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# Cascade Reliability of Stress-Strength system when Strength follows mixed Exponential distribution

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***Abstract:*** *Cascade reliability model is a special type of Stress- Strength model. The n- Cascade system is a hierarchical standby redundancy system, where the standby component taking the place of failed component with decreased value of stress and independently distributed strength.*

*In assessing system reliability it is first necessary to define and categorize different modes of system failures. The individual distributions that are combined to form the mixture distribution are called mixer components. In this paper it has been discussed that the reliability of n- cascade system when strength follows mixed exponential distribution and stress follows exponential distribution.*

## I. Introduction

An $n$ - cascade system is defined as a special type of standby system with $n$ components by Sriwastav et al.,[2]. Cascade redundancy is defined as a hierarchical standby redundancy where a standby component takes the place of a failed component with a changed stress. This changed stress is $k$ times the preceding stress. $k$ is the attenuation factor.

Sriwastav and Pandit[2] derived the expressions for reliability of an n-cascade system when stress and strength follow exponential distribution. They computed reliability values for a 2-cascade system with gamma and normal stress and strength distributions. Raghava Char et al [3] studied the reliability of a cascade system with normal stress and strength distribution. T.S.Uma Maheswari et al [4] studied the reliability comparison of n-cascade system with addition of n-strengths system when stress and strength follow exponential distribution. They concluded that when the attenuation factor is less than 0.5 the cascade model has more reliability and at each attack if the stress decreases more than cascade model is more reliable. Uma Maheswari et al [5] derived the reliability of n-cascade system with normal stress and exponential strength. They concluded that the marginal reliability rate increases for higher values of strength parameter. Rekha and Shyam Sunder [6] studied the reliability of a cascade system with exponential strength and gamma stress. They showed that for higher parametric values and lower attenuation factors a high degree of reliability could be attained. Rekha and Chenchu Raju [7] studied the stress attenuated cascade reliability when both stress and strength follow rayleigh distribution. They concluded that for lower attenuation factors a high degree of reliability could be obtained.

## II. Statistical Model

If $X$ denotes the strength of the component and $Y$ is the stress imposed on it, then the reliability of the component is given by[1] ,

$$R = P(X > Y) = \int_0^\infty \left\{ \int_0^x g(y)dy \right\} f(x)dx$$

Let $X_1, X_2, X_3, \dots, X_n$ be the strengths of the components $C_1, C_2, C_3, \dots, C_n$ as arranged in order of activation respectively. All the $X_i's$ are independent and identically distributed random variables with probability density functions$f_i(x_i); i = 1,2, \dots, n$. Also let $Y_1$ be the stress on the first component which is also randomly distributed with the density function $g(y_1)$.

If $Y_1 < X_1$, the first component which is also randomly distributed stress varies with the density function $g(y_1)$ . If $Y_1 < X_1$ , the first component $C_1$ works and hence the system survives. $Y_1 \geq X_1$ leads to the failure of $C_1$ ; thus the second component in line viz., $C_2$ ,takes its place and has a strength $X_2$. However, the stress $Y_2$ on $C_2$ will normally be different from $Y_1$ . Let $Y_2 = K_2^* Y_1, where\ K_2^*$ is the cumulative attenuation factor on the second component and $K_2^* = K_1 K_2$ where by definition $K_1 = 1$.Although the system has suffered the loss of one component , it survives if $Y_2 < X_2$ and so on . In general, if the $(i-1)^{th}$ component $C_{i-1}$ fails then the $i^{th}$ component $C_i$ , with the strength $X_i$, gets activated and will be subjected to the stress.

$$Y_i = K_i Y_{i-1} = K_i^* Y_1 \qquad (1)$$

$$where \qquad K_i^* = K_1 K_2 \dots \dots K_i \qquad (2)$$

represents the cumulative attenuation factor on the $i^{th}$ component $C_i$.

The system could survive with a loss of the first $(n-1)$ components if and only if $X_i \leq Y_i\,; i = 1,2,3,\dots\dots, n-1$ $and$ $X_n > Y_n$ . The system totally fails if all the components fail when $X_i \leq Y_i; i = 1,2,\dots\dots n.$

The probability $R(n)$ of the system to survive with the first $(n-1)$ components failed and the $n^{th}$ component active is

$$R(n) = P\left[\left\{\bigcap_{i=1}^{n-1}(X_i \leq Y_i)\right\} \cap (X_n > Y_n)\right] \qquad (3)$$

$R(2), R(3), \dots\dots\dots, R(n)$ are the increments in reliability due to the addition of $2^{nd}, 3^{rd}, \dots\dots, n^{th}$ components respectively.

Then

$R(n) = P[X_1 \leq K_1^* Y_1, X_2 \leq K_2^* Y_1, \dots\dots\dots, Y_1, X_{n-1} \leq K_{n-1}^* Y_1, X_n > K_n^* Y_1]$ (4)

we can obviously associate the $n^{th}$ component attenuation factor with $Y_1$ .

The distribution of $Y_1$ is the specified if the distribution of $Y_2, Y_3 \dots\dots$ are fixed. Hence it is necessary to specify the distribution of $Y_1$ . Let $g(y_1)$ and $f_i(x_i)$ be the probability density function of $Y_1$ and $X_i$ $(i = 1,2,\dots,n$ respectively.

The equation (4) can now be written as

$$R(n) = \int_0^\infty \left[\int_0^{K_1^* y_1} f_1(x_1)dx_1 \times \int_0^{K_2^* y_1} f_2(x_2)dx_2 \times \dots\dots \int_0^{K_{n-1}^* y_1} f_{n-1}(x_{n-1})dx_{n-1} \times \int_{K_n^* y_1}^\infty f_n(x_n)dx_n\right] g(y_1)dy_1 \qquad (5)$$

(or)

$$\int_0^\infty [F_1(K_1^* y_1)F_2(K_2^* y_1)\dots\dots\dots F_{n-1}(K_{n-1}^* y_1)\bar{F}_n(K_n^* y_1)]g(y_1)dy_1 \qquad (6)$$

$$where \quad F_i(K_i^* y_1) = \int_0^{K_i^* y_1} f_i(x_i)dx_i \qquad and$$

$$\bar{F}_i(K_i^* y_1) = 1 - F_i(K_i^* y_1) \qquad (7)$$

Let $X_1$ and $X_2$ be the two strengths additively acted on a system and X be the strength composed on it with pdf's

$$g(y_i) = \mu e^{-\mu y_i} \qquad i = 1,2 \dots.n$$

$$f(x) = \lambda e^{-\lambda x} \qquad \lambda \geq 0$$

Let $X_1$ and $X_2$ are additive in the ratio $p_1, p_2$ acted on a single stress system then the combined strength X= $p_1 X_1 + p_2 X_2$ and $p_1 + p_2 = 1$

Its density function is

$$f(x) = \int_0^y f_1(p_1 x_1) f_2(p_2 x_2) dx_1 dx_2$$

Transformation

$$U = p_1 X_1 + p_2 X_2$$

$$V = X_2 \qquad X_1 = \frac{U - p_2 V}{p_1}$$

$$J = \begin{vmatrix}\frac{\partial X_1}{\partial U} & \frac{\partial X_2}{\partial V}\end{vmatrix} = \begin{vmatrix}\frac{\partial X_1}{\partial u} & \frac{\partial X_2}{\partial u} \\ \frac{\partial X_1}{\partial v} & \frac{\partial X_2}{\partial v}\end{vmatrix} = \begin{vmatrix}\frac{1}{p_1} & 0 \\ \frac{-p_2}{p_1} & 1\end{vmatrix} = \frac{1}{p_1}$$

$$f(u,v) = \lambda^2 e^{-\lambda x_1 - \lambda x_2}$$

$$= \lambda^2 e^{-\lambda\left(\frac{u - p_2 v}{p_1}\right) - \lambda v} \lambda^2 e^{-\frac{\lambda u}{p_1}} e^{\left(\frac{\lambda p_2}{p_1} - \lambda\right)v}$$

$$f(u) = \lambda^2 e^{-\frac{\lambda u}{p_1}} \int_0^u e^{\left(\frac{\lambda p_2}{p_1} - \lambda\right) v} \, dv = \frac{\lambda^2 e^{-\frac{\lambda u}{p_1}}}{\lambda p_2 - \lambda p_1} \left[ e^{-\left(\frac{\lambda p_2 - \lambda p_1}{p_1}\right) u} - 1 \right]$$

$$= \frac{\lambda^2}{\lambda p_2 - \lambda p_1} \left[ e^{-\lambda\left(\frac{1-p_2}{p_1}\right) u} - e^{-\frac{\lambda u}{p_1}} \right]$$

$$f(u) = \frac{\lambda^2}{\lambda p_2 - \lambda p_1} \left[ e^{-2\lambda u} - e^{-\frac{\lambda u}{p_1}} \right]$$

$$f(x) = \frac{\lambda}{p_2 - p_1} \left[ e^{-2\lambda x} - e^{-\frac{\lambda x}{p_1}} \right]$$

Then n- cascade Reliability

$$R(n) = P\left[ \bigcap_{i=1}^{n-1} (X_i < Y_i) \cap (X_n > Y_n) \right] \qquad (8)$$

Then 1-Cascade is

$$R(1) = P(X_1 > Y_1) \qquad (9)$$

$$R(1) = \int_{x=0}^{\infty} \left( \int_{y=0}^{x} g(y) dy \right) f(x) dx = \int_{x=0}^{\infty} (G(x)) f(x) dx$$

Where $G(x) = \int_{y=0}^{x} g(y) dy = \int_0^x \mu e^{-\mu y} \, dy = (1 - e^{-\mu x})$ (10)

$$R(1) = \int_{x=0}^{\infty} (1 - e^{-\mu x}) \left( \frac{\lambda}{p_2 - p_1} \right) \left( e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_0^{\infty} \left( e^{-(\mu + 2\lambda) x} - e^{-\left(\frac{\lambda}{p_1} + \mu\right) x} + e^{-2\lambda x} - e^{\frac{\lambda}{p_1} x} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \left[ \frac{1}{2\lambda} - \frac{p_1}{\lambda} - \frac{1}{\mu + 2\lambda} + \frac{p_1}{p_1 \mu + \lambda} \right] \qquad (11)$$

2-Cascade system

$$R(2) = P(X_1 < Y_1, X_2 > Y_2) = P(X_1 < Y, \, X_2 > K_1^* Y)$$

$$= \int_{x=0}^{\infty} (1 - G(x))(G(K^*{}_1 X)) f(x) dx$$

$$= \int_{x=0}^{\infty} e^{-\mu x} (1 - e^{-\mu x K_1^*}) \frac{\lambda}{p_2 - p_1} \left( e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_{x=0}^{\infty} \left( e^{-\mu x} - e^{-\mu x (K_1^* + 1)} \right) \left( e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_0^{\infty} \left( e^{-(\mu + 2\lambda) x} - e^{-\left(\mu + \frac{\lambda}{p_1}\right) x} - e^{-(\mu (1 + K_1^*) + 2\lambda) x} + e^{-(\mu (1 + K_1^*) + \frac{\lambda}{p_1}) x} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \left[ \frac{1}{\mu + 2\lambda} - \frac{p_1}{\mu p_1 + \lambda} - \frac{1}{\mu (1 + K_1^*) + 2\lambda} + \frac{p_1}{p_1 \mu (1 + K_1^*) + \lambda} \right] \qquad (12)$$

3-Cascade system

$$R(3) = P(X_1 < Y_1, \, X_2 < Y_2, X_3 > Y_3)$$

$$= \int_{x=0}^{\infty} \left(1 - G(X)\right) \left(1 - G(K_1^* X)\right) (G(K_2^* X)) f(x) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_{x=0}^{\infty} e^{-\mu x} e^{-K_1^* x \mu} \left(1 - e^{-K_2^* x \mu}\right) \left( e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}} \right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_{y=0}^{\infty} \left(e^{-(\mu(1+K_1^*)x}\right)\left(1 - e^{-K_2^* x\mu}\right)\left(e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}}\right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_{y=0}^{\infty} \left(e^{-(\mu(1+K_1^*)x} - e^{-(K_1^*+K_2^*+1)x\mu}\right)\left(e^{-2\lambda x} - e^{\frac{-\lambda x}{p_1}}\right) dx$$

$$= \frac{\lambda}{p_2 - p_1} \int_{y=0}^{\infty} e^{-(\mu(1+K_1^*)+2\lambda)x} - e^{-(\mu(1+K_1^*)+\frac{\lambda}{p_1})x} - e^{-(K_1^*+K_2^*+1+2\lambda)x\mu} + e^{-(K_1^*+K_2^*+1+\frac{\lambda}{p_1})x\mu} dx$$

$$R(3) = \frac{\lambda}{p_2 - p_1}\left[\frac{1}{\mu(1+K_1^*)+2\lambda} - \frac{p_1}{p_1\mu(1+K_1^*)+\lambda} - \frac{1}{\mu(1+K_1^*+K_2^*)+2\lambda} + \frac{p_1}{p_1\mu(K_1^*+K_2^*+1)+\lambda}\right] \quad (13)$$

4- Cascade system

$$R(4) = \frac{\lambda}{p_2 - p_1}\left[\frac{1}{(1+K_1^*+K_2^*)\mu+2\lambda} - \frac{p_1}{p_1\mu(1+K_1^*+K_2^*)+\lambda} - \frac{1}{\mu(1+K_1^*+K_2^*+K_3^*)+2\lambda} + \frac{p_1}{p_1\mu(1+K_1^*+K_2^*+K_3^*)+\lambda}\right] \quad (14)$$

n- Cascade system

$$R(n) = \frac{\lambda}{p_2 - p_1}\left[\frac{1}{(1+K_1^*+K_2^*+\cdots..K_{n-2}^*)\mu+2\lambda} - \frac{p_1}{p_1\mu(1+K_1^*+K_2^*+\cdots K_{n-2}^*)+\lambda} - \frac{1}{\mu(1+K_1^*+K_2^*+K_3^*+\cdots+K_{n-1}^*)+2\lambda} + \frac{p_1}{p_1\mu(1+K_1^*+K_2^*+K_3^*+\cdots..K_{n-1}^*)+\lambda}\right] \quad (15)$$

$$R(n) = \frac{\lambda}{p_2 - p_1}\left[\frac{1}{\mu(\sum_{i=0}^{n-2} K_i^*)+2\lambda} - \frac{p_1}{p_1\mu(\sum_{i=0}^{n-2} K_i^*)+\lambda} - \frac{1}{\mu(\sum_{i=0}^{n-1} K_i^*)+2\lambda} + \frac{p_1}{p_1\mu(\sum_{i=0}^{n-1} K_i^*)+\lambda}\right]$$

## III. Reliability Computations

**Table 1**

| λ | p1 | p2 | μ | k1* | k2* | R(1) | R(2) | R(3) | R(4) |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.9 | 5 | 2 | 3 | 0.455128 | 0.02794 | 0.0129 | 0.0223 |
| 0.2 | 0.2 | 0.8 | 5 | 2 | 3 | 0.382716 | 0.05361 | 0.0676 | 0.0752 |
| 0.3 | 0.3 | 0.7 | 5 | 2 | 3 | 0.241071 | 0.07727 | 0.2266 | 0.3522 |
| 0.4 | 0.4 | 0.6 | 5 | 2 | 3 | 0.17816 | 0.09912 | 0.4033 | 0.5153 |

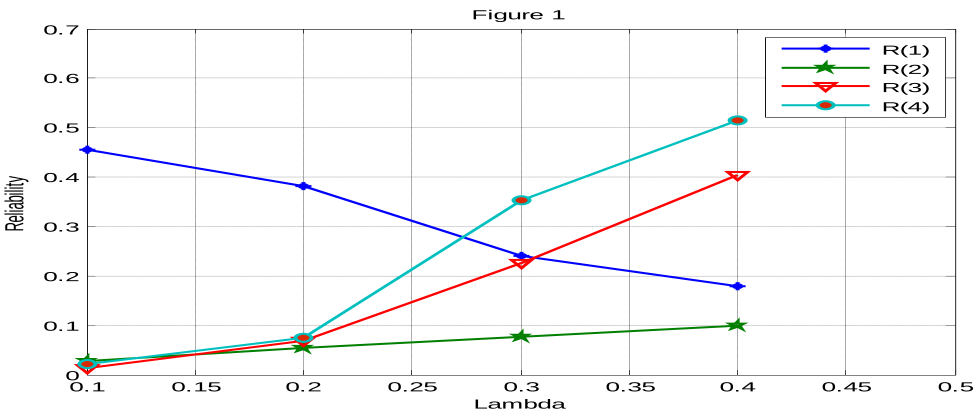
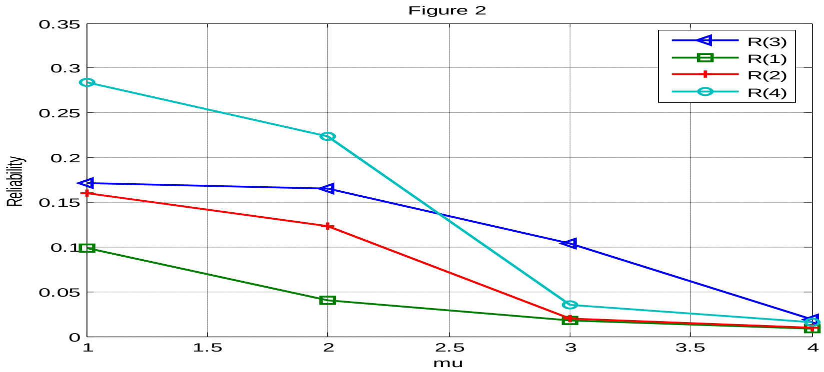



**Table 2**

| λ | p1 | p2 | μ | k1* | k2* | R(1) | R(2) | R(3) | R(4) |
|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.1 | 0.9 | 1 | 2 | 3 | 0.1718 | 0.0989 | 0.1598 | 0.2835 |
| 0.3 | 0.2 | 0.8 | 2 | 2 | 3 | 0.1648 | 0.0403 | 0.1231 | 0.2231 |
| 0.3 | 0.3 | 0.7 | 3 | 2 | 3 | 0.1041 | 0.0177 | 0.0199 | 0.0352 |
| 0.3 | 0.4 | 0.6 | 4 | 2 | 3 | 0.0188 | 0.0087 | 0.0095 | 0.0162 |




## IV. Conclusion

The reliability of n- cascade system when stress follows mixed exponential distribution and strength follows exponential distribution. In this paper we find out formula for the Reliability of n- cascade system.

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