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Semi-Compatible Maps On Intuitionistic Fuzzy Metric Space

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Abstract: *In this paper, we prove common fixed point theorem for semi-compatible mappings on intuitionistic fuzzy metric space with different some conditions of Park and Kim ([10], 2008). This research extended and generalized the results of Singh and Chauhan ([14], 2000).*

The concept of fuzzy set was developed extensively by many authors and used in various fields. Several authors have defined fuzzy metric space Kramosil and Michalek(([5],1975) etc.) with various methods to use this concept in analysis. Jungck (([3],1986), ([4],1988)) researched the more generalized concept compatibility than commutativity and weak commutativity in metric space and proved common fixed point theorems, and Singh and Chauhan ([14],2000) introduced the concept of compatibility in fuzzy metric space and studied common fixed point theorems for four compatible mappings.

Recently, Park et. al. ([7], 2006) defined the upgraded intuitionistic fuzzy metric space and Park et.al. (([8], 2008), ([9], 1999), ([11], 2007), ([12], 2005)) studied several theories in this space. Also, Park and Kim ([10], 2008) proved common fixed point theorem for self maps in intuitionistic fuzzy metric space.

I. Introduction:

In this paper, we prove common fixed point theorem for semi-compatible mappings on intuitionistic fuzzy metric space with different some conditions of Park and Kim ([10], 2008). This research extended and generalized the results of Singh and Chauhan ([14], 2000).

We give some definitions and properties of intuitionistic fuzzy metric space. Throughout this paper, Ñ will denote the set of all positive integers.

Let us recall Schweizer and Sklar (see ([13], 1960)) that a continuous t-norm is a binary operation* : $[0, 1] \times [0, 1] \to [0, 1]$ which satisfies the following conditions:

(a) * is commutative and associative;
(b) * is continuous;
(c) $a * 1 = a$ for all $a \in [0, 1]$;
(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

Similarly, a continuous t-conorm is a binary operation ◊: $[0, 1] \times [0, 1] \to [0, 1]$ which satisfies the following conditions :

(a) ◊ is commutative and associative;
(b) ◊ is continuous;
(c) $a \lozenge 0 = a$ for all $a \in [0, 1]$;
(d) $a \lozenge b \geq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

Also, let us recall (see [6] that the following conditions are satisfied :

(a) For any any $r_1, r_2 \in (0, 1)$ with $r_1 > r_2$ there exist $r_3, r_4 \in (0, 1)$ such that $r_1 * r_3 \geq r_2$ and $r_4 \lozenge r_2 \leq r_1$;
(b) For any $r_5 \in (0, 1)$, there exist $r_6, r_7 \in (0, 1)$ such that $r_6 * r_6 \geq r_5$ and $r_7 \lozenge r_7 \leq r_5$.

1.1 Definition:- (Park and Kwun ([7], 2006)). The 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norms, ◊ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions; for all $x, y, z \in X$, such that -

(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1 \Leftrightarrow x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(e) $M(x, y, .) : (0, \infty) \to (0, 1]$ is continuous,
(f) $N(x, y, t) > 0$,
(g) $N(x, y, t) = 0 \Leftrightarrow x = y$,
(h) $N(x, y, t) = N(y, x, t)$,

(i) N (x, y, t) ◊ N (y, z, s) ≥ N (x, z, t + s),
(j) N (x, y, .) : $(0, \infty) \to (0, 1]$ is continuous.

Note that (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x, y, t) and N (x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

1.2 Definition:- (Park and Kwun ([12], 2005)). Let X be an intuitionistic fuzzy metric space. Then (a) A sequence $\{x_n\} \subset X$ is convergent to x in X if and only if for each $\varepsilon > 0$, $t > 0$, there exists $n_0 \in \tilde{N}$ such that M $(x_n, x, t) > 1 - \varepsilon$, N $(x_n, x, t) < \varepsilon$ for all $n \geq n_0$.

(b) A sequence $\{x_n\} \subset X$ is called Cauchy sequence if and only if for each $\varepsilon > 0$, $t > 0$, there exists $n_0 \in \tilde{N}$ such that M $(x_n, x_m, t) > 1 - \varepsilon$, N $(x_n, x_m, t) < \varepsilon$ for all $n, m \geq n_0$.

(c) X is complete if every Cauchy sequence in X is convergent.

1.3 Definition:- (Park and Kim ([10], 2008)). Let A, B be mappings from intuitionistic fuzzy metric space X into itself.

(a) (A, B) are said to be compatible if and only if $\lim_{n \to \infty}$ M $(ABx_n, BAx_n, t) = 1$, $\lim_{n \to \infty}$ N $(ABx_n, BAx_n, t) = 0$, for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Bx_n = x$ for some $x \in X$.

(b) (A, B) are said to be semi compatible if and only if $\lim_{n \to \infty}$ M $(ABx_n, Bx, t) = 1$, $\lim_{n \to \infty}$ N $(ABx_n, Bx, t) = 0$, for all $t > 0$, whenever $\{x_n\} \subset X$ such that $\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Bx_n = x$ for some $x \in X$.

1.4 Lemma:- (Park[10],2008)). Let A, B to be self mappings on intuitionistic fuzzy metric space X. If B is continuous, then (A,B) is semi-compatible if and only if (A,B) is compatible.

II. Main Result

2.1 Theorem:- Let P, Q, S and T be self maps of complete intuitionistic fuzzy metric space X with t - norm*and t – conorms (defined by a*b = min {a, b} and a(b = max {a, b}, a, b ∈ [0, 1], satisfying

(a) (P, S) and (Q, T) are semi-compatible pairs of maps,
(b) S and T are continuous,
(c) $P^p(x) \subset T^t(x)$, $Q^q(x) \subset S^s(X)$,
(d)
$$M(P^p x, Q^q y, kt) \geq \text{Min}\{M(S^s_x, T^t_y, t), M(P^p_x, S^s_x, t), M(Q^q_y, T^t_y, t), M(P^p_x, T^t_y, \alpha t), M(Q^q_y, S^s_x, (2-\alpha)t)\},$$
$$N(P^p_x, Q^q_y, kt) \leq \max\{N(S^s_x, T^t_y, t), N(P^p_x, S^s_x, t), N(Q^q_y, T^t_y, t), N(P^p_x, T^t_y, \alpha t), N(Q^q_y, S^s_x, (2-\alpha)t)\}.$$
(e) $\lim_{t \to \infty} M(x, y, t) = 1$,
$\lim_{t \to \infty} N(x, y, t) = 0$

for all $x, y \in x$, $\alpha \in (0, 2)$, $t > 0$ and p, q, s, t ∈ Ñ.

Then P, Q, S and T have a unique common fixed point in X.

Proof. Let x_o be an arbitrary point in X. we can inductively construct a sequence $\{y_n\} \subset X$ such that $y_{2n-1} = T^t x_{2n-1} = P^p x_{2n-2}$, , $y_{2n} = S^s x_{2n} = Q^q x_{2n-1}$ for n = 1, 2, 3, ...

First, we prove that $\{y_n\}$ is a Cauchy sequence, from (d) with $\alpha = 1$, we have.

$M(y_{2n+1}, y_{2n+2}, Kt) = M(P^p_{x2n}, Q^q_{x2n+1}, Kt)$
$\geq \min \{M(S^s_{x2n}, T^t_{x2n+1}, t), M(P^p_{x2n}, S^s_{x2n}, t),$
$M(Q^q_{x2n+1}, T^t_{x2n+1}, t), M(P^p_{x2n}, T^t_{x2n+1}, t),$
$M(Q^q_{x2n+1}, S^s_{x2n}, t)\}$
$\geq \text{Min} \{M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+2}, y_{2n+1}, t),$
$M(y_{2n+1}, y_{2n+1}, t), M(y_{2n+2}, y_{2n}, t)\}$
$\geq \text{Min} \{M(y_{2n}, y_{2n+1}, t), M(y_{2n+2}, y_{2n+1}, t), 1\}$
$N(y_{2n+1}, y_{2n+2}, Kt) = (P^p_{x2n}, Q^q_{x2n+1}, Kt)$

$\leq \max \{N(S^s_{x2n}, T^t_{x2n+1}, t), N(P^p_{x2n}, S^s_{x2n}, t),$
$N(Q^q_{x2n+1}, T^t_{x2n+1}, t), N(P^p_{x2n}, T^t_{x2n+1}, t),$
$N(Q^q_{x2n+1}, S^s_{x2n}, t)\}$
$\leq \max \{N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t),$
$N(y_{2n+2}, y_{2n+1}, t), N(y_{2n+1}, y_{2n+1}, t),$
$N(y_{2n+2}, y_{2n}, t)\}$
$\leq \text{Max} \{N(y_{2n}, y_{2n+1}, t), N(y_{2n+2}, y_{2n+1}, t), 0\}$
which implies
$M(y_{2n+1}, y_{2n+2}, k\,t) \geq M(y_{2n}, y_{2n+1}, t),$
$N(y_{2n+1}, y_{2n+2}, k\,t) \leq N(y_{2n}, y_{2n+1}, t),$
Generally, $M(y_n, y_{n+1}, k\,t) \geq M(y_{n-1}, y_n, t),$
$N(y_n, y_{n+1}, k\,t) \leq N(y_{n-1}, y_n, t).$
Therefore,

$$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, \frac{t}{k}) \geq \ldots \geq M(y_0, y_1, \frac{t}{k^n})$$

Taking limit $n \to \infty$ then it tends to $\to 1$ as

$$N(y_n, y_{n+1}, t) \leq N(y_{n-1}, y_n, \frac{t}{k}) \leq \ldots \leq N(y_0, y_1, \frac{t}{k^n}) \quad \to 0 \text{ as } n \to \infty$$

Hence for $t > 0$ and $\varepsilon \in (0, 1)$, we can choose $n_o \in Ñ$ such that

$$M(y_n, y_{n+1}, t) > 1-\varepsilon, N(y_n, y_{n+1}, t) < \varepsilon$$

for all $n \geq n_o$.
Suppose that for m,

$$M(y_n, y_{n+m}, t) > 1-\varepsilon, N(y_n, y_{n+m}, t) < \varepsilon$$

for all $n \geq n_o$ and for every $m \in Ñ$.
Then

$$M(y_n, y_{n+m+1}, t) \geq \min \{M(y_n, y_{n+m}, \frac{t}{2}), M(y_{n+m}, y_{n+m+1}, \frac{t}{2})\}$$

$> 1 - \varepsilon,$

$$N(y_n, y_{n+m+1}, t) \leq \max \{N(y_n, y_{n+m}, \frac{t}{2}), N(y_{n+m}, y_{n+m+1}, \frac{t}{2})\}$$

$< \varepsilon.$
Therefore $\{y_n\} \subset X$ is a cauchy sequence.
Second, we prove that P^p, Q^q, S^s, and T^t have a unique common fixed point.
Since $\{y_n\}$ converges to some point x from completeness of X,

$$P^p x_{2n} \to x, S^s x_{2n} \to x, Q^q{}_{x2n-1} \to x \text{ and } T^t{}_{x2n-1} \to x$$

Since S is continuous, hence

$$S^s(P^p x_{2n}) \to S^S(x)$$

Thus for $t > 0$ and $\varepsilon \in (0, 1)$, there exists an $n_o \in Ñ$ such that

$$M(S^s(P^p x_{2n}), S^s(x), \frac{t}{2}) > 1 - \varepsilon,$$

$$N(S^s(P^p x_{2n}), S^s(x), \frac{t}{2}) < \varepsilon$$

for all $n \geq n_o$. Also since (P, S) and (Q, T) are semi – compatible pairs, by Lemma 1.4, (P, S) and (Q, T) are compatible pairs.
Therefore (P^p, S^s) and (Q^q, T^t) are compatible pairs for all P, q, s, t $\in$ Ñ. From (a), we have

$$\text{Lim}_{n \to \infty} M(P^p(S^s x_{2n}), S^s(P^p x_{2n}), \frac{t}{2}) = 1$$

$\lim_{n \to \infty} N(P^p (S^s x_{2n}), S^s (P^p_{x2n}) \frac{t}{2}) = 0$

Hence,

$M (S^s (P^p x_{2n}), S^s (x), t) \geq \min \{M (P^p (S^s x_{2n}), S^s (P^p x_{2n}), \frac{t}{2}), M (S^s P^p (x_{2n}), S^s x, \frac{t}{2})\}$

$> 1 - \varepsilon,$

$N (S^s (P^p x_{2n}), S^s (x), t) \leq \max \{N (P^p (S^s x_{2n}), S^s (P^p x_{2n}), \frac{t}{2}), N (S^s P^p (x_{2n}), S^s x, \frac{t}{2})\}$

$< \varepsilon$

for all $n \geq$ no.

Therefore $\lim_{n \to \infty} (P^p S^s x_{2n}), = S^s x$.

Also since $\lim_{n \to \infty} Q^q x_{2n-1} = x$ and T is continuous,

$\lim_{n \to \infty} T^t (Q^q x_{2n-1}) = T^t x$.

Thus for $t > 0$ and $\varepsilon \in (0, 1)$, there exists an $n_o \in$ Ñ such that

$M (T^t (Q^q_{x2n-1}), T^t (x), t/2) > 1-\varepsilon$, $N (T^t(Q^q_{x2n-1}), T^t (x), t/2) < \varepsilon$

for all $n \geq n_o$.

From (a), We have

$\lim_{n \to \infty} M (Q^q (T^t x_{2n-1}), T^t (Q^q x_{2n-1}), t/2) = 1$

$\lim_{n \to \infty} N (Q^q (T^t x_{2n-1}), T^t (Q^q x_{2n-1}), t/2) = 0$

Hence

$M (Q^q (T^t x_{2n-1}), T^t x, t) \geq \min \{M (Q^q (T^t x_{2n-1}), T^t (Q^q x_{2n-1}), t/2), M (T^t (Q^q x_{2n-1}), T^t x, t)\}$

$\geq 1 - \varepsilon$

$N (Q^q (T^t x_{2n-1}), T^t x, t) \leq \max \{N (Q^q (T^t x_{2n-1}), T^t (Q^q x_{2n-1}), t/2), N (T^t (Q^q x_{2n-1}), T^t x, t)\}$

$\leq \varepsilon$

for all $n \geq n_o$,

Therefore $\lim_{n \to \infty} Q^q (T^t x_{2n-1}) = T^t x$.

Using (d) with $\alpha = 1$, we have

$M(P^p (S^s x_{2n}), Q^q (T^t x_{2n-1}), K t) \geq \text{Min}\{M (S^s (S^s x_{2n}), T^t (T^t x_{2n-1}), t), M (P^p (S^s x_{2n}), S^s (S^s x_{2n}), t),$
$M (Q^q (T^t x_{2n-1}), T^t (T^t x_{2n-1}), t), M (P^p (S^s x_{2n}), T^t (T^t x_{2n-1}), t), M (Q^q (T^t x_{2n-1}), S^s (S^s x_{2n}), t)\}$

$N(P^p (S^s x_{2n}), Q^q (T^t x_{2n-1}), K t) \leq \text{Max} \{ N (S^s (S^s x_{2n}), T^t (T^t x_{2n-1}), t), N (P^p (S^s x_{2n}), S^s (S^s x_{2n}), t),$
$N (Q^q (T^t x_{2n-1}), T^t (T^t x_{2n-1}), t), N (P^p (S^s x_{2n}), T^t (T^t x_{2n-1}), t), N (Q^q (T^t x_{2n-1}), S^s (S^s x_{2n}), t)\}$

Taking limit as $n \to \infty$ and Using above results,

$M (S^s x, T^t x, K t) \geq \min \{M (S^s x, T^t x, t), M (S^s x, S^s x, t), M (T^t x, T^t x, t),$
$M (S^s x, T^t x, t), M (T^t x, S^s x, t)\}$

$\geq M (S^s x, T^t x, t)$

$N (S^s x, T^t x, K^t) \leq \text{Max} \{N (S^s x, T^t x, t), N (S^s x, S^s x, t), N (T^t x, T^t x, t),$
$N (S^s x, T^t x, t), N (T^t x, S^s x, t)\}$

$\leq N (S^s x, T^t x, t).$

which implies $S^s x = T^t x.$

Now from (d) with $\alpha = 1$,

$M (P^p x, Q^q (T^t x_{2n-1}), k t) \geq \min \{ M (S^s x, T^t (T^t x_{2n-1}), t), M (P^p x, S^s x, t),$
$M (Q^q (T^t x_{2n-1}), T^t (T^t x_{2n-1}), t), M (P^p x, T^t (T^t x_{2n-1}), t), M (Q^q (T^t x_{2n-1}), S^s x, t)\}$

$N (P^p x, Q^q (T^t x_{2n-1}), k t) \leq \max \{ N (S^s x, T^t (T^t x_{2n-1}), t), N (P^p x, S^s x, t),$
$N(Q^q (T^t x_{2n-1}), T^t (T^t x_{2n-1}), t), N (P^p x, T^t (T^t x_{2n-1}), t), N (Q^q (T^t x_{2n-1}), S^s x, t)\}$

Taking the limit as $n \to \infty$ and using above results

$M(P^p x, T^t x, K t) \geq \min\{M (T^t x, T^t x t), M(P^p x, T^t x, t), M(T^t x, T^t x, t), M(P^p x T^t x, t), M(T^t x, T^t x, t)\},$

$\geq M (P^p x, T^t x, t)$

$N(P^p x, T^t x, k t) \leq \max \{N(T^t x, T^t x, t), N(P^p x, T^t x, t), N(T^t x, T^t x, t), N(P^p x, T^t x, t), N(T^t x T^t x, t)\}$

$\leq N(P^p_x, T^t_x, t)$

Which implies $P^p_x = T^t_x$. Also since

$M(P^p_x, Q^q_x, K t) \geq M(P^p_x, Q^q_x, t), N(P^p_x, Q^q_x, kt) \leq N(P^p_x, Q^q_x, t)$

Hence $P^p_x = Q^q_x$. Therefore $P^p_x = Q^q_x = S^s_x = T^t_x$.

Furthermore using (d) with $\alpha = 1$, we have

$M(P^p_{x2n}, Q^q_x, K t) \geq \min\{M(S^s_{x2n}, T^t_x, t), M(P^p_{x2n}, S^s_{x2n}, t), M(Q^q_x, T^t_x, t),$
$M(P^p_{x2n}, T^t_x, t), M(Q^q_x, S^S_{x2n}, t)\}$

$N(P^p_{x2n}, Q^q_x, K t) \leq \max\{N(S^s_{x2n}, T^t_x, t), N(P^p_{x2n}, S^s_{x2n}, t), N(Q^q_x, T^t_x, t),$
$N(P^p_{x2n}, T^t_x, t), N(Q^q_x, S^s_{x2n}, t)\}$

Taking limit as $n \to \infty$ we have

$M(x, Q^q_x, K t) \geq \min\{M(x, Q^q_x, t), M(x, Q^q_x, t), M(Q^q_x, Q^q_x, t), M(x, Q^q_x, t), M(Q^q_x, x, t)\}$
$\geq M(x, Q^q_x, t),$

$N(x, Q^q_x, K t) \leq \max\{N(x, Q^q_x, t), N(x, Q^q_x, t), N(Q^q_x, Q^q_x, t), N(x, Q^q_x, t), N(Q^q_x, x, t)\}$
$\leq N(x, Q^q_x, t).$

Which implies $x = Q^q_x$

Therefore $x = Q^q_x = P^p_x = S^s_x = T^t_x$

That is, x is a common fixed point of P^p, Q^q, S^s and T^t. Let z be another common fixed point of maps. Then from (d) with $\alpha = 1$

$M(p^p_x, Q^q z, k t) \geq \text{Min}\{M(S^s_x T^t_z, t), M(P^p_x, S^s_x, t), M(Q^q_z, T^t_z, t),$
$M(P^p_x, T^t z, t), M(Q^q_z, S^s_x, t)\}$
$\geq \text{Min}\{M(x, z, t), M(x, x, t), M(z, z, t), M(x, z, t), M(z, x, t)\}$
$\geq M(x, z, t)$

$N(p^p_x, Q^q z, K t) \leq \text{Max}\{N(S^s_x, T^t z, t), N(P^p_x, S^s_x, t), N(Q^q_z, T^t_z, t),$
$N(P^p_x, T^t_z, t), N(Q^q_z, S^s_x, t)\}$
$\leq \text{Max}\{N(x, z, t), N(x, x, t), N(z, z, t), N(x, z, t), N(z, x, t)\}$
$\leq N(x, z, t)$

Which implies $x = z$.

Hence x is a unique common fixed point of maps.

Third, we prove that this point x is a common fixed point of P, Q, S and T.

Since $P_x = P(P^p_x) = P^p(P_x)$ and $P_x = P(S^s_x) = S^s(P_x)$

from (a), hence P_x is a common fixed point of P^p and S^s. Also since $Q_x = Q(Q^q_x) = Q^q(Q_x)$ and $Q_x - Q(T^t_x) = T^t(Q_x)$ from (a), hence Q_x is a common fixed point of Q^q and T^t. Now letting $x = P_x$ and $y = Q_x$ and $\alpha=1$ in (d), we have $M(P_x, Q_x, K t) = M(P^p(P_x), Q^q(Q_x), K t)$

$\geq \text{Min}\{M(S^s(P_x), T^t(Q_x), t), M(P^p(P_x), S^s(P_x), t),$
$M(Q^q(Q_x), T^t(Q_x), t), M(P^p(P_x), T^t(Q_x), t), M(Q^q(Q_x), S^s(P_x), t)\}$
$= \text{Min}\{M(P_x, Q_x, t), M(P_x, P_x, t), M(Q_x, Q_x, t), M(P_x, Q_x, t), M(Q_x, P_x, t)\}$
$\geq M(P_x, Q_x, t)$

$N(P_x, Q_x, k t) = N(P^p(P_x), Q^q(Q_x), K t)$
$\geq \text{Max}\{N(S^s(P_x), T^t(Q_x), t), N(P^p(P_x), S^s(P_x), t), N(Q^q(Q_x), T^t(Q_x), t),$
$N(P^p(P_x), T^t(Q_x), t), N(Q^q(Q_x), S^s(P_x), t)\}$
$= \text{Max}\{N(P_x, Q_x, t), N(P_x, P_x, t), N(Q_x, Q_x, t), N(P_x, Q_x, t), N(Q_x, P_x, t)\}$
$\geq N(P_x, Q_x, t)$

Therefore $P_x = Q_x$.

Also from (d) with $\alpha = 1$, we have

$M(S_x, T_x, K t) = M(S^S(S_x), T^t(T_x), K t)$
$\geq \text{Min}\{M(S^s(S_x), T^t(T_x), t), M(P^p(S_x), S^s(S_x), t), M(Q^q(T_x), T^t(T_x), t),$
$M(P^p(S_x), T^t(T_x), t), M(Q^q(T_x), S^s(S_x), t)\}$
$= \text{Min}\{M(S_x, T_x, t), M(S_x, S_x, t), M(T_x, T_x, t), M(S_x, T_x, t), M(T_x, S_x, t)\}$
$\geq M(S_x, T_x, t)$

$N(S_x, T_x, K t) = N(S^S(S_x), T^t(T_x), K t)$
$\leq \text{Max}\{N(S^s(S_x), T^t(T_x), t), N(P^p(S_x), S^s(S_x), t), N(Q^q(T_x), T^t(T_x), t),$
$N(P^p(S_x), T^t(T_x), t), N(Q^q(Tss_x), S^s(S_x), t)\}$
$= \text{Max}\{N(S_x, T_x, t), N(S_x, S_x, t), N(T_x, T_x, t), N(S_x, T_x, t), N(T_x, S_x, t)\}$

$\geq N(S_x, T_x, t)$

Therefore, $Sx = Tx$. Since x is a unique common fixed point of P^p, Q^q, S^s, T^t. Hence $Px = Qx$ is a common fixed points of P^p, S^s and $Sx = Tx$ is a common fixed points of Q^q, T^t. Hence $x = Px = Qx = Sx = Tx$. That is, x is common fixed point of P, Q, S and T.

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