IOSR Journal of Mathematics (IOSR-JM)
ISSN: 2278-5728. Volume 4, Issue 1 (Nov. - Dec. 2012), PP 65-71
www.iosrjournals.org

MHD VISCO-Elastic Fluid Flow and Heat Transfer over A Stretching Sheet

Anis Fatima[1] & Subhas Abel[2]
1 Department Of Mathematics Govt.College Gulbarga
2 Department Of Mathematics Gulbarga University Gulbarga

Abstract: *MHD visco elastic Fluid flow and heat transfer over a stretching surface in presence of viscous dissipation, internal heat generation or absorption and work due to deformation has been investigated. The flow is influenced by linearly stretching sheet, in presence of heat generation/absorption of the wall. The problem has been solved numerically by shooting technique with fourth order RUNGE-KUTTA integration scheme. Heat Transfer analysis has been carried out for both prescribed surface temperature (PST) and prescribed power law heat flux (PHF) to get the effect of visco –elasticity (k_1),Eckert number (Ec), heat source /sink (β), and Prandle number (Pr) in presence and absence of magnetic field (Mn).*

Keywords: *visco-elasticity, magnetic parameter, internal heat generation/absorption and Eckert number.*

I. Introduction:

During the past three decades there have been several studies of boundary layer flows of non-Newtonian fluids. In recent years, the study of boundary layer flow over a moving continuous solid surface has gained momentum because of its numerous application in the various engineering disciplines. Keeping in view its applications in drawing of polymer sheets. Sakiadis [1] has initiated the study of such boundary layer problem assuming the velocity of the boundary sheet as constant. However, in reality, one has to deal with the stretching of plastic sheet. Hence, Crane [2] has reinvestigated this problem assuming the stretching sheet's velocity varying linearly with the distance from the slit. Gupta and Gupta [3] have analyzed the heat and mass transfer characteristics in the boundary layer flow, in the presence suction/blowing.

So many other researchers carried out extensive analysis of heat and mass transfer phenomena associated with such flow, but their investigations are restricted only to the flow of Newtonian fluid.

However, in reality most of the liquids used in industrial applications particularly polymer processing applications are of non-Newtonian in nature. The non-Newtonian fluids are being considered more important and appropriate in technological applications in comparison with Newtonian fluids. A large class of real fluids does not exhibit the linear relationship between stress and rate of strain. Because of non-linear dependence, the analysis and behavior of non- Newtonian fluids tends to be more complicated in comparison to Newtonian fluids.

In view of the importance of these applications, Rajagopal et.al 4 has studied the flow behavior of visco-elastic fluid over a stretching sheet and gave an approximate solution for the flow. Vajravelu and Rollins [5] examined the effects of viscous dissipation and internal heat generation or absorption in viscoelastic boundary layer flow. Sarma and Rao [6] analyzed the effects of work done due to deformation in the energy equation. Vajravelu and Roper [7] examined the effects of viscous dissipation, internal heat generation or absorption and work done due to deformation.

In this chapter, we study the flow and heat transfer in MHD visco-elastic fluid over a stretching sheet, including the effects of viscous dissipation, internal heat generation or absorption and work due to deformation.

II. Mathematical Analysis:

Consider a steady state two-dimensional boundary layer flow of incompressible visco-elastic fluid of the type Walter's liquid *B* over a semi infinite stretching sheet coinciding with the plane *y=0*. Two equal and opposite forces are introduced along *x-axis*. so that the wall is stretching keeping the origin fixed. The basic boundary layer equation governing the flow, heat and mass transfer take the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2}{\rho}u-\frac{\gamma}{k'}u \tag{2}$$

Here *u* and *v* are velocity components in x and y directions respectively, σ is the electrical conductivity, B_0 is the applied magnetic field, k_0 is the visco-elastic parameter of the Walter's liquid *B*. k' permeability of porous medium, *Q* is the volumetric rate of heat generation, *k* is the thermal conductivity. The other quantities have their usual meanings.

The appropriate boundary conditions governing the flow are

$u = bx \quad v = 0 \quad at \quad y=0, \quad b>0$ (3)

$u \to 0 \quad as \quad y \to \infty$

Here *u* and *v* are velocity components along *x* and *y* directions respectively, x is being along the stretching sheet and *y* is normal to the surface, *A* and *B* are arbitrary constants, which depend on the nature of the boundary surface. T_w, T_∞ are temperature of the stretching sheet and temperature of the flow region far away from the sheet respectively.

III. Solution of momentum equation:

In order to obtain the mathematical form of the velocity, we introduce the following similarity transformations,

$$u = bxf'(\eta), \quad v = -\sqrt{b\nu}\, f(\eta) \quad \text{Where} \quad \eta = \sqrt{\frac{b}{\nu}}.y \tag{4}$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation.

$$f'^2 - ff'' = f''' - k_1\{2f'f''' - ff'''' - f''^2\} - Mnf' \tag{5}$$

Where

$$k_1 = \frac{k_o b}{\gamma}, \quad Mn = \frac{\sigma B_o^2}{b\rho}$$

are non-dimensional visco-elastic and Magnetic parameters respectively and the boundary condition takes the form

$$f = 0 \quad f' = 1 \quad at \quad \eta=0$$

$$f' \to 0 \quad f'' \to 0 \quad as \quad \eta \to \infty \tag{6}$$

Where prime denotes differentiation w.r.t η. The exact solution of equation (5) corresponding to the boundary conditions (6) is obtained as

$$f = \frac{1}{\alpha}(1 - e^{-\alpha\eta}), \text{ Where } \alpha = \sqrt{\frac{1+Mn}{1-k_1}} \tag{7}$$

The solutions for velocity field is obtained as

$$u = bxe^{-\alpha\eta}, \quad v = -\sqrt{b\gamma}\,\frac{1-e^{-\alpha\eta}}{\alpha} \tag{8}$$

IV. Heat Transfer Analysis:

The governing boundary layer equation with viscous dissipation (or frictional heating) and work done due to deformation for the two dimensional flow is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial^2 y}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\rho\lambda\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right]+Q(T-T_\infty) \tag{9}$$

Where ρ is the density, c_p is the specific heat at constant pressure, *k* is the thermal conductivity and *Q* is volumetric rate of heat generation. Thermal boundary conditions depend upon the type of the heating process. Here we considered two different types of heating process namely:

(1) Prescribed surface temperature

(2) Prescribed power law heat flux

CASE (1): Prescribed Surface Temperature

For this circumstance, the boundary conditions are

$$T = T_\omega = T_\infty + A\left(\frac{x}{l}\right)^s \quad at \quad y = 0.$$

$$T \to T_\infty \quad \text{as} \quad y \to \infty \tag{10}$$

T_w is variable wall temperature, A is a constant and l is the characteristic length.

We now define a non-dimensional temperature $\theta(\eta)$ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{11}$$

Using relation (7), (8) and (11) in equation (9) leads the following equation

$$\theta'' + Pr\, f\, \theta' - Pr(s\, f' - \beta)\, \theta = Pr\, E(xl)^{s-2}\, [(f'')^2 + k_1 f''(f' f'' - f f''')] \tag{12}$$

Obviously, we get an x-independent similarity equation from the above when s=2 and this yields

$$\theta'' + Pr\, f\, \theta' - Pr(2\, f' - \beta)\, \theta = Pr\, E\, [(f'')^2 + k_1 f''(f' f'' - f f''')] \tag{13}$$

and the corresponding boundary conditions in terms of θ can be obtained as

$$\theta(\eta) = 1 \quad at \quad \eta = 0$$

$$\theta \to 0 \quad as \quad \eta \to \infty \tag{14}$$

Where prime denotes differentiation w.r.t η, $E = \frac{bl^2}{c_p A}$, $\beta = \frac{Q}{\rho c_p b}$ and Pr = $\frac{\mu c_o}{k}$ are respectively Eckert number, heat source/sink parameter and Prandtl number.

CASE(2): Prescribed Power Law Heat Flux

The power law heat flux on the wall surface is considered to be power of x in the form

$$-k\frac{\partial T}{\partial y} = q_w = D\left(\frac{x}{l}\right)^s \quad \text{at} \quad y = 0$$

$$T \to T_\infty \quad as \quad y \to \infty \tag{15}$$

Where *D* is constant and *k* is thermal conductivity.

We now define a non-dimensional temperature *g* (η) as

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ , \tag{16}$$

Where

$$T - T_\infty = \frac{D}{k}\left(\frac{x}{l}\right)^s \sqrt{\frac{\upsilon}{c}}\, g(\eta)$$

With this change of variable equation (9) and corresponding boundary conditions takes the form

$$g'' + Pr\, f\, g' - Pr(sf' - \beta)\, g = Pr\, E(xl)^{s-2}\, [(f'')^2 + k_1 f''(f' f'' - f f''')] \tag{17}$$

Obviously, we get an *x*-independent similarity equation from the above when *s=2* and this yields

$$g'' + Pr\, f\, g' - Pr(2\, f' - \beta)\, g = Pr\, E\, [(f'')^2 + k_1 f''(f' f'' - f f''')] \tag{18}$$

and the corresponding boundary conditions in terms of g can be obtained as

$$g'(\eta) = -1 \quad at \quad \eta = 0$$

$$g(\eta) \to 0 \quad as \quad \eta \to \infty \tag{19}$$

Since equation (13) and (18) is linear (but with variable coefficients) subject to the boundary conditions (14) and (19) can be solved analytically in terms of Kummer's functions (confluent hyper geometric functions). However, it is solved here numerically for several chosen values of the parameters k_1 *E*, β *and Pr*, and interesting results involving $\theta(\eta)$ and $g(\eta)$ are presented in Figures 1-8.

V. Results and discussion:

In order to have a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. Results for prescribed Surface temperature (PST) are drawn in Fig1-4 and for prescribed power law heat flux (PHF) are drawn in Fig 5-8.

Fig-1 shows the variation of temperature profile against the space variable η for different values of visco-elastic parameter $k_{1,}$ it can be seen from Fig-1 that temperature profile increases when k_1 increase. This is due to the fact that the thickness of thermal boundary layer occurs due to the increase of Visco-elastic normal stress. This behavior is even true in the absence of *Mn*., which is represented in Fig-1 as dotted line.

The effect of heat source/sink parameter (β) on temperature profile $\theta(\eta)$ in the boundary layer is shown in Fig-2 in presence and absence of magnetic field *Mn*. It is observed that the effect of heat source ($\beta > 0$) in the boundary layer generates the energy, which causes the temperature to increase, while the presence of heat sink ($\beta < 0$) in the boundary layer absorbs the energy, which causes the temperature to decrease. This behavior is even true in the absence of *Mn*, which is represented in Fig-2 as dotted line.

The graph for temperature distribution $\theta(\eta)$ Vs. distance η from the sheet for different values of Eckert number is plotted in Fig-3 The analysis of the graph reveals that the effect of increasing the values Eckert number is to increase temperature distribution $\theta(\eta)$ in the flow region. This is due to the fact that heat energy is stored in the fluid due to frictional heating. This behavior is even true in the absence of *Mn*., which is represent in Fig-3 as dotted line.

Fig-4 represents the temperature profile $\theta(\eta)$ *vs.* η for different values of *Pr*. we infer from this figure that the temperature profile decreases with increase in Prandtl number *(Pr).* This is because of the fact that the thermal boundary layer thickness decreases with increase in Prandtl number *(Pr).* This behavior is even true in the absence of *Mn.*, which is represented in Fig-3 as dotted line.

The graphs for the situation when the boundary has been prescribed with heat flux (PHF) are shown in fig.5-8 for both in presence and absence of magnetic field *Mn*. It is noticed from these figures that the wall temperature $g(\eta)$ is not unity; it is changed at the wall with the change of physical parameters like Visco-elastic parameter (k_1), heat source/sink (β), Eckert number (*Ec*) and Prandl number (*Pr*) have same qualitative effects as those we found in PST case but quantitatively wall temperature is more in PHF case.

VI. Summaries and Conclusion:

This work presents heat transfer analysis in a boundary-layer flow of an electrically conducting visco-elastic fluid under the influence of transverse magnetic field over a linearly stretching non-isothermal flat sheet. Effects due to dissipation, work done due to deformation and heat generation/absorption are considered. The wall is assumed to be non-isothermal with a temperature distribution, in one case, and a prescribed wall heat flux, in the other case, varying quadratically with the distance. The mathematical problem has been solved numerically by Runge-Kutta fourth order method.

The important findings of our study are as follows:

1. The effect of magnetic parameter is to decrease the horizontal velocity in the boundary layer.
2. The effect of magnetic parameter is to increase temperature profile in both cases of prescribed surface temperature and prescribed wall heat flux.
3. In case of prescribed surface temperature, wall temperature remains constant at unity with the change of non-dimensional physical parameters. Whereas in prescribed wall heat flux wall temperature changes with the change of non-dimensional parameters.
4. The effect of Eckert number is to increase temperature distribution in the flow region, due to the fact that heat energy is stored in the fluid due to frictional heating in both the cases of PST and PHF.
5. The effect of Prandtl number is to decrease the thickness of the thermal boundary layer in both the cases of PST and PHF.
6. The effect of magnetic parameter is to decrease the temperature gradient in both the cases of PST and PHF.

Nomenclature:

x , y	Distances along and perpendicular to the surface, respectively
u , v	Components of velocities along and perpendicular to the surface, respectively
k_1	Visco-elastic parameter
C_p	Specific heat at constant pressure
k	Thermal conductivity

Pr	Prandtl number
T	Temperature

Greek symbols:

η	Dimensionless normal distance
θ	Dimensionless temperature
ρ	Fluid density
ν	Fluid kinematics viscosity

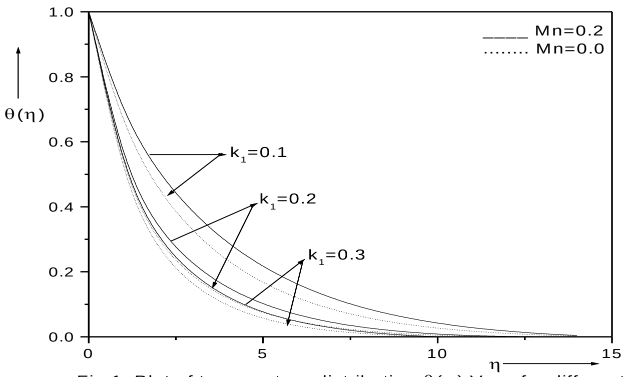

Fig 1: Plot of temperature distribution $\theta(\eta)$ Vs.η for different values of visc0-elastic parameter k_1,when β=0.05, Pr=1.0 and E_c=.02

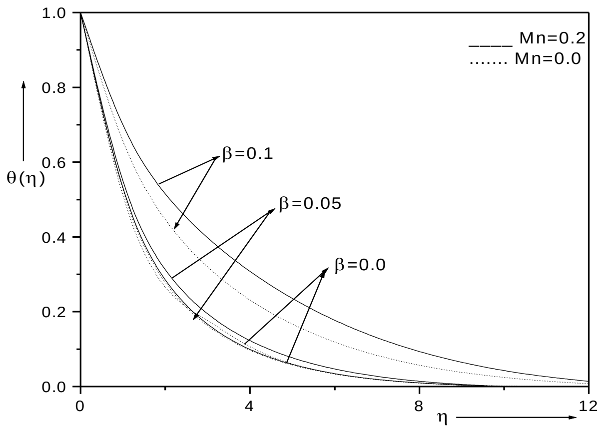

Fig 2: Plot of temperature distribution $\theta(\eta)$ Vs.η for different values of heat source/sink parameter β,whenk$_1$=0.1, Pr=1.0 and E_c=.02

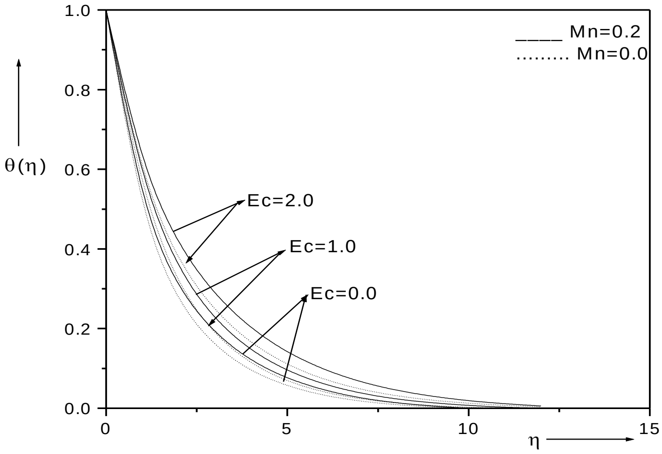

Fig 3: Plot of temperature distribution $\theta(\eta)$ Vs.η for different values of Eckert number Ec,when k_1=0.1,β=0.05 and Pr=1.0

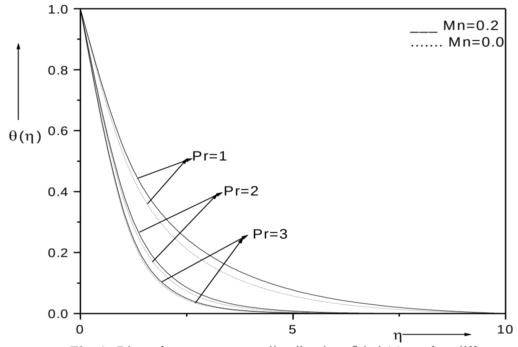

Fig 4: Plot of temperature distribution $\theta(\eta)$ Vs.η for different values of Pr,when k_1=0.1,β=0.05 and E_c=.02

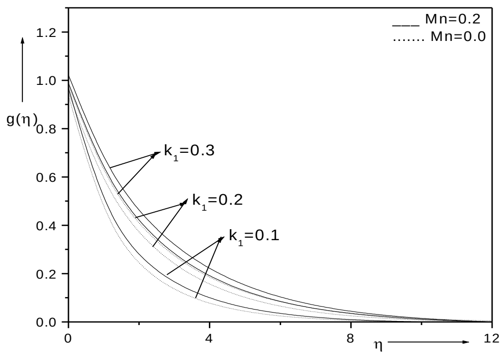

Fig 5: Plot of temperature distribution $g(\eta)$ Vs.η for different values of visc-elastic parameter k_1,when β=0.01, Pr=1.0 and E_c=.02(PHFcase)

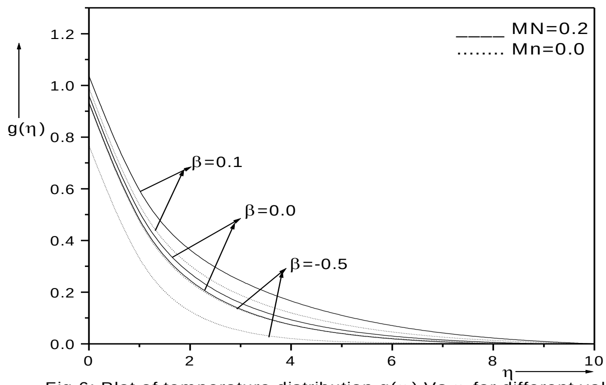

Fig 6: Plot of temperature distribution g(η) Vs.η for different values of heat source/sink parameter β,when k_1=0.1, Pr=1.0 and E_c=.02(PHFcase)

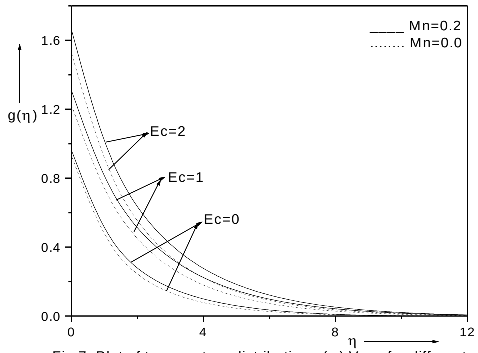

Fig 7: Plot of temperature distribution g(η) Vs.η for different values of Eckert number Ec,when k_1=0.1,β=0.01 and Pr=1.0 (PHFcase)

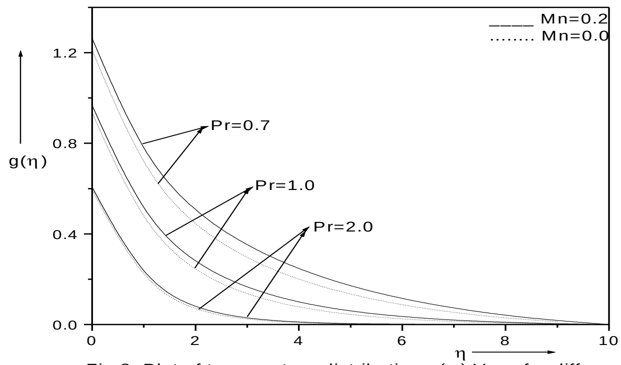

Fig 8: Plot of temperature distribution g(η) Vs.η for different values of Pr,when k_1=0.1,β=0.01 and E_c=.02

References:

[1] Sakiadis.B.C: Boundary layer behavior on continuous solid surface. A.I.Ch. E.J7. (1961a) 26-28.
[2] Sakiadis.B.C: Boundary layer behaviour on continuous flat surfaces. A.I.Ch. E.J.7 (1961b) 221-225
[3] Crane.L.J: Flow past a stretching plate.ZAMP.21 (1970) 645-647
[4] Gupta.P.S, Gupta.A.S: Heat and mass transfer on a stretching sheet with suction and blowing. Canad. J.chem. Engg.55 (1977) 744-746.
[5] Rajgopal.K.R, Na.T.Y, Gupta.A.S: Flow of a visco-elastic fluid over a stretching sheet.
[6] Rheol.Acta 23(1984)213-215.
[7] Vajravelu.K, Rollins.D: Heat transfer in electrically conducting fluid over stretching surface, Int.J.Non- linear Mech , 27(1992)265-277
[8] Sarme.M.S, Rao.B.N: Heat transfer in a visco-elastic fluid over a stretching sheet, J.Math.Anal.Appl.222 (1998) 268-275.
[9] Vajravelu.K, Roper.T: Flow and heat transfer in a second grade fluid over a stretching sheet, Int.J.Non- linear Mech , 34(1999)1031-1036