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Inventory Model: Deteriorating Items with Time-Dependent Deterioration Rate for Quadratic Demand Rate With Unit Production Cost and Shortage

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Abstract : *In this paper, an order level inventory unit production system for deteriorating items with a quadratic demand is developed. It is assumed that shortages in inventory are permitted and are completely backlogged. The finite production rate is proportional to the time dependent demand rate and deterioration rate is time proportional. The unit production cost is taken to be inversely related to the demand rate. Sensitivity of the decision variable to changes in the parameter values is examined and numerical example is presented to illustrate the model is developed.*

Keywords - *Deterioration; Inventory model; Quadratic demand rate; Shortage;*

I. INTRODUCTION

In general, deterioration is defined as the damage, spoilage, dryness, vaporization, etc., that result in decrease of usefulness of the original one. Goods deteriorate and their value reduces with time. Electronic products may become absolute as technology changes. Fashion tends to depreciate the value of clothing over time .Batteries dies out as they age. The effect of time is even more critical for perishable goods such as foods stuffs and cigarettes

Whitin (1957) considered fashion goods deteriorating at the end of a prescribed shortage period. Ghare and Schrader (1963) develop a model for an exponentially decaying inventory. Various types of order-level inventory models for items deteriorating at a constant were discussed by Shah and Jaiswal (1977), Dave and Patel (1981), Roychowdhury and Chaudhuri (1983), Dave (1986) and Bahari-kashani (1989). Inventory models with a time-dependent rate of deterioration were studied by Covert and Philip (1973), Philip (1974), Mishra (1975) and Deb and Chaudhuri (1986).

Numerous inventory models for deteriorating items with various features of the inventory systems have been discussed; it is not possible to list them all. Interested readers may consult the review articles by Nahmias (1982) and Raafat (1991). In the classical inventory models, the demand rate is assumed to be a constant. In reality, demand for physical goods may be time dependent, stock dependent and price dependent. The first analytic model for linearly time-dependent demand was developed by Donaldson (1977). Most of these papers take the replenishment rate to be infinite.

In this present paper, we discuss an economic order quantity (EOQ) model taking into account the following factors

1. The production rate is finite and proportional to the time dependent quadratic demand rate
2. The unit cost of production depends on the demand rate.
3. The deterioration rate is time proportional.

II. PROPOSED ASSUMTIONS & NOTATIONS

1. Assumptions

A deterministic order-level model with a finite rate of replenishment is developed with the following assumptions and notations.

1.1 c_1 is the constant holding cost per item per unit of time.

1.2 The shortage cost c_2 is infinite.

1.3 c_3 is the constant deterioration cost per unit per unit of time.

1.4 $R = f(t) = a + bt + ct^2$ is the quadratic demand rate at any time $t \geq 0,\ a \geq 0,\ b > 0,\ c > 0.$

1.5 $K = \beta\, f(t)$ is the production rate where $\beta(>1)$ is a constant.

1.6 The lead time is zero.

1.7 A variable fraction $\theta(t) = \alpha t,\ (0 < \alpha << 1,\ t \geq 0)$ of the on-hand inventory deteriorates per unit of time.

1.8 C is the total average cost for a production cycle.

1.9 The unit production cost v is inversely related to the demand rate as $v=\alpha_1 R^{-\gamma}$, where $\alpha_1>0, \gamma>0$ *and* $\gamma\neq 2$. α_1 is obviously positive since v and R are both non-negative; also higher demands result in lower unit costs of production. This implies that v and R are inversely related and, hence, γ must be positive.

Now, $$\frac{dv}{dR}=-\alpha_1\gamma R^{-(\gamma+1)}<0,$$

$$\frac{d^2v}{dR^2}=\alpha_1\gamma(\gamma+1)R^{-(\gamma+2)}>0.$$

Thus marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. For this reason, the manufacturer is encouraged to produce more as the demand for the item increases. The necessity of the restriction $\gamma\neq 2$ arises from the nature of the solution of problem. The amount of stock is zero at time $t=0$. Production starts at time $t=0$ and stops at time t_1 when the stock attains a level S. During $[t_1, t_2]$, the inventory level gradually decreases mainly to meet demands and partly because of deterioration. By this process, the stock reaches zero level at time t_2.The cycle then repeats itself after the scheduling period t_2. The intensity of deterioration is very low during the early stage of inventory because t is small. However, the intensity increases with time, but $\theta(t)$ remains bounded for $t>>1$ since $0<\alpha<<1$.

In this case, we intend to develop an order-level model for deteriorating items with a finite rate of replenishment with the assumptions described and also with the additional assumption that shortages in inventory are permitted and are completely backlogged. c_2 is the constant shortage per unit per unit of time. Here the amount of stock is zero at time $t=0$. Production starts at time $t=0$ and continues up to $t=t_1$ when the stock reaches a level S. Inventory accumulated in $[0, t_1]$ after meeting the quadratic demands is used in $[t_1, t_2]$. The stock reaches the zero level at time t_2. Now shortages start to develop and accumulate to the level P at $t=t_3$. Production starts at time t_3. The running demands as well as the backlog for $[t_2, t_3]$ are satisfied in $[t_3, t_4]$. The inventory again falls to the zero level at time t_4. The cycle then repeats itself after a time t_4. Our problem is to determine the optimum values of t_1, t_2, t_3, t_4 *and* C with the assumptions stated above.

III. INDENTATIONS AND EQUATIONS

MATHEMATICAL FORMULATION:

Let $Q(t)$ represent the instantaneous inventory level at any time t $(0\le t\le t_4)$.The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, t_4]$ are given by

$$\frac{dQ(t)}{dt}+\theta(t)Q(t)=K-f(t), \quad 0\le t\le t_1, \tag{1}$$

with the conditions $Q(0)=0$ and $Q(t_1)=S$;

$$\frac{dQ(t)}{dt}+\theta(t)Q(t)=-f(t), \quad t_1\le t\le t_2. \tag{2}$$

with the conditions $Q(t_1)=S$ and $Q(t_2)=0$.

$$\frac{dQ(t)}{dt}=-f(t), \quad t_2\le t\le t_3, \tag{3}$$

with the conditions $Q(t_2)=0$ and $Q(t_3)=-P$;

$$\frac{dQ(t)}{dt}=K-f(t),\quad t_3\le t\le t_4. \tag{4}$$

with the conditions $Q(t_3)=-P$ and $Q(t_4)=0$.

Using $\theta(t)=\alpha t$ and $f(t)=a+bt+ct^2$, equation (1)-(4) become respectively

$$\frac{dQ(t)}{dt}+\alpha tQ(t)=(\beta-1)\left(a+bt+ct^2\right),\quad 0\le t\le t_1, \tag{5}$$

with the conditions $Q(0)=0$ and $Q(t_1)=S$;

$$\frac{dQ(t)}{dt}+\alpha tQ(t)=-\left(a+bt+ct^2\right),\quad t_1\le t\le t_2, \tag{6}$$

with the conditions $Q(t_1)=S$ and $Q(t_2)=0$;

$$\frac{dQ(t)}{dt}=-\left(a+bt+ct^2\right),\quad t_2\le t\le t_3, \tag{7}$$

with the conditions $Q(t_2)=0$ and $Q(t_3)=-P$;

$$\frac{dQ(t)}{dt}=(\beta-1)\left(a+bt+ct^2\right),\quad t_3\le t\le t_4. \tag{8}$$

with the conditions $Q(t_3)=-P$ and $Q(t_4)=0$.

The solutions of (5)-(8) are respectively.

$$Q(t)=(\beta-1)\left(at-\frac{1}{3}a\alpha t^3+\frac{1}{2}bt^2-\frac{1}{8}b\alpha t^4+\frac{1}{3}ct^3-\frac{1}{15}c\alpha t^5\right),\qquad 0\le t\le t_1, \tag{9}$$

$$Q(t)=\begin{cases} S\left(1+\frac{1}{2}\alpha t_1^2-\frac{1}{2}\alpha t^2\right)+a\left(t_1-t+\frac{1}{6}\alpha t_1^3-\frac{1}{2}\alpha t^2t_1+\frac{1}{3}\alpha t^3\right) \\ \quad +b\left(\frac{1}{2}t_1^2-\frac{1}{2}t^2+\frac{1}{8}\alpha t_1^4-\frac{1}{4}\alpha t^2t_1^2+\frac{1}{8}\alpha t^4\right) \\ \quad +c\left(\frac{1}{3}t_1^3-\frac{1}{3}t^3+\frac{1}{10}\alpha t_1^5-\frac{1}{6}\alpha t^2t_1^3+\frac{1}{15}\alpha t^5\right),\qquad t_1\le t\le t_2. \end{cases} \tag{10}$$

$$Q(t)=a(t_2-t)+\frac{1}{2}b\left(t_2^2-t^2\right)+\frac{1}{3}c\left(t_2^3-t^3\right),\quad t_2\le t\le t_3, \tag{11}$$

$$Q(t)=(\beta-1)\left[a(t-t_4)+\frac{1}{2}b\left(t^2-t_4^2\right)+\frac{1}{3}c\left(t^3-t_4^3\right)\right],\quad t_3\le t\le t_4. \tag{12}$$

Since $Q(t_2)=0$, then from (10),

$$S=a\left(t_2-t_1+\frac{1}{3}\alpha t_1^3+\frac{1}{6}\alpha t_2^3-\frac{1}{2}\alpha t_1^2t_2\right)$$

$$+b\left(\frac{1}{2}t_2^2-\frac{1}{2}t_1^2+\frac{1}{8}\alpha t_1^4+\frac{1}{8}\alpha t_2^4-\frac{1}{4}\alpha t_1^2t_2^2\right)$$
$$+c\left(\frac{1}{3}t_2^3-\frac{1}{3}t_1^3+\frac{1}{15}\alpha t_1^5+\frac{1}{10}\alpha t_2^5-\frac{1}{6}\alpha t_1^2t_2^3\right).$$

for a first-order approximation of α.

The total inventory in the cycle $[0,\ t_2]$ is

$$\int_0^{t_1} Q(t)\,dt + \int_{t_1}^{t_2} Q(t)\,dt = \frac{1}{12}a\left[6\beta t_1^2 - \alpha\beta t_1^4 + 6t_2^2 - 12t_1t_2 + 2\alpha t_1^3 t_2 + \alpha t_2^4 - 2\alpha t_1 t_2^3\right]$$

$$+\frac{1}{120}b\left[20\beta t_1^3 - 3\alpha\beta t_1^5 + 40t_2^3 - 60t_1t_2^2 + 10\alpha t_1^3 t_2^2 + 8\alpha t_2^5 - 15\alpha t_1 t_2^4\right]$$

$$+\frac{1}{180}c\left[15\beta t_1^4 - 2\alpha\beta t_1^6 + 45t_2^4 - 60t_1t_2^3 + 10\alpha t_1^3 t_2^3 + 10\alpha t_2^6 - 18\alpha t_1 t_2^5\right]$$

for a first-order approximation of α .

The total number of deteriorated items in $[0,\ t_2]$ is given by

Production in $[0,\ t_1]$-demand in

$$[0,\ t_2] = \beta\int_0^{t_1}\left(a+bt+ct^2\right)dt - \int_0^{t_2}\left(a+bt+ct^2\right)dt$$

$$= a\left(\beta t_1 - t_2\right) + \frac{1}{2}b\left(\beta t_1^2 - t_2^2\right) + \frac{1}{3}c\left(\beta t_1^3 - t_2^3\right) \tag{13}$$

Since the production in $[u, u+du]$ is $K\,du$, the cost of production in $[u, u+du]$ is

$$Kv\ du = \frac{\alpha_1\beta f(t)}{R^{\gamma}}\,du = \frac{\alpha_1\beta}{\left(a+bu+cu^2\right)^{\gamma-1}}\,du$$

Hence the production cost in $[0, t_1]$ is

$$\int_0^{t_1}\frac{\alpha_1\beta}{\left(a+bu+cu^2\right)^{\gamma-1}}\,du = \frac{\alpha_1\beta}{6a^{\gamma}}\left[6at_1 + (1-\gamma)\left(3bt_1^2 + 2ct_1^3\right)\right],\quad \gamma\neq 2. \tag{14}$$

The total number of deteriorated items in $[0,\ t_4]$ is the same as given in (13) since there will be no deteriorated items during the period $[t_2,\ t_4]$.

The total shortage in $[t_2,\ t_4]$ is given by

$$\int_{t_2}^{t_4}\left[-Q(t)\right]dt = \int_{t_2}^{t_3}\left[-Q(t)\right]dt + \int_{t_3}^{t_4}\left[-Q(t)\right]dt$$

$$= \frac{1}{2}a\left[t_2^2 + \beta t_3^2 + (\beta-1)t_4^2 - 2t_2t_3 - 2(\beta-1)t_3t_4\right]$$

$$+\frac{1}{6}b\left[2t_2^3 + \beta t_3^3 + 2(\beta-1)t_4^3 - 3t_2^2t_3 - 3(\beta-1)t_3t_4^2\right]$$

$$+\frac{1}{12}c\left[3t_2^4 + \beta t_3^4 + 3(\beta-1)t_4^4 - 4t_2^3t_3 - 4(\beta-1)t_3t_4^3\right]$$

The production cost in $[t_3, t_4]$ is

$$\int_{t_3}^{t_4} Kv\, du = \alpha_1\beta\int_{t_3}^{t_4}\left(a+bu+cu^2\right)^{1-\gamma} du$$

$$=\frac{\alpha_1\beta}{6a^{\gamma}}\left[6a\left(t_4-t_3\right)+(1-\gamma)\left(3b\left(t_4^{\,2}-t_3^{\,2}\right)+2c\left(t_4^{\,3}-t_3^{\,3}\right)\right)\right],\quad \gamma\neq 2.$$

Therefore, the production cost in $[0,\ t_4]$ is

$$\frac{\alpha_1\beta}{6a^{\gamma}}\left[6at_1+(1-\gamma)\left(3bt_1^{\,2}+2ct_1^{\,3}\right)\right]+\frac{\alpha_1\beta}{6a^{\gamma}}\left[6a\left(t_4-t_3\right)+(1-\gamma)\left(3b\left(t_4^{\,2}-t_3^{\,2}\right)+2c\left(t_4^{\,3}-t_3^{\,3}\right)\right)\right]$$

$$=\frac{\alpha_1\beta}{6a^{\gamma}}\left[6a\left(t_1+t_4-t_3\right)+3b(1-\gamma)\left(t_1^{\,2}+t_4^{\,2}-t_3^{\,2}\right)+2c(1-\gamma)\left(t_1^{\,3}+t_4^{\,3}-t_3^{\,3}\right)\right],\quad \gamma\neq 2.$$

The total average cost of the system in $[0,\ t_4]$ is

$$\begin{aligned}
C=\frac{1}{t_4}\Bigg[&\frac{1}{12}ac_1\left[6\beta t_1^{\,2}-\alpha\beta t_1^{\,4}+6t_2^{\,2}-12t_1t_2+2\alpha t_1^{\,3}t_2+\alpha t_2^{\,4}-2\alpha t_1t_2^{\,3}\right]\\
&+\frac{1}{120}bc_1\left[20\beta t_1^{\,3}-3\alpha\beta t_1^{\,5}+40t_2^{\,3}-60t_1t_2^{\,2}+10\alpha t_1^{\,3}t_2^{\,2}+8\alpha t_2^{\,5}-15\alpha t_1t_2^{\,4}\right]\\
&+\frac{1}{180}cc_1\left[15\beta t_1^{\,4}-2\alpha\beta t_1^{\,6}+45t_2^{\,4}-60t_1t_2^{\,3}+10\alpha t_1^{\,3}t_2^{\,3}+10\alpha t_2^{\,6}-18\alpha t_1t_2^{\,5}\right]\\
&+\frac{1}{2}ac_2\left[t_2^{\,2}+\beta t_3^{\,2}+(\beta-1)t_4^{\,2}-2t_2t_3-2(\beta-1)t_3t_4\right]\\
&+\frac{1}{6}bc_2\left[2t_2^{\,3}+\beta t_3^{\,3}+2(\beta-1)t_4^{\,3}-3t_2^{\,2}t_3-3(\beta-1)t_3t_4^{\,2}\right]\\
&+\frac{1}{12}cc_2\left[3t_2^{\,4}+\beta t_3^{\,4}+3(\beta-1)t_4^{\,4}-4t_2^{\,3}t_3-4(\beta-1)t_3t_4^{\,3}\right]\\
&+ac_3\left(\beta t_1-t_2\right)+\frac{1}{2}bc_3\left(\beta t_1^{\,2}-t_2^{\,2}\right)+\frac{1}{3}cc_3\left(\beta t_1^{\,3}-t_2^{\,3}\right)\\
&+\frac{\alpha_1\beta}{6a^{\gamma}}\left[6a\left(t_1+t_4-t_3\right)+3b(1-\gamma)\left(t_1^{\,2}+t_4^{\,2}-t_3^{\,2}\right)+2c(1-\gamma)\left(t_1^{\,3}+t_4^{\,3}-t_3^{\,3}\right)\right]
\end{aligned}\qquad (15)$$

The optimums values of $t_1,\ t_2,\ t_3$ *and* t_4 which minimize the cost function C are the solutions of the equations

$$\frac{\partial C}{\partial t_1}=0,\ \frac{\partial C}{\partial t_2}=0,\ \frac{\partial C}{\partial t_3}=0 \text{ and } \frac{\partial C}{\partial t_4}=0 \qquad (16)$$

provided that these values of $t_i\ (i=1,2,3,4)$ satisfy the conditions $D_i>0\ (i=1,2,3,4)$, where D_i is the Hessian determinant of order i given by

$$D_i=\begin{vmatrix} c_{11} & c_{12} & \cdots & c_{1i}\\ c_{21} & c_{22} & \cdots & c_{2i}\\ \vdots & \vdots & \cdots & \vdots\\ c_{i1} & c_{i2} & \cdots & c_{ii}\end{vmatrix},$$

$$c_{ij} = \frac{\partial^2 C}{\partial t_i \partial t_j} \quad (i, j = 1,\ 2,\ 3,\ 4).$$

The expanded forms of the (16) are

$$\frac{1}{12} ac_1 \left[12\beta t_1 - 4\alpha\beta t_1^3 - 12t_2 + 6\alpha t_1^2 t_2 - 2\alpha t_2^3 \right]$$

$$+\frac{1}{120} bc_1 \left[60\beta t_1^2 - 15\alpha\beta t_1^4 - 60t_2^2 + 30\alpha t_1^2 t_2^2 - 15\alpha t_2^4 \right]$$

$$+\frac{1}{180} cc_1 \left[60\beta t_1^3 - 12\alpha\beta t_1^5 - 60t_2^3 + 30\alpha t_1^2 t_2^3 - 18\alpha t_2^5 \right]$$

$$+ac_3\beta + bc_3\beta t_1 + cc_3\beta t_1^2 + \frac{\alpha_1\beta}{a^\gamma}\left[a + (1-\gamma) bt_1 + (1-\gamma) ct_1^2 \right] = 0, \gamma \neq 2, \tag{17}$$

$$\frac{1}{12} ac_1 \left[12t_2 - 12t_1 + 2\alpha t_1^3 + 4\alpha t_2^3 - 6\alpha t_1 t_2^2 \right]$$

$$+\frac{1}{120} bc_1 \left[120t_2^2 - 120t_1 t_2 + 20\alpha t_1^3 t_2 + 40\alpha t_2^4 - 60\alpha t_1 t_2^3 \right]$$

$$+\frac{1}{180} cc_1 \left[180t_2^3 - 180t_1 t_2^2 + 30\alpha t_1^3 t_2^2 + 60\alpha t_2^5 - 90\alpha t_1 t_2^4 \right]$$

$$+ac_2 (t_2 - t_3) + bc_2 \left(t_2^2 - t_2 t_3 \right) + cc_2 \left(t_2^3 - t_2^2 t_3 \right) - ac_3 - bc_3 t_2 - cc_3 t_2^2 = 0, \tag{18}$$

$$ac_2 \left[\beta t_3 - t_2 - (\beta - 1) t_4 \right] + \frac{1}{2} bc_2 \left[\beta t_3^2 - t_2^2 - (\beta - 1) t_4^2 \right]$$

$$+\frac{1}{3} cc_2 \left[\beta t_3^3 - t_2^3 - (\beta - 1) t_4^3 \right] - \frac{\alpha_1\beta}{a^\gamma}\left[a + (1-\gamma) bt_3 + (1-\gamma) ct_3^2 \right] = 0, \tag{19}$$

$$ac_2 \left[(\beta - 1) t_4 - (\beta - 1) t_3 \right] + bc_2 \left[(\beta - 1) t_4^2 - (\beta - 1) t_3 t_4 \right] + cc_2 \left[(\beta - 1) t_4^3 - (\beta - 1) t_3 t_4^2 \right]$$

$$+\frac{\alpha_1\beta}{a^\gamma}\left[a + (1-\gamma) bt_4 + (1-\gamma) ct_4^2 \right] - C = 0. \tag{20}$$

Example - Let a = 0.4, b = 0.23, c = 0.05, c_1 = 40, c_2 = 120, c_3 = 150, α = 0.02, β = 8, α_1 = 50 and γ = 1.5 in appropriate units. By applying Mathematica 4.1, we obtain the optimum solutions for t_1, t_2, t_3 *and* t_4 of equations (17)-(20) as t_1^*=12.79, t_2^*=18.62, t_3^* =23.51 and t_4^*=25.32. Substituting t_1^*, t_2^*, t_3^* and t_4^* in equation (15), we obtain the optimum average cost as C^*=34,026.70.

Table 1

Changing Parameter	(%) change	t_1^*	t_2^*	t_3^*	t_4^*	C^*
α	+50	12.63	18.70	22.17	23.15	35,207.20
	+25	12.86	18.58	22.77	24.13	34,385.20
	-25	12.95	18.49	25.14	27.19	33,384.13
	-50	13.54	18.35	26.31	31.52	31,148.60
β	+50	13.36	19.37	25.56	27.45	41,639.60
	+25	13.21	19.02	24.62	26.41	39,528.40
	-25	12.65	17.79	22.62	24.17	34,030.20
	-50	12.13	17.26	21.54	22.35	31,481.40
α_1	+50	12.54	17.66	24.14	26.51	44,629.60
	+25	12.74	18.07	23.83	25.71	39,241.70
	-25	13.16	19.52	23.25	25.15	28,378.10
	-50	13.67	21.09	22.82	24.62	22,312.50
γ	+50	14.35	18.61	22.90	24.20	40,232.10
	+25	13.50	18.32	23.29	24.77	38,935.20
	-25	12.52	18.15	24.15	25.23	29,346.10
	-50	11.37	17.86	24.96	26.45	23,806.40
c_1	+50	12.79	18.62	23.65	25.32	52,026.50
	+25	12.79	18.62	23.65	25.32	46,040.70
	-25	12.79	18.62	23.65	25.32	27,020.30
	-50	12.79	18.62	23.65	25.32	19,025.30
c_2	+50	12.79	18.62	23.65	25.32	34,024.30
	+25	12.79	18.62	23.65	25.32	34,026.70
	-25	12.79	18.62	23.65	25.32	34,104.60
	-50	12.79	18.62	23.65	25.32	34,629.10
c_3	+50	10.56	14.75	18.67	19.29	30,215.00
	+25	11.27	16.24	20.87	23.24	33,502.00
	-25	15.31	21.91	25.48	28.37	44,681.90
	-50	19.76	27.14	32.89	37.38	55,714.80

IV. CONCLUSION

An order level inventory model for deteriorating items with time-dependent deterioration rate, quadratic demand rate and unit production cost have been present here. The quadratic demand rate is assumed to be time-dependent and the unit production cost is inversely related to the quadratic demand rate and we assumed a, b, c is constant.

This paper is solved by allowing shortages in inventory. Solving highly nonlinear algebraic equations using fourth-order, the average system cost is minimized satisfying the optimization criterion.

VI. Sensitivity Analysis

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 25 % and 50 % and taking one parameter at a time, keeping the remaining parameters at their original values. From table, the following points are noted

On the basis of the results of table, the following observation can be made.

1. Decrease in the value of the parameter α , then t_1^*, t_3^*, t_4^* is increased and t_2^*, C^* is decreased.
2. Decrease in the value of the parameter β then $t_1^*, t_2^*, t_3^*, t_4^*$ and C^* is decreased.
3. Decrease in the values of either of the parameters c_1, c_2 then $t_1^*, t_2^*, t_3^*, t_4^*$ are unchanged but C^* value is decreased or increased.
4. Decrease in the value of the parameter γ then t_1^*, t_2^*, C^* is decreased and t_3^*, t_4^* is increased.
5. Decrease in the values of either of the parameter α_1 then t_1^*, t_2^* is increased and t_3^*, t_4^*, C^* is decreased.
6. Decrease in the value of parameter c_3 then $t_1^*, t_2^*, t_3^*, t_4^*$ and C^* is increased.

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