that Grashof numbers decrease in skin friction.

References

[1]. (Anco, S.C. & Dar, A. 2009). Classification and Conservation Laws in fluid flow.

[2]. Hossain, M.A. and Mandal, C.A. “Effect of Mass Transfer and free convection on the unsteady MHD flow past a vertical porous plate with variable suction”. IC/84/192 International centre for Theoretical Physics, Triestile, Italy Publications, 1984.

[3]. Isreal-Cookey, C. and Sigalo, F.B.,(2002). “On the unsteady MHD free – convection flow past semi – infinite heated porous vertical plate with time dependent suction and radiative heat transfer”.

[4]. Kai – long, H., (2010). Heat and Mass transfer for a viscous flow with radiation effect past a non linear stretching sheet. World Academic Sci. Eng. Technology, 62(3): 320—330.

[5]. Muthucumaraswamy, R. and G.K. Senthil,(2004). Heat and mass effects on moving vertical plate in the presence of thermal radiation. Int. J. Appl. Theoretical Mec., 17(20): 801-820.

[6]. Noushima, H.G., M.V. Ramana Murthy Rafiudinn and R.M. Chennakrishna, 2009. Unsteady MHD Memory flow and Heat transfer over a moving continuous porous horizontal surface. Bull cal Math. Soc., 1010 (3):281-290.

[7]. Uwanta I.J. and Omokhuale (2012.) Viscoelastic fluid flow in a fixed plane with Heat and Mass Transfer Res. J. Math. Stat., 4(3): 63-69. .