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An EOQ Inventory Model with Fuzzy Deterioration Rate and Finite Production Rate

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Abstract: *In this paper, we have developed an economic production models with finite production rate and fuzzy deterioration rate. In the development of the model, lost of production quantity due to faulty machine aged machine, manufacturing defect etc. from the actual production quantity have also been taken into account. We developed the corresponding fuzzy model. The solution form minimizing the fuzzy cost function have been derived The sensitivity of the optimal solution with respect to the changes in the different parameter values is also discussed. all the inventory models have been developed under the assumption that the deterioration rate is constant or it is dependent on time.*

I. Introduction

The basic EOQ model has the specific requirements of constant demand rate and lack of deterioration of the items in stock. Deterioration of an items can be defined as decay, evaporation obsolescence, loss of utility or marginal value of a commodity that results in the decreasing usefulness of the inventory from the original condition. Vegetables, meat, fertilizers, gasoline, different types of oils, medicines, mild etc. are examples of deteriorating items. Inventory models for deteriorating items has been studied by several researchers in recent decades. Gharese and Schrader[1]-[2] developed an EOQ model for items with an exponentially decaying inventory. An EOQ models for items with a variable rate of deterioration was discussed Covert and Philip[3] who used a two parameter weibull distribution for the time to deterioration. Covert and Philip[4] adopted a three parameters weibull distribution for the time to deterioration time. Mishra [5] formulated an inventory models with a variable rate of deterioration, a finite rate of deterioration, a finite rate of production. Several researcher like Cohen[6], Kang and Kim[7], Aggrawal and Jaggi [8-9]. Hui-Ming Wee [10], B.C. Giri [11] and K.S. Choudhuri [12] and San Chyi Chang[13] etc. developed economic production lot-size models with different assumption on the patterns on deterioration rate. Till now, all the inventory models have been developed under the assumption that the deterioration rate is constant or it is dependent on time. For solving the economic production equanimity, we always consider the demands rate, production rate and deterioration rate as constant in the crisp model. But, in the real life situation, these quantities will have little deviations from the exact value. The variables whose values are not crisp but uncertain in nature. Hence these variables should be treated as fuzzy variables. Yao and Lee [14] developed as economic order quantity model by considering order quantity as fuzzy and allowing shortages. The same authors Yao and Lee [15-16] developed another inventory model with fuzzy demand quantity and fuzzy production quantity.

In this , we have developed an economic production models with finite production rate and fuzzy deterioration rate. In the development of the model, lost of production quantity due to faulty machine aged machine, manufacturing defect etc. from the actual production quantity have also been taken into account. We developed the corresponding fuzzy model. The solution form minimizing he fuzzy cost function have been derived with the help of Zimmerman [17] and Kaufmann and Gupta[18] and the solution procedure is illustrated by one numerical example. The sensitivity of the optimal solution with respect to the changes in the different parameter values is also discussed.

II. Notation And Assumption Of The Model

The following notations have been used to develop the model.

1. K = Production Quantity per unit time.
2. d = Demand rate per unit time.
3. θ = deterioration rate of the on hand inventory per unit time.
4. $\tilde{\theta}$ = fuzzy deterioration rate of the on hand inventory per unit time.
5. ϕ = deterioration fraction of production, rate per unit time.
6. C_1 = constant inventory holding cost per unit time
7. r = constant purchase cost of raw materials per unit item.
8. b = constant setup cost per order.

9. p = constant selling price of the production
10. t_1 = time at which production is stopped.
11. t_2 = time at which inventory level reaches zero.
12. T = the whole period for the plan.
13. q_1 = inventory level at any time t, where $o \le t \le t_1$
14. q_2 = inventory level at any time t, where $o \le t \le t_2$
15. q = actual production quantity received per cycle.
16. K (1-ϕ) = actual production rte per unit time.

III. Assumptions Of The Model

The following assumption are made in developing the model

1. Replenishment are instantaneous with constant lead time.
2. Shortage are not allowed
3. The membership function of the fuzzy deterioration rate $\bar{\theta}$

$$\mu_{\bar{\theta}}(\theta) = \begin{cases} \dfrac{\theta - \theta_1}{\theta_0 - \theta_1} & \text{for } \theta_1 \le \theta < \theta_2 \\ \dfrac{\theta_2 - \theta_1}{\theta_2 - \theta_0} & \text{for } \theta_0 \le \theta < \theta_2 \\ 0 & \text{elsewhere} \end{cases}$$

Where θ_1, θ_0 and θ_2 are positive numbers.

IV. Development Of The Crisp Model

At time t = 0 the production starts at the beginning of each cycle and continues upto time t = t_1. The actual production rate becomes less than original production reduce to faulty machine since some quantities deteriorate at the time of production.

The inventory accumulated during the production period t_1 after meeting up demand during the period and deterioration reaches to the zero level at time t = 1_2. Then the cycle repeat itself for total planning period T.

The following differential equation gives the instantaneous states of q_1 and q_2 over the cycle t_1 & t_2

$$\frac{dq_1}{dt} + \theta q_1 = K(1-\phi) - d; \qquad 0 \le t \le t_1 \qquad \text{....... (2.1)}$$

$$\frac{dq_2}{dt} + \theta q_2 = -d; \qquad t_1 \le t \le t_2 \qquad \text{....... (2.2)}$$

With the condition

$q_1(0) = 0$; $q_1(t_1) = 0$, $q_1(t_1) = q_2(t_1)$ and $q_2(t_2) = 0$ .... (2.3)

Eq. (2.1) is a linear differential equation with integrating factor (IF) given by

$$I.F. = e^{\int \theta dt} = e^{\theta t}$$

Its solution is given by

$$e^{\theta t}\ q_1(t) = \frac{(k(1-\phi)-d)}{\theta} e^{\theta t} + c$$

When $t = 0, \quad q_1(t) = 0, \quad$ we have

$$C = \frac{-(k(1-\phi)-d)}{\theta} \qquad \text{....... (2.4)}$$

$$e^{\theta t}\ q_1(t) = \frac{(k(1-\phi)-d)}{\theta}(e^{\theta t}-1)$$

$$\therefore q_1(t) = \frac{(k(1-\phi)-d)}{\theta}(1-e^{-\theta t}) \quad ; \quad 0 \le t \le t_1$$

From equation (2.2), we get

$$\frac{dq_2}{dt} + \theta q_2 = -d$$

Which is also a linear differential equation, its solution is given by

$$\Rightarrow q_2(t) = \frac{d}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)$$

From equation (1.24) we have

$$q_1(t_1) = \frac{\left(k(1-\varphi)-d\right)}{\theta}\left(1-e^{-\theta t_1}\right)$$

$$= \left(k(1-\varphi)-d\right)\ \theta t_1\left(t_1 - \frac{\theta t_1^2}{2} + \frac{\theta^2 t_1^3}{6}\right)$$

From equation (2.5), we have

$$q_2(t_1) = \frac{d}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right)$$ ($\therefore$ Neglecting higher power) …….

$$= \frac{d}{\theta}\left(1 + \theta(t_2 - t_1) + \frac{\theta^2(t_2-t_1)^2}{2} - 1\right)$$

$$= \frac{d}{\theta}\left(\theta t_2 - \theta t_1 + \frac{\theta^2(t_2^2 + t_1^2 - 2t_1t_2)}{2}\right)$$ …………(2.6)

…………(2.7)

$$q_1(t_1) = \frac{\left(k(1-\varphi)-d\right)}{\theta}\left(1-e^{-\theta t_1}\right)$$

$$= \frac{\left(k(1-\varphi)-d\right)}{\theta}\left(1-e^{-\theta t_1}\right)$$

$$q_2(t_1) = \frac{d}{\theta}\left(e^{\theta(t_2-t_1)} - 1\right) = \frac{d}{\theta}\frac{\left(e^{\theta t_2} - e^{\theta t_1}\right)}{e^{\theta t_1}}$$

given $q_1(t_1) = q_2(t_1)$

$$t_2 = \frac{q}{d}\left\{1 + \frac{\theta t_1}{2} + \frac{\theta^2 t_1^2}{C}\right\}$$ …………(2.8)

Where q = k (1-φ)t_1 is the actual production quantity during the period t_1

Let HC denote the holding cost per cycle and is given by

$$HC = C_1\left[\int_0^{t_1} q_1(t)dt + \int_{t_1}^{t_2} q_2(t)dt\right]$$

$$= C_1\int_0^{t_1}\frac{\left(k(1-\varphi)-d\right)}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{t_1}^{t_2}\frac{d}{\theta}\left\{e^{\theta(t_2-t)} - 1\right\}dt$$

$$= C_1\left[\int_0^{t_1}\left(k(1-\varphi)-d\right)\left(t - \frac{\theta t^2}{2}\right)dt + d\int_{t_1}^{t_2}\left((t_2-t) + \theta\frac{(t_2-t)^2}{2}\right)dt\right]$$

$$\text{HC} \quad = C_1\left[\left(k(1-\varphi)-d\right)\left(\frac{t_1^2}{2}-\frac{\theta t_1^3}{6}\right)+d\left\{\frac{\left(t_2-t_1\right)^2}{2}+\frac{\theta\left(t_2-t_1\right)^3}{6}\right\}\right]$$
………(2.9)

Let LS denote the cost for loss of stock due to deterioration in each cycle and is given by

$$LS = p\left[\theta\left\{\int_0^{t_1} q_1(t)dt+\int_{t_1}^{t_2} q_2(t)dt\right\}\right]$$
…….(2.10)

$$= p\{k(1-\phi)t_1 - dt_2\}$$

Let LP denote the cost for loss of production quantity per cycle due to faulty machine and is given by

LP = r k ϕ t₁ Also the number of cycles in the entire planning horizon T is T/t_2. Hence the total inventory cost, TC for the whole planning horizon T is given by

$$TC = [HC+LC+LP+b].T/t_2$$
……(2.11)

$$=\left[\begin{array}{l} C_1\left[\{K(1-\phi)-d\}\left(\frac{t_1^2}{2}-\frac{\theta t_1^3}{6}\right)+d\left\{\frac{(t_2-t_1)^2}{2}+\frac{\theta(t_2-t_1^3)}{6}\right\}\right] \\ + p\{(k(1-\phi)t_1-dt_2\}+rk\phi t_1+b \end{array}\right]$$

$$= f_1(q)+\theta f_2(q)$$

Where

$$f_1(q)=\frac{C_1T}{2}\left\{1-\frac{d}{k(1-\phi}\right\}q+\left(b+\frac{rq\phi}{1-\phi}\right)\frac{dT}{q}$$
…….(2.12)

…….(2.13)

$$f_2(q)=\frac{T}{2}\left\{\left(b+pq+\frac{rq\phi}{1-\phi}\right)+\frac{C_1q^2}{6}\left\{\frac{d}{k^2(1-\theta)^2}-\frac{1}{d}\right\}\right\}$$

The model developed above gives the following particular cases
…….(2.14)

Case (i), If $\phi \to 0, \quad k \to \infty, \quad \theta \to 0,$ we obtain

$$TC = \frac{C_1qT}{2}+\frac{bdT}{q}$$

This equation is same as the average total cost of a classical EOQ model with constant demand and no deterioration

CASE (II) when $\phi \to 0,$ and R = ∞

$$TC = \frac{C_1qT}{2}+\frac{bdT}{q}+\frac{\theta T}{2}\left(b+pq-\frac{C_1q}{6}\right)$$

Which is the classical EPQ (Economic Production Quality) model under deterioration and constant demand.

CASE (III) when $\theta \to 0,$ **we get**

$$TC = \frac{C_1qT}{2}\left(1-\frac{d}{k}\right)+\frac{bdT}{q}+\frac{\theta T}{2}\left(1-\frac{d}{k}\right)\left\{b+pq-\left(1+\frac{d}{k}\right)\frac{C_1q^2}{6}\right\}$$
…….(2.9)

Which is the classical EPQ (Economic Production Quality) model under deterioration and constant demand.

CASE (IV) : For $\phi \to 0, \quad \theta \to 0,$ **we get**

$$f_1(q) = \frac{C_1 T}{2}\left(1 - \frac{d}{k}\right)q + b.\frac{dT}{q}$$

$$TC = \frac{C_1 q T}{2}\left(1 - \frac{d}{k}\right) + b.\frac{dT}{q}$$

Which is same as obtained by Lee & Yao (1998).

V. Development Of Fuzzy Model

Earlier authors have assumed that the deterioration rate is constant fraction of the on hand inventory level in the development of Economic Order Quantity models. But in real world it is not always easy to determine the deterioration rate exactly but in most cases it is uncertain in nature. The Economic Order Quantity model have been developed with the assumption that deterioration rate is a fuzzy number $\tilde{\theta}$ instead of considering deterioration rate as constant.

Thus the function of (2.12) can be redefined as

$$Min\ TC = f_1(q) + \tilde{\theta}\ f_2(q)\ where\ q \geq 0 \qquad \text{……….(2.15)}$$

Where Wavy bar ~ represents the fuzzification of the parameters.

We express the fuzzy deterioration rate $\tilde{\theta}$ as the normal triangular fuzzy number $(\theta_1, \theta_0, \theta_2)$

Suppose the membership function of the fuzzy deterioration rate $\tilde{\theta}$ is as follows :-

$$\mu_{\tilde{\theta}}(\theta) = \begin{cases} \dfrac{\theta - \theta_1}{\theta_0 - \theta_1} & for\, \theta_1 \leq \theta \leq \theta_0 \\ \dfrac{\theta_2 - \theta}{\theta_2 - \theta_0} & for\, \theta_0 \leq \theta \leq \theta_2 \\ 0 & elsewhere \end{cases} \qquad \text{…….(2.16)}$$

Where θ_1, θ_0 and θ_2 are positive values and $0 \leq \theta_1 < \theta_0 < \theta_2$

The centroid of $\mu_{\tilde{\theta}}(\theta)$ is given by

$M_0(\theta_1, \theta_0, \theta_2) = (\theta_1 + \theta_0 + \theta_2) / 3$

For any positive numbers C_1, d, b, T, φ, p, r, k and every fixed value of $q > 0$.

Let $G_q(\theta)$ denotes the total cost function TC (q, θ) for q, θ.

$G_q(\theta) = f_1(q) + \theta f_2(q) = Z.$

Then we have

$$\theta = \frac{z - f_1(q)}{f_2(q)} \qquad \text{for } f_2(q) \neq 0 \qquad \text{……(2.17)}$$

$$\Rightarrow z \geq f_1(q) = z_1(q)$$

By extension principle, the membership function for fuzzy cost function $G_q(\tilde{\theta})$ is given by

$$\mu_{G_q(\tilde{\theta})}(z) = Sup_{\theta \in G^{-1}(z)} \mu_{\tilde{\theta}} \qquad if\ \ G_q^{-1}(z) \neq \phi \qquad \text{……….(2.18)}$$

$$= 0 \qquad \text{if } G_q^{-1}(z) = \phi$$

Thus if $z \geq z_1(q)$ then

$$\mu_{G_q(\tilde{\theta})}(z) = \mu_{\tilde{\theta}}(\theta)$$

$$= 0 \quad else.$$

Now let

$$C_j(q) = f_1(q) + \theta_j f_2(q)$$

$$So \quad C_j(q) - z_1(q) = \theta_j f_2(q) \geq 0$$

$$\Rightarrow z_1(q) \leq C_j(q)$$

$$and \quad C_i(q) - C_j(q) = (\theta_i - \theta_j) f_2(q) \qquad \text{for } j = 1,0,2$$

$$and \quad 0 \leq \theta_1 < \theta_0 < \theta_2 \qquad \text{…….(2.19)}$$

Further

$$\theta_i \geq \theta_j \Leftrightarrow C_i(q) \geq C_j(q)$$

$$\text{for } (i = 0,2,\ j = 1) \text{ or } (i = 2,\ j = 0)$$

$$and \qquad \theta_i \leq \theta_j \Leftrightarrow C_i(q) \geq C_j(q)$$

$$\text{for } (i = 0,1,\ j = 2) \text{ or } (i = 1,\ j = 0) \qquad \text{…….(2.19a)}$$

$$\mathrm{P} = \int_{-\infty}^{\infty} \mu_{G_q(\tilde{\theta})}(z)dz$$

and

$$R = \int_{-\infty}^{\infty} \mu_{G_q(\tilde{\theta})}(z)dz$$

The centroid for $\mu_{G_q(\tilde{\theta})}(z)$ is given by R/P which is the estimate of the total cost.

We have $Z_1(q) \leq C_1(q) < C_0(q) < C_2(q)$
Using the condition (2.18) to (2.19a)

The membership function of $G_q(\tilde{\theta})$ can be defined as

$$\mu_{G_q(\tilde{\theta})}(z) = \frac{z - f_1(q) - \theta_1 f_2(q)}{(\theta_0 - \theta_1) f_2(q} \qquad \text{for } C_1(q) \leq z \leq C_0(q)$$

$$= \frac{f_1(q) + \theta_2 f_2(q) - 2}{(\theta_0 - \theta_1) f_2(q)} \qquad \text{for } C_0(q) \leq z \leq C_2(q) \qquad \text{……….(2.20)}$$

$$= 0, \quad \text{else where}$$

Therefore P and R can be written as

$$P = \frac{1}{(\theta_0 - \theta_1) f_2(q)} \int_{C_1(q)}^{C_0(q)} (z - f_1(q) - \theta_1 f_2(q)) dz$$

$$+ \frac{1}{(\theta_2 - \theta_0) f_2(q)} \int_{C_1(q)}^{C_2(q)} \{f_1(q) + \theta_2 f_2(q) - z\} dz \qquad \text{……….(2.21)}$$

$$= \frac{(\theta_2 - \theta_1) f_2(q)}{2}$$

$$R = \frac{1}{(\theta_0 - \theta_1) f_2(q)} \int_{C_1(q)}^{C_0(q)} z\{z - f_1(q) - \theta_1 f_2(q)\} dz$$

$$+ \frac{1}{(\theta_2 - \theta_0) f_2(q)} \int_{C_1(q)}^{C_2(q)} \{f_1(q) - \theta_2 f_2(q) - z\} dz$$

$$= (\theta_2 - \theta_1) f_2(q).\{3 f_1(q) + (\theta_0 + \theta_1 + \theta_2) f_2(q)\}/6$$

After evaluating the integral of R and P and with simple algebra, we obtain the centroid of $\mu_{G_q(\tilde{\theta})}(z)$ as

$$\frac{R}{P} = f_1(q) + (\theta_1 + \theta_0 + \theta_2) f_2(q)/3 \qquad \text{……….(2.23)}$$

VI. Optimal Solution Of The Inventory Model

For given θ_0, we have to find $(\theta_1^*, \theta_2^*, q^*)$ such that the centroid of the fuzzy total cost is minimal i.e.

$$Min \frac{R(\theta_1, \theta_2, q)}{P(\theta_1, \theta_2, q)} = \frac{R(\theta_1^*, \theta_2^*, q^*)}{P(\theta_1^*, \theta_2^*, q^*)} \qquad \text{……….(2.24)}$$

Since the equation (2.23) is a highly non linear. It is difficult to minimize the cost function analytically. We have used the standard software package LINGO for minization of the cost function (2.23).
Then we regards as the Economic production quantity.
Also

$$\hat{\theta}_0 = (\theta_1^* + \theta_0^* + \theta_2^*)/3$$

Is the centroid of the normal fuzzy triangular number $(\theta_1^*, \theta_0, \theta_2^*)$.

VII. Numerical Illustration Of The Model

The following numerical example has been considered to illustrate the Inventory model.

Let,

C_1	=	10
d	=	2
b	=	500
T	=	40
p	=	3
r	=	1
k	=	10
θ	=	0.003
ϕ	=	.005

In appropriate units.With these parameter values the solution of the equation (2.24).
Using the software LINGO are obtains as

$$q^* = 15.89, \theta_1^* = 0.001, \theta_2^* = 0.005$$

$$and \quad \frac{\mathrm{R}}{\mathrm{P}} = 5071.059$$

$$\text{Thus}\ \ \theta_0^* = 0.003 \quad if \qquad \theta_0 = 0, \theta_1 = 0, \theta_2 = 0$$

$$\text{Then}\ \ q^* = 15.821$$

$$and \qquad \frac{\mathrm{R}}{\mathrm{P}} = 5056.866$$

VIII. Sensitivity Analysis Of The Model

We have examined the sensitivity of each of the decision variables q*, θ_1*, θ_2*, using he numerical example and the centroid of the fuzzy total cost R/P to changes in each of the eight parameters e,d,b,p,r,k, θ_0 and φ.

Where each parameters has been varied from -50% to +50% keeping other parameters constant. The results obtained are displaced in table 1.1

Table 1.1 : Effect of changes in parameters on the decision variables.

Parameters	Changes (%)	q*	θ_1*	θ_2*	Total cost (R/P)
C	+50	12.964	0.00000	0.00600	6207.063
	+20	14.501	0.00000	0.00590	5553.479
	-20	17.774	0.00097	0.00503	4537.498
	-50	22.507	0.00000		3590.985
D	+50	22.809	0.00156	0.00444	5801.410
	+20	17.859	0.00300	0.00300	5411.154
	-20	13.872	0.00091	0.00508	4650.951
	-50	10.601	0.00028	0.00572	3811.510
B	+50	19.483	0.00043	0.00557	6213.923
	+20	17.415	0.00300	0.00300	5556.267
	-20	14.205	0.00000	0.00599	4534.615
	-50	11.220	0.00300	0.00300	3583.439
P	+50	15.887	0.00108	0.00492	5072.201
	+20	15.889	0.00106	0.00494	5071.516
	-20	15.892	0.00104	0.00493	5070.602
	-50	15.894	0.00101	0.00499	5069.926
R	+50	15.890	0.00105	0.00495	5071.262
	+20	15.890	0.00105	0.00495	5071.140
	-20	15.890	0.00105	0.00495	5070.978
	-50	15.890	0.00105	0.00495	5070.856
K	+50	15.256	0.00300	0.00300	5281.584
	+20	15.564	0.00107	0.00493	5177.367
	-20	16.421	0.00099	0.00500	4907.379
	-50	18.399	0.00037	0.00563	4380.649
θ_0	+50	15.925	0.00158	0.00742	5048.119
	+20	15.904	0.00126	0.00594	5073.886
	-20	15.876	0.00240	0.00240	5068.228
	-50	15.856	0.00150	0.00150	5063.975
φ	+50	15.895	0.00300	0.00300	5069.640
	+20	15.892	0.00105	0.00495	5070.492
	-20	15.888	0.0105	0.00495	5071.624
	-50	15.885	0.00300	0.00300	5072.470

We see from table 1.1 that all he decision variables are sensitive to changes in C_1 total cost decreases from 29% for a 50% decrease in C_1 and increases by 22% for a 50% increase in C_1. All the decision variables are sensitive to changes in d.

For a 50% increases, q and total cost increases by 31% and 14% respectively. On the other hand when d is reduced by 50%, q and total cost are decreased by 33% and 24% respectively. For changes in the parameters b, we observe that q and total cost are sensitive whereas θ_1 and θ_2 are very less sensitive. The decision variable q and total cost increases and decreases by 22% and 29% respectively, for a 50% increment and decrement in b.

IX. Conclusion

In the inventory model, in order to solve the economic production quantity precycle we always treat the deterioration rate as constant. But, in the real situation, this quantity probably will have little disturbances. Therefore, we should fuzzily this quantity to solve the economic production quantities it the fuzzy sense. In this article, a perishable inventory model with finite production rate, constant demands rate and fuzzy deterioration rate is developed for finite planning horizon. Also loss of production incurred due to faulty/aged machines have been taken into account by considering a fraction of production rate deteriorates per unit time. The theory for minimizing the fuzzy total cost function is developed by constructing the membership function of the cost function. The solution procedure is illustrated with analysis of the parameters of the system is carried out. It is observed that c_1, b,d, and k are sensitive parameters of the system. The other parameters have no significant sensitivity.

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