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β^* Closed sets in topological spaces

P.G.Palanimani[1], R.Parimelazhagan[2]
[1](*Research Scholar,Karpagam University,India*)
[2](*Department of Science and Humanities, Karpagam College of Engineering, Anna University, India*)

Abstract : *In this paper, the authors introduce and study the concept of a new class of closed sets called weakly generalized β^* closed sets (briefly β^* closed set). Also we investigate some of their properties*

Keywords - *g -closed set, gs-closed set, αg-closed set,wg-closed*

I. Introduction

Levine.N[1970] introduced the concept of generalized closed (briey g-closed) sets in topological spaces. S.Arya and Nour[1990], Bhattacharya.P and Lahiri.B.K[1987],Levine.N[1963], Maki et.al[1982] introduced and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets.

Ever since general topologists extend the study of generalized closed sets on the basis of generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semipre-open sets. Dontchev and Maki have introduced the concept of generalized closed sets, In 1997 Park et.al., introduced the notion of δ -semi open sets and investigated several properties of open sets. In 1986 Maki continued the work of Levine and Dunham on generalised closed sets and closure operations by introducing the notion of generalized Δ -set in topological spaces(X, τ).

Extension research of generalized closedness was done in recent years as the notion of generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets were investigated. The aim of this paper is to continue the study of generalized closed sets in general and in particular, the notion of generalized β^* closed sets and its various characterizations were studied.

II. PRELIMINARIES

Before entering into our work we recall the following definitions in our sequel.

Definition 2.1 [1]: A subset A of a topological space (X,τ) is called a semi open set if A $\subseteq$ cl (int (A)) and semi closed set if int (cl (A)) $\subseteq$ A .

Definition 2.2 [2]: A subset A of a topological space (X,τ) is called a pre-open set if A$\subseteq$ int (cl (A)) and pre-closed set if cl (int (A)) $\subseteq$ A .

Definition 2.3 [3]: A subset A of a topological space (X,τ) is called an α -open set if A$\subseteq$ int (cl (int (A))) and an α -closed set if cl (int (cl (A))) $\subseteq$ A.

Definition 2.4 [4]: A subset A of a topological space (X,τ) is called a semi-preopen set (β -open set) if A$\subseteq$ cl (int (cl (A))) and semi-preclosed set if int (cl (int (A))) $\subseteq$A.

Definition 2.5 [5]: A subset A of a topological space (X,τ) is called a generalized closed set(briefly g- closed) if cl(A) $\subseteq$U, whenever A$\subseteq$U and U is open in X.

Definition 2.6 [6]: A subset A of a topological space (X,τ) is called a semi generalized closed set (simply sg-closed) if scl (A) $\subseteq$ U ,whenever A $\subseteq$ U and U is semi open in X.

Definition 2.7 [7]: A subset A of a topological space (X,τ) is called a generalized semi closed set (simply gs-closed) if scl(A) $\subseteq$ U whenever A$\subseteq$ U and U is semi open in X.

Definition 2.8 [8]: A Subset of A topological space (X,τ) is called a α -generalized closed set (briefly α g-closed) if α cl(A) $\subseteq$ U, whenever A$\subseteq$ U and U is open in (X,τ).

Definition 2.9 [9]: A subset A of a topological space (X,τ) is called generalized α closed set (briefly gα -closed if cl(A) $\subseteq$ U, whenever A$\subseteq$ U and U is α open in (X,τ).

Definition 2.10 [10]: A subset A of a topological space (X,τ) is called a a generalized semi pre closed (briefly gsp -closed) if spcl(A) $\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).

Definition 2.11 [11: A subset A of a topological space (X,τ) is called a weakly generalized closed set (briefly wg-closed) if cl (int (A)) $\subseteq$U, whenever A$\subseteq$U and U is open in X.

Definition 2.12 [12] subset A of a topological space (X,τ) is called semi weakly closed set(briefly swg-closed) if cl(int(A)) $\subseteq$ U, whenever A$\subseteq$ U and U is semi open in X.

Definition 2.13 [13] A subset A of a topological space (X,τ) is called regular open set if A = int (cl (A))and regular closed set cl(int(A)) = A.

III. SOME BASIC PROPERTIES OF β^* - CLOSED SETS

In this section we introduce the concept of β^* closed sets in topological space.

Definition 3.1: A subset A of a topological space (X,τ) is called a weakly β^* closed set if cl(int(A)) $\subseteq$ U, whenever A$\subseteq$ U and U is g open.

Theorem 3.2: If a subset A of a topological space X is g-closed then it is β^* closed set in X .

Proof: Suppose A is g-closed, let U be an g open set containing A in X, then U$\supset$ cl(A). Now U $\supset$ cl(A) $\supset$ cl (int (A)). Thus A is β^* closed set.

Remark: The converse of the above theorem is true as seen from the following example.

Example 3.3: Let X= $\{a,b,c\}$ with the topology $\tau=\{X,\phi,\{a\},\{a,b\}\}$. In this topological space the subset $\{c\}$ is β^* closed and g closed.

Theorem 3.4: A set A is β^* closed iff cl(int(A)) - A containing no non empty closed set.

Proof: Suppose that F is a non empty closed subset of cl(int(A)).Now F$\subseteq$ cl(int(A))-A. F $\subseteq$ cl (int (A))$\cap$ A^c. Since cl(int(A))- A = cl(int (A)) $\cap$ A^c ,then F$\subseteq$cl(int(A)).F$\subseteq$ A^c that implies A$\subseteq$ F^c .Here F^c is open and A is β^* closed .We have cl (int (A)) $\subseteq$ F^c . F$\subseteq$ cl(int(A))$\cap$ $(\text{cl(int(A))})^c$ $=\phi$ implies int(cl(A)) - A contains no nonempty closed set.

Sufficiency: Let A$\subseteq$ G, G is g-open. Suppose that cl(int (A)) is not contained in G, then cl(int(A))$\cap$$G^c$ is a non empty closed set of cl(int(A)) - A which is contradiction .Therefore cl (int (A)) $\subseteq$ G and hence A is β^* closed.

Theorem 3.5: Suppose that B$\subseteq$ A $\subseteq$ X, B is β^*-closed set relative to A and that A is both g-open and β^* closed subset of X, then B is β^* -closed set relative to X.

Proof: Let B$\subseteq$ G and G be a g-open set in X. But given that B $\subseteq$ A $\subseteq$ X, therefore B$\subseteq$ A and B$\subseteq$ G. This implies B $\subseteq$ A$\cap$G. Since B is β^*-closed relative to A, cl(int(B)) $\subseteq$ A $\cap$ G.(i.e) A$\cap$cl (int (B)) $\subseteq$A$\cap$G implies A $\cap$ cl(int(B))$\subseteq$G. Thus A$\cap$cl(int(B))$\cup$ $[\text{cl(int(B))}]^c$ $\subseteq$G$\cup$$[\text{cl(int(B))}]^c$ implies A $\cup$ $[\text{cl(int(B))}]^c$ $\subseteq$ G$\cup$$[\text{cl(int(B))}]^c$. Since A is β^* closed in X, we have cl(int(A))$\subseteq$ G $\cup$$[\text{cl(int(B))}]^c$. Also B $\subseteq$ A implies cl(int(B)) $\subseteq$cl(int(A)). Thus cl(int(B)) $\subseteq$ cl(int(A)) $\subseteq$ G $\cup$ $[\text{cl(int(B))}]^c$. Therefore cl(int(B)) $\subseteq$G, since cl(int(B)) is not contained in $[\text{cl(int(B))}]^c$. Thus B is β^* closed set relative to X.

Corollary 3.6: Let set A be β^* closed and suppose F is closed then A$\cap$F is β^* closed.

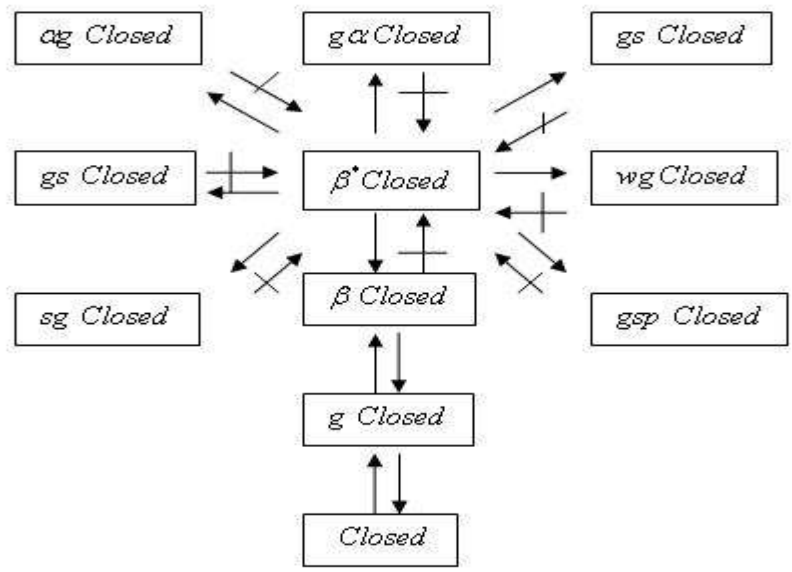
Proof: To show A $\cap$F is β^* closed, we have to show that cl(int(A) $\subseteq$ U ,whenever A$\cap$F$\subseteq$U,U is g-open , A$\cap$F is closed in A and so β^* closed in A. Therefore A$\cap$F is β^* closed in X, since A$\cap$F$\subseteq$A$\subseteq$X .

Theorem 3.7: If A is β^* closed set and A$\subseteq$ B$\subseteq$cl (int (A)) then B is β^* closed.

Proof: Given that B $\subseteq$ cl(int(A)),then cl(int(B)) $\subseteq$ cl(int(A)), then cl(int(B))-B $\subseteq$cl (int(A))- A. Therefore

$A \subseteq B$,A is β^* closed.Then by theorem cl(int(A))-A containing no non empty set, cl(int(B))-B containing no non empty set, B is β^* closed.

We have the following implications for properties of subsets:



IV. β^* CLOSED SETS

Theorem 4.1: If a subset A of a topological space X is β^* closed then it is gsp-closed but not conversely.

Proof: Suppose A is β^* closed set in X, Let U be g-open containing A then cl(int(A)) $\subseteq$ U, int(cl(int(A))) $\subseteq$ U, U is open which implies $A \cup$ int(cl(int(A))) $\subseteq A \cup U$, that is spcl(A) $\subseteq$ U,then A is gsp-closed in X.

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The converse of the above theorem need not be true as seen from the following example

Example 4.2: Let X= $\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$. In this topological space the subset $\{a\}$ is gsp- closed but not β^* closed set in X .

Remark 4.3: If A and B are β^* closed set. Then $A \cap B$ will be a β^* closed set Consider X = $\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$ In this topological space the subset A = $\{b,c\}$,B = $\{c\}$ are β^* closed sets in X. $A \cap B$ = $\{c\}$ is also a β^* closed set.

Theorem 4.4: Every closed set in a topological space X is β^* closed.

Proof: Suppose A is closed set in X,Let U be g-open set containing A in X such that A $\subseteq$ U and A $\subseteq$ cl(int(A)) $\subseteq$ U,clearly gcl(A) $\subseteq$ cl(A) $\subseteq$ U that implies A $\subseteq$ cl (int (A)) $\subseteq$ cl(A) $\subseteq$ U ,U is g-open ,thus A $\subseteq$ gcl(A) $\subseteq$ U,A is β^* closed.

Remark: The converse of the above theorem is true as seen from the following example.

Example 4.5: Let X= $\{a,b,c\}$with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$.In this topological space the subset $\{c\}$ is β^* closed set in X and also closed in (X,τ).

Theorem 4.6: Every g-closed set in a topological space is a β^* closed.

Proof: Let A is g-closed set in X, then cl(int(A)) $\subseteq$ U, whenever A $\subseteq$ U,U is open, Now A = gcl(A) as A is g-closed that implies A is β^* closed.

Remark: The converse of the above theorem is true as seen from the following example.

Example 4.7: Let X= $\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,c\}\}$. In this topological space the subset $\{b\}$is β^* closed set in X and also g-closed in (X,τ) .

Theorem 4.8: Every β^* closed set in a topological space is rwg- closed.

Proof: The set cl(int(A)) $\subseteq$U,whenever A$\subseteq$U,U is semi open, Every open set is regular open, so by definition every β^* closed set in a topological space is rwg-closed.

Remark 4.9: The converse of the above theorem need not be true from the following example.

Example 4.10: Let X=$\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$.In this topological space the subset $\{a,b\}$is rwg- closed set in X but not β^* closed set in (X,τ) .

Remark 4.11: Every_ β^* closed set is sg-closed set.

Example 4.12: Let X=$\{a,b,c\}$with the topology τ = $\{X,\phi,\{a\}\}$. In this topological space the subset$\{a\}$ is sg-closed but not β^* closed set in X .

Remark 4.13: Every β^* closed set is α g-closed set.

Example 4.14: Let X=$\{a,b,c\}$with the topology τ = $\{X,\phi,\{b\}\}$.In this topological space the subset $\{b\}$is g closed but not_ β^* closed set in X .

Remark 4.15: Every β^* closed set is gα closed set.

Example 4.16: Let X=$\{a,b,c\}$with the topologyτ = $\{X,\phi,\{a,c\}\}$. In this topological space the subset $\{a,c\}$ is gα closed but not_ β^* closed set in X .

Remark 4.17: Everyβ^* closed set is gsp-closed set.

Example 4.18: Let X=$\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$. In this topological space the subset $\{a,b\}$is gsp-closed set but not β^* closed set in X .

Remark 4.19: Every β^* closed set is wg-closed set.

Example 4.20: Let X=$\{a,b,c\}$with the topology τ = $\{X,\phi,\{b\}\}$. In this topological space the subset $\{b\}$is wg -closed set but not β^* closed set in X .

Remark 4.21: Everyβ^* closed set is swg-closed set.

Example 4.22: Let X=$\{a,b,c\}$ with the topology τ = $\{X,\phi,\{a\}.\{a,b\}\}$. In this topological space the subset $\{a,b\}$ is swg-closed set but β^* closed set in X .

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