Gracefulness of $^Nc_4$ Merging With Paths

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Abstract: Gracefulness of nc4 merging with paths

I. Introduction:
Most graph labeling methods trace their origin to one introduced by Rosa [2] or one given Graham and Sloane [1]. Rosa defined a function f, a β-valuation of a graph with q edges if f is an injective map from the vertices of G to the set {0, 1, 2, …, q} such that when each edge xy is assigned the label $|f(x)-f(y)|$, the resulting edge labels are distinct.


A. Solairaju and others [5,6,7,8,9] proved the results that (1) the Gracefulness of a spanning tree of the graph of Cartesian product of $P_m$ and $C_n$ was obtained (2) the Gracefulness of a spanning tree of the graph of cartesian product of $S_m$ and $S_n$, was obtained (3) edge-odd Gracefulness of a spanning tree of Cartesian product of $P_2$ and $C_n$ was obtained (4) Even-edge Gracefulness of the Graphs was obtained (5) ladder $P_2 \times P_n$ is even-edge graceful, and (6) the even-edge gracefulness of $P_{k0} \times nC_5$ is obtained.

II. Section – I: Preliminaries
Definition 1.1 Let G = (V,E) be a simple graph with p vertices and q edges. A map f: V(G) -> {0,1,2,……,q} is called a graceful labeling if
i) F is one-to-one
ii) The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its end, a graph having a graceful labeling is called a graceful graph.

Example 1.1: $k = 11$ (odd); P: $V \rightarrow 19$; Q: $e \rightarrow 20$

Example 2.2: $k = 14$ (even); P: $V \rightarrow 22$; Q: $e \rightarrow 23$
Gracefulness of nc4 merging with path generalization

Case I
K is odd
n is copies; p = V(G), q = e(G)
Define : f : V(G) \mapsto \{0, ..., q\} by
\begin{align*}
f(T_1) &= 0, \\
f(T_2) &= 2, \\
f(V_1) &= q, \\
f(V_2) &= (q) - 1 \\
f(V_i) &= f(V_1) - 1 \quad \text{if } i \text{ is odd.} \\
f(V_i) &= f(V_{i-1}) - 1 \quad \text{if } i = 4l + 2 \\
f(V_i) &= f(V_{i-1}) + 1 \quad \text{if } i = 4l \\
f(T_i) &= f(T_{i-1}) + K \quad \text{if } i = (4l + 3, 4l + 1) \\
f(T_i) &= f(T_{i-1}) + 2 \quad \text{if } i = (4l, 4l + 2)
\end{align*}

Case II
K is even
n is copies; p = V(G), q = e(G)
Define : f : V(G) \mapsto \{0, ..., q\} by
\begin{align*}
f(V_0) &= q, \\
f(V_1) &= q - 1, \\
f(T_1) &= 0, \\
f(T_2) &= 2, \\
f(V_3) &= 6, \\
f(V_4) &= 7.
\end{align*}
Gracefulness Of $C_4$ Merging With Paths

\[ f(V_i) = \]
\[ f(V_{i+1}) = \]
\[ f(T_i) = f(T_{i+1}) + 11; \quad \text{if } i = 4l + 1, \ 4l + 2 \]
\[ f(T_3) = q - (K/2 + 2); \quad \text{if } i = 5, 9, 13, ... \]
\[ f(T_{4}) = f(T_3) + 2; \quad \text{if } i = 6, 10, 14, ... \]
\[ f(T_{i}) = f(T_{i-1}) - 13; \quad \text{if } i = 4l + 3, \]
\[ f(T_{i}) = f(T_{i-1}) - 2; \quad \text{if } i = 7, 11, 15, ... \]
\[ f(T_{i}) = f(T_{i-1}); \quad \text{if } i = 8, 12, 16, ... \]

References

[9] “The Java Complete Reference” by Herbert Scheidt