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Fixed Point with Intimate Mappings

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Abstract: *The aim of this paper is to introduce the concept of Intimate mapping in metric space and prove a lemma and a common fixed point theorem for six mappings with Intimate mappings.*
Key Words: *Intimate mapping*

I. Introduction

In 1976, Jungk [2] gave common fixed point theorem for commuting maps, which generalizes Banach's fixed point theorem. This result was further generalized and extended in various ways by many authors. On the other hand Sessa [9] defined commutativity and proved common fixed point theorems for weakly commuting maps. Further, Jungck [2] introduced more generalized commutativity, so called compatibility, which is more general than that of weak commutativity.

Further, Kang-Cho-Jungck [4] and others extended the previous known results for commuting maps using compatible maps. Further Jungck-Murthy-P.P. and Cho [3] introduced the concept of compatible mappings of type (A) in metric space, which improved the results of various authors. Recently, Sahu, Dhagat and Srivastva [10] generalized the concept of compatible mappings of type (A) so called **Intimate mappings.**

In this chapter we shall prove a fixed point theorem for Intimate mappings in complete metric space which extend the results of Lohani and Badshah[5], Savita[11], Prasad[8], Pathak[7] and Jungck and Pathak .

Preliminaries:

Definition (1) : The pair (A, S) is said to be compatible if

$$\lim_{n\to\infty} d(ASx_n, SAx_n) = 0 \quad \text{whenever } \{x_n\} \text{ is a sequence in X s.t.}$$

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t \text{ for some } t \in X.$$

Definition (2) : The mappings A and S are said to be compatible of type (A) if

$$\lim_{n\to\infty} d(ASx_n, SSx_n) = 0 \text{ and}$$

$$\lim_{n\to\infty} d(ASx_n, AAx_n) = 0$$

Whenever $\{x_n\}$ is a sequence in X , such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some t $\in$ X.

Definition (3) : Let A and S be self maps of metric space (X, d). The pair {A,S} is said to be S-Intimate iff $\alpha d(SAx_n, Sx_n) \le \alpha d(AAx_n, Ax_n)$ where α = lim sup or lim inf, $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some t $\in$ X.

EXAMPLE: Let X = {0, 1} with d(x,y) = $|x-y|$ and A, S are self maps on X. Defined

$$Ax = \frac{2}{x+2} \quad \text{and} \quad Sx = \frac{1}{x+1} \quad \text{for all } x \in [0, 1]$$

Now the sequence $\{x_n\}$ in X defined by $x_n = \frac{1}{n}$, n $\in$ N. Then we have

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = 1 \text{, then } |ASx_n, Ax_n| \to \frac{1}{3} \text{ as } n\to\infty \text{ and } |SSx_n, Sx_n| \to \frac{1}{2} .$$

Then clearly we have

$$\lim_{n\to\infty} |ASx_n - Ax_n| < \lim_{n\to\infty} |SSx_n - Sx_n| .$$

Thus {A, S} is A-intimate.

But $\left|ASx_n, SSx_n\right| \to \frac{1}{6}$ as n → ∞.

Thus {A, S} is not compatible of type {A}.

PROPOSITION (4): If the pair (A, S) is compatible of type (A) then it is both A and S-intimate.

Proof: Since

$$d(ASx_n, Ax_n) \le d(ASx_n, SSx_n) + d(SSx_n, Sx_n) \text{ for } n \in N.$$

Therefore

$$\alpha d(ASx_n, Ax_n) \le \alpha.0 + \alpha d(SSx_n, Sx_n)$$

Implies $\alpha d(SAx_n, Ax_n) \le d(SSx_n, Sx_n)$,

Whenever $\{x_n\}$ is a sequence in metric space X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some t ∈ X.

Thus the pair {A, S} is S-intimate.

Similarly, we can show that the pair {A, S} is A-intimate. But its converse need not be true.

PROPOSITION (5) : A and S be self maps of a metric space (X, d), if the pair {A, S} is S-intimate and At = St = p ∈ X for some t ∈ X.

Then d(Sp, p) ≤ d(Ap, p).

Proof : Suppose x_n = t for all n ≥ 1

So $Ax_n = Sx_n \to At = St = p.$

Since the pair {A, S} is S-intimate then

d(SAt, St) = $\lim_{n\to\infty} d(SAx_n, Sx_n)$

$\le \lim_{n\to\infty} d(AAx_n, Ax_n)$

= d(AAt, At)

Implies d(Sp, p) ≤ d(Ap, p).

Intimate mapping condition is more improved that the well known mappings conditions such as weakly commuting (Sessa , compatible (Jungck), D-compatibility, Semi-compatibility (Cho, Sharma and Sahu]) and R-commutativity (Pant). Infact, newly defined mapping is a generalization of the compatibility of type (A) mapping condition considered by Murthy, Chag, Cho and Sharma[6].

The most important feature of intimate mapping condition is that for all the above said mapping pair, it is necessary to commute at coincidence point but for intimate mapping condition such necessity is not required i.e. the mapping pair does not necessarily commute at coincidence point.

Jungck([2]) introduced compatible mappings and obtained several fixed point theorems for them. Later on Cho[1] , Murthy, Chang, Cho and Sharma[6], Jungck, Murthy and Cho[3] proved some common fixed point theorems for compatible mappings of type(A). Intimate mapping is a generalization of compatibility of type (A).

In 2003, Savita[11] improves the results of Lohani and Badshan[5] and gave the following theorem for four intimate mappings.

Theorem A : Let A, B, S and T be mappings from a metric space (X, d) in to itself satisfying:

(1) A(X) ⊂ T(X) and B(X) ⊂ S(X)

(2) The pair {A,S}is S-intimate and {B,T}is T-intimate,

(3) S(X) is complete,

(4) d(Ax,By) ≤ $\alpha \frac{d(Ty,By)[1+d(Sx,Ax)]}{1+d(Sx,Ty)} + \beta[d(Sx,Ax)+d(Ty,By)] +$
$\gamma[d(Sx,By)+d(Ty,Ax)] + \delta[d(Sx,Ty)]$

(5) Where $\alpha, \beta, \gamma, \delta \ge 0$, $0 \le \alpha + 2\beta + 2\gamma + \delta < 1$. Then A, B, S and T have a unique common fixed point in X.

[1.2] MAIN RESULTS:
Firstly we prove a lemma:

LEMMA 1 : [Singh and Meade-(1977)]: For every t > 0, γ (t) < t, if and only if $\lim_{n\to\infty}\gamma^n(t)=0$ where γ^n denotes the n-times composition of γ.

LEMMA 2 : Let A, B, S, T, L and M be the six mappings from metric space (X, d) in to itself such that

(1.2.1) A(X) $\subset$ T(X) $\cup$ L(X) and B(X) $\subset$ S(X) $\cup$ M(X)

(1.2.2) d(Ax, By) ≤

$$\alpha_1\frac{d(Ty,By)[1+d(Sx,Ax)]}{[1+d(Mx,Ly)]}+\alpha_2[d(Mx,Ax)+d(Ly,By)]+$$
$$\alpha_3\frac{d(Sx,Ax)[1+d(Ty,By)]}{[1+d(Ly,By)]}+\alpha_4[d(Sx,Ty)+d(Ty,Ax)]+$$
$$\alpha_5[d(Mx,By)+d(Ax,Ly)]+\alpha_6\frac{d(Ty,By)[1+d(Ax,Mx)]}{[1+d(Ly,Sx)]}$$

For all x, y in X where $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5,\alpha_6\geq 0,$ and

$0\leq\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6<1.$

Then for any arbitrary point x_0 in X, by (1.2.1), there exists a point $x_1\in$ X such that A x_0 = T x_1 = L x_1 and for this point x_1, we can choosen a point $x_2\in X$ such that B x_1 = S $x_2=Mx_2$ and so on.

Inductively, we can defined a sequence { y_n } in X such that

(1.2.3) $y_{2n}=Tx_{2n+1}=Lx_{2n+1}=Ax_{2n}$ and

$y_{2n+1}=Sx_{2n+2}=Mx_{2n+2}=Bx_{2n+1}$, for n = 0, 1, 2, 3,…………………….

Then $\lim_{n\to\infty}d(y_n,y_{n+1})=0$ and { y_n }is a Cauchy sequence in X.

Proof : First, we show that $\lim_{n\to\infty}d(y_n,y_{n+1})=0$, where { y_n }is defined in (1.2.3) for Let us suppose that $d_n=d(y_n,y_{n+1})$, for n = 0 , 1, 2, 3, ……………Now, we shall prove that the sequence $\{d_n\}$ is non-increasing in R^+, i.e. $d_{n+1}\leq d_n$ for n = 1, 2, 3, …..

By using (1.2.1), we have

$$d_{2n}=d(y_{2n},y_{2n+1})$$

$$= d(Ax_{2n},Bx_{2n+1})$$

$$\leq\alpha_1\frac{d(Tx_{2n+1},Bx_{2n+1})[1+d(Sx_{2n},Ax_{2n})]}{[1+d(Mx_{2n},Lx_{2n+1})]}+\alpha_2[d(Mx_{2n},Ax_{2n})+d(Lx_{2n+1},Bx_{2n+1})]+$$

$$\alpha_3\frac{d(Sx_{2n},Ax_{2n})[1+d(Tx_{2n+1},Bx_{2n+1})]}{1+d(Lx_{2n+1},Bx_{2n+1})}+\alpha_4[d(Sx_{2n},Tx_{2n+1})+d(Tx_{2n+1},Ax_{2n})]+$$

$$\alpha_5[d(Mx_{2n},Bx_{2n+1})+d(Ax_{2n},Lx_{2n+1})]+\alpha_6\frac{d(Tx_{2n+1},Bx_{2n+1})[1+d(Ax_{2n},Mx_{2n})]}{[1+d(Lx_{2n+1},Sx_{2n})]}$$

$$\leq \alpha_1 \frac{d(y_{2n}, y_{2n+1})[1+d(y_{2n-1}, y_{2n})]}{[1+d(y_{2n-1}, y_{2n})]} + \alpha_2[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] +$$

$$\alpha_3 \frac{d(y_{2n-1}, y_{2n})[1+d(y_{2n}, y_{2n+1})]}{[1+d(y_{2n}, y_{2n+1})]} + \alpha_4[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n})] +$$

$$\alpha_5[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n})] + \alpha_6 \frac{d(y_{2n}, y_{2n+1})[1+d(y_{2n}, y_{2n-1})]}{[1+d(y_{2n}, y_{2n-1})}$$

$$\leq \alpha_1 d(y_{2n}, y_{2n+1}) + \alpha_2 d(y_{2n}, y_{2n+1})] + \alpha_2 d(y_{2n-1}, y_{2n}) + \alpha_3 d(y_{2n-1}, y_{2n}) +$$

$$\alpha_4 d(y_{2n-1}, y_{2n}) + \alpha_5 .0 + \alpha_6 d(y_{2n}, y_{2n+1})$$

$$\leq (\alpha_1 + \alpha_2 + \alpha_6) d(y_{2n}, y_{2n+1}) + (\alpha_2 + \alpha_3 + \alpha_4) d(y_{2n-1}, y_{2n})$$

$$(1 - \alpha_1 - \alpha_2 - \alpha_6) d(y_{2n}, y_{2n+1}) \leq (\alpha_2 + \alpha_3 + \alpha_4) d(y_{2n-1}, y_{2n})$$

$$d(y_{2n}, y_{2n+1}) \leq \frac{(\alpha_2 + \alpha_3 + \alpha_4)}{(1 - \alpha_1 - \alpha_2 - \alpha_6)} d(y_{2n-1}, y_{2n})$$

$$d_{2n} \leq \frac{(\alpha_2 + \alpha_3 + \alpha_4)}{(1 - \alpha_1 - \alpha_2 - \alpha_6)} d_{2n-1}$$

$d_{2n} \leq h d_{2n-1}$ Where $h = \frac{(\alpha_2 + \alpha_3 + \alpha_4)}{(1 - \alpha_1 - \alpha_2 - \alpha_6)} d_{2n-1}$, Now

$$d(y_n, y_{n+1}) \leq h d(y_{n-1}, y_n) \leq \ldots\ldots\ldots\ldots \leq h^n d(y_0, y_1).$$

$$\leq (1 + h + h^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + h^{p-1}) d(y_n, y_{n+1})$$

For every integer p>0, we get

$$d(y_n, y_{n+p}) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots\ldots\ldots\ldots\ldots\ldots + d(y_{n+p-1}, y_{n+p})$$

$$\leq (1 + h + h^2 + \ldots\ldots\ldots\ldots\ldots\ldots + h^{p-1}) d(y_n, y_{n+1})$$

In general we have

$$d(y_n, y_{n+p}) \leq \frac{h^n}{1-h} d(y_0, y_1)$$

$d_n \leq \frac{h^n}{1-h+} d_0$, which implies that if d_0>0 then by lemma (1) we have

$\lim_{n\to\infty} \frac{h^n}{1-h}(d_0) = 0$. Therefore, it follows that

$\lim_{n\to\infty} d_n = 0,$ for $d_0 = 0$

Since $\{d_n\}$ is non-decreasing, we have clearly

$\lim_{n\to\infty} d_n = \lim_{n\to\infty} d(y_n, y_{n+1}) = 0$.

Now we shall prove that $\{y_n\}$ is a Cauchy sequence in X.

Since $\lim_{n\to\infty} d_n = 0,$ it is sufficient to show that a subsequence $\{y_{2n}\}$ of $\{y_n\}$ is a Cauchy sequence in X.

Suppose that the sequence $\{y_{2n}\}$ is not Cauchy sequence in X. Then there exists a number $\in > 0$ and stricktly increasing sequence $\{m_k\}$, $\{n_k\}$ of positive integers such that

$$d(y_{2n_k}, y_{2m_k}) \geq \in \qquad \text{…… (1.2.4)}$$

For each even integer 2k, let $2m_k$ be the least even integer exceeding $2n_k$ satisfying (1.2.4), i.e.

$$d(y_{2n_k}, y_{2m_{k-2}}) > \in \text{ and } d(y_{2n_k}, y_{2m_k}) > \in \qquad \text{…… (1.2.5)}$$

Then for each even integer 2k , we have

$$\in \leq d(y_{2n_k}, y_{2m_k})$$

$$\leq d(y_{2n_k}, y_{2m_{k-2}}) + d(y_{2m_{k-2}}, y_{2m_{k-1}}) + d(y_{2m_{k-1}}, y_{2m_k})$$

Then by (1.2.4) we have

$$d(y_{2n_k}, y_{2m_k}) \to \in \quad \text{as } k \to \infty \qquad \text{……(1.2.6)}$$

By the triangular inequality, we have

$$\left| d(y_{2n_k}, y_{2m_{k-1}}) - d(y_{2n_k}, y_{2m_k}) \right| \leq d(y_{2m_{k-1}}, y_{2m_k})$$

And

$$\left| d(y_{2n_{k+1}}, y_{2m_{k-1}}) - d(y_{2n_k}, y_{2m_k}) \right| \leq d(y_{2m_{k-1}}, y_{2m_k}) + d(y_{2n_k}, y_{2n_{k+1}}),$$

then by (1.2.5) and (1.2.6) we have as k→∞

$$d((y_{2n_k}, y_{2m_{k-1}}) \to \in \quad \text{and} \quad d(y_{2n_{k+1}}, y_{2m_k}) \to \in. \qquad \text{…….. (1.2.7)}$$

Thus by (1.2.2) we have

$$d(y_{2m_k}, y_{2n_{k+1}}) = d(Ax_{2m_k}, Bx_{2n_{k+1}})$$

$$\leq \alpha_1 \frac{d(Tx_{2n_{k+1}}, Bx_{2n_{k+1}})[1 + d(Sx_{2m_k}, Ax_{2m_k})]}{[1 + d(Mx_{2m_k}, Lx_{2n_{k+1}})]} + \alpha_2[d(Mx_{2m_k}, Ax_{2m_k}) + d(Lx_{2n_{k+1}}, Bx_{2n_{k+1}})] +$$

$$+ \alpha_3 \frac{d(Sx_{2m_k}, Ax_{2m_k})[1 + d(Tx_{2n_{k+1}}, Bx_{2n_{k+1}})]}{[1 + d(Lx_{2n_{k+1}}, Bx_{2n_{k+1}})]} + \alpha_4[d(Sx_{2m_k}, Tx_{2n_{k+1}}) + d(Tx_{2n_{k+1}}, Ax_{2m_k})]$$

$$+ \alpha_5[d(Mx_{2m_k}, Bx_{2n_{k+1}}) + d(Ax_{2m_k}, Lx_{2n_{k+1}})] + \alpha_6 \frac{d(Tx_{2n_{k+1}}, Bx_{2n_{k+1}})[1 + d(Ax_{2m_k}, Mx_{2m_{k+1}})]}{[1 + d(Lx_{2n_{k+1}} Sx_{2m_k})]}$$

Or equivalently

$$\leq \alpha_1 \frac{d(y_{2n_k}, y_{2n_{k+1}})[1 + d(y_{2m_{k-1}}, y_{2m_k})]}{[1 + d(y_{2m_{k-1}}, y_{2n_k})]} + \alpha_2[d(y_{2m_{k-1}}, y_{2m_k}) + d(y_{2n_k}, y_{2n_{k+1}})] +$$

$$+ \alpha_3 \frac{d(y_{2m_{k-1}}, y_{2m_k})[1 + d(y_{2n_k}, y_{2n_{k+1}})]}{[1 + d(y_{2n_k}, y_{2n_{k+1}})]} + \alpha_4[d(y_{2m_{k-1}}, y_{2n_k}) + d(y_{2n_k}, y_{2m_k})]$$

$$+ \alpha_5[d(y_{2m_{k-1}}, y_{2n_{k+1}}) + d(y_{2m_k}, y_{2n_k})] + \alpha_6 \frac{d(y_{2n_k}, y_{2n_{k+1}})[1 + d(y_{2m_k}, y_{2m_{k-1}})]}{[1 + d(y_{2n_k}, y_{2m_{k-1}})]}$$

Using (1.2.4), (1.2.6) and (1.2.7) and taking k → ∞ we have

$$\in \leq (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6) \in$$

Since $(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6)<1$
Therefore
$\in \leq \in$ as k→∞, which is a contradiction. Therefore $\{y_{2n}\}$ is a Cauchy sequence in X and so is $\{y_n\}$.This completes the proof.

Theorem 1 : Let A, B, S, T, L and M be mappings from a metric space (X, d) in to itself satisfying (1.2.1),(1.2.2),(1.2.3) and the following:

(1.2.4) The pair {A,S} is S-intimate ,{B,T} is T-intimate and

(1.2.5) S(X) and M(X) are complete.

Then A, B, S, T, L and M have unique common fixed point in X.

Proof : By Lemma(2) we can observe that the sequence $\{y_n\}$ defined in (1.2.3) is a Cauchy sequence in X. Since S(X) and M(X) are complete and $\{Sx_{2n}\}$ and $\{Mx_{2n}\}$ is Cauchy.

Then it converges to a point p = Su = Mu for some u in X. Then $y_n \to p$ and

Hence $Ax_{2n}, Sx_{2n}, Mx_{2n}, Bx_{2n+1}, Tx_{2n+1}, Lx_{2n+1} \to p$

Now from (1.2.2) we have

$$d(Au, Bx_{2n+1}) \leq$$

$$\leq \alpha_1 \frac{d(Tx_{2n+1}, Bx_{2n+1})[1+d(Su,Au)]}{[1+d(Mu,Lx_{2n+1})]} + \alpha_2[d(Mu,Au)+d(Lx_{2n+1},Bx_{2n+1})]+$$

$$\alpha_3 \frac{d(Su,Au)[1+d(Tx_{2n+1},Bx_{2n+1})]}{[1+d(Lx_{2n+1},Bx_{2n+1})]} + \alpha_4[d(Su,Tx_{2n+1})+d(Tx_{2n+1},Au)]+$$

$$\alpha_5[d(Mu,Bx_{2n+1})+d(Au,Lx_{2n+1})]+\alpha_6 \frac{d(Tx_{2n+1},Bx_{2n+1})[1+d(Au,Mu)]}{[1+d(Lx_{2n+1}Su)]}$$

Taking limit as n$\to\infty$ we obtain

$$d(Au,p) \leq$$

$$\leq \alpha_1 \frac{d(p,p)[1+d(p,Au)]}{[1+d(p,p)]} + \alpha_2[d(p,Au)+d(p,p)] + \alpha_3 \frac{d(p,Au)[1+d(p,p)]}{[1+d(p,p)]} +$$

$$\alpha_4[d(p,p)+d(p,Au)]+\alpha_5[d(p,p)+d(Au,p)]+\alpha_6 \frac{d(p,p)[1+d(Au,p)]}{[1+d(p,p)]}$$

$$d(Au,p) \leq (\alpha_2+\alpha_3+\alpha_4+\alpha_5)d(Au,p)$$

< d(Au, p), since $(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6)<1$

Which is a contradiction.

Hence Au = p.

Since A(X) $\subset$ T(X) $\cup$ L(X), there exist v$\in X$ such that

Tv = Lv = p

Hence from (1.2.2)

$$d(p,Bv)=d(Au,Bv)$$

$$\leq \alpha_1 \frac{d(Tv,Bv)[1+d(Su,Au)]}{[1+d(Mu,Lv)]}+\alpha_2[d(Mu,Au)+d(Lv,Bv)]+\alpha_3 \frac{d(Su,Au)[1+d(Tv,Bv)]}{[1+d(Lv,Bv)]}+$$

$$\alpha_4[d(Su,Tv)+d(Tv,Au)]+\alpha_5[d(Mu,Bv)+d(Au,Lv)]+\alpha_6 \frac{d(Tv,Bv)[1+d(Au,Mu)]}{[1+d(Lv,Su)]}$$

$$\leq \alpha_1 \frac{d(p,Bv)[1+d(p,p)]}{[1+d(p,p)]}+\alpha_2[d(p,p)+d(p,Bv)]+\alpha_3 \frac{d(p,p)[1+d(p,Bv)]}{[1+d(p,Bv)]}+$$

$$\alpha_4[d(p,p)+d(p,p)]+\alpha_5[d(p,Bv)+d(p,p)]+\alpha_6 \frac{d(p,Bv)[1+d(p,p)]}{[1+d(p,p)]}$$

$$d(p,Bv)\leq(\alpha_1+\alpha_2+\alpha_5+\alpha_6)d(p,Bv)$$

$d(p,Bv)< d(p,Bv)$, since $(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6)<1$

Which is a contradiction.
Hence, p = Bv.
Since Au = Su = Mu = p and the pair {A,S}is S-intimate ,

We have $d(SAx_{2n},Sx_{2n})\leq d(AAx_{2n},Ax_{2n})$

$d(Sp,p)\leq d(Ap,p)$.

Suppose Ap $\neq$ p,
Then from (1.2.2) we have
d(Ap, p) = d(Ap, Bv)

$$\leq \alpha_1 \frac{d(Tv,Bv)[1+d(Sp,Ap)]}{[1+d(Mp,Lv)]}+\alpha_2[d(Mp,Ap)+d(Lv,Bv)]+\alpha_3 \frac{d(Sp,Ap)[1+d(Tv,Bv)]}{[1+d(Lv,Bv)]}+$$

$$\alpha_4[d(Sp,Tv)+d(Tv,Ap)]+\alpha_5[d(Mp,Bv)+d(Ap,Lp)]+\alpha_6 \frac{d(Tv,Bv)[1+d(Ap,Mp)]}{[1+d(Lv,Sp)]}$$

$$\leq \alpha_1 \frac{d(p,p)[1+d(Sp,Ap)]}{[1+d(Mp,p)]}+\alpha_2[d(Mp,Ap)+d(p,p)]+\alpha_3 \frac{d(Sp,Ap)[1+d(p,p)]}{[1+d(p,p)]}+$$

$$\alpha_4[d(Sp,p)+d(p,Ap)]+\alpha_5[d(Mp,p)+d(Ap,p)]+\alpha_6 \frac{d(p,p)[1+d(Ap,Mp)]}{[1+d(p,Sp)]}$$

$$\leq \alpha_1 \frac{d(p,p)[1+d(Ap,p)]}{[1+d(Ap,p)]}+\alpha_2[d(Ap,Ap)+d(p,p)]+\alpha_3 \frac{d(Ap,Ap)[1+d(p,p)]}{[1+d(p,p)]}+$$

$$\alpha_4[d(Ap,p)+d(p,Ap)]+\alpha_5[d(Ap,p)+d(Ap,p)]+\alpha_6 \frac{d(p,p)[1+d(Ap,Ap)]}{[1+d(p,Ap)]}$$

$$d(Ap,p)\leq 2(\alpha_4+\alpha_5)d(Ap,p),$$

$d(Ap,p)<d(Ap,p)$, which is a contradiction

So, Ap = p

Hence Sp = p and Mp = p

Similarly, Since {B, T} is T-intimate. Then we have

$$d(TBx_{2n+1}, Tx_{2n+1}) \leq d(BBx_{2n+1}, Bx_{2n+1})$$

Taking limit $n \to \infty$ d(Tp, p) ≤ d(Bp, p)

Suppose Bp ≠ p then from (1.2.2)

d(p, Bp) = d(Ap, Bp)

$$\leq \alpha_1 \frac{d(Tp,Bp)[1+d(Sp,Ap)]}{[1+d(Mp,Lp)]} + \alpha_2[d(Mp,Ap)+d(Lp,Bp)] + \alpha_3 \frac{d(Sp,Ap)[1+d(Tp,Bp)]}{[1+d(Lp,Bp)]} +$$

$$\alpha_4[d(Sp,Tp)+d(Tp,Ap)] + \alpha_5[d(Mp,Bp)+d(Ap,Lp)] + \alpha_6 \frac{d(Tp,Bp)[1+d(Ap,Mp)]}{[1+d(Lp,Sp)]}$$

$$\leq \alpha_1 \frac{d(Bp,Bp)[1+d(p,p)]}{[1+d(p,Bp)]} + \alpha_2[d(p,p)+d(Bp,Bp)] + \alpha_3 \frac{d(p,p)[1+d(Bp,Bp)]}{[1+d(Bp,Bp)]} +$$

$$\alpha_4[d(p,Bp)+d(Bp,p)] + \alpha_5[d(p,Bp)+d(p,Bp)] + \alpha_6 \frac{d(Bp,Bp)[1+d(p,p)]}{[1+d(Bp,p)]}$$

$d(p,Bp) \leq 2(\alpha_4+\alpha_5)d(p,Bp)$, since $(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6) < 1$

$(\alpha_4+\alpha_5) < \frac{1}{2}$

$d(p,Bp) < d(p,Bp)$, which is a contradiction

Therefore Bp = p. Hence Tp = p and Lp = p
Hence p is a common fixed point of A, B, S, T, L and M.

Now we prove the uniqueness of fixed point p.
Suppose q be another common fixed point of A, B, S, T, L and M

Then from (1.2.2), we have

d(p, q) = d(Ap, Bq)

$$\leq \alpha_1 \frac{d(Tq,Bq)[1+d(Sp,Ap)]}{[1+d(Mp,Lq)]} + \alpha_2[d(Mp,Ap)+d(Lq,Bq)] + \alpha_3 \frac{d(Sp,Ap)[1+d(Tq,Bq)]}{[1+d(Lq,Bq)]} +$$

$$\alpha_4[d(Sp,Tq)+d(Tq,Ap)] + \alpha_5[d(Mp,Bq)+d(Ap,Lq)] + \alpha_6 \frac{d(Tq,Bq)[1+d(Ap,Mp)]}{[1+d(Lq,Sp)]}$$

$$d(p,q) \leq 2(\alpha_4+\alpha_5)d(p,q)$$

$$d(p,q) \leq d(p,q),$$

Which implies that p = q.
This completes the proof of theorem.

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