Uniform Order Continuous Block Hybrid Method for the Solution of First Order Ordinary Differential Equations

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Abstract: We know that for any numerical method to be efficient and computational reliable, it must be convergent, consistent, and stable. This paper adopted the method of interpolation of the approximate solution and collocation of its differential system at grid and off grid points to yield a continuous linear multistep method with a constant step size. The continuous linear multistep method is solved for the independent solution to yield a continuous block method which is evaluated at selected grid and off grid points to yield a discrete block method. The basic property of this method is verified to be convergent consistent and satisfies the conditions for stability. The method was tested on numerical examples and found to compete favorably with the existing methods in term of accuracy and error variation.

Keywords: interpolation, IVP, ODEs, colocation, approximate solution, independent solution, block method, convergent.

I. INTRODUCTION

It has been established that a given linear or non-linear equations does not have a complete solution that can be expressed in terms of a finite number of elementary functions (Ross, 1964; Humi and Miller, 1988). It has also been established that such problems could be solved by seeking an approximate solution by adopting interpolation and collocation method.

In this paper, we consider a numerical method for solving first order initial value problems of the form

\[
y' = f(x, y), \quad y(x_0) = y_0
\]


In this paper, we propose a continuous block method which when evaluated at selected grid points gives a discrete block which the authors mentioned above had proposed. The continuous block possesses the same properties as the continuous linear multistep method. This paper is partitioned into sections as follows: Section two is methodology involved in deriving the continuous multistep method and the continuous block method. Section three considers the analysis of the block method viz; the order, zero stability and the region of absolute stability. Section four considers the numerical examples where we test our method on first order ordinary differential equation and compare our result with existing methods.

II. METHODOLOGY

Consider a monomial power approximate solution in the form

\[
y(x) = \sum_{j=0}^{s+r-1} a_j x^j
\]

where \( r \) and \( s \) are interpolation and collocation points respectively. The first derivative of (2) gives

\[
y'(x) = \sum_{j=0}^{s+r-1} ja_j x^{j-1}
\]

Substituting (3) into (1) gives

\[
f(x, y) = \sum_{j=0}^{s+r-1} ja_j x^{j-1}
\]
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collocating (4) at \( x_{n+p}, s = 0(\frac{1}{12}) \) and interpolating (2) at \( x_n \) gives and a system of non-linear equation in the form

\[ AX = U \quad (5) \]

where

\[ A = [a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}]^T \]

\[ X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 & x_n^{10} & x_n^{11} & x_n^{12} & x_n^{13} \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 & 10x_n^9 & 11x_n^{10} & 12x_n^{11} & 13x_n^{12} \\ 0 & 1 & 2x_n^{1/2} & 3x_n & 4x_n^{3/2} & 5x_n^2 & 6x_n^{5/2} & 7x_n^3 & 8x_n^{7/2} & 9x_n^4 & 10x_n^{9/2} & 11x_n^5 & 12x_n^{11/2} & 13x_n^6 \\ 0 & 1 & 2x_n^{3/2} & 3x_n^2 & 4x_n^{5/2} & 5x_n^3 & 6x_n^{7/2} & 7x_n^4 & 8x_n^{9/2} & 9x_n^5 & 10x_n^{11/2} & 11x_n^6 & 12x_n^7 & 13x_n^8 \\ 0 & 1 & 2x_n^2 & 3x_n & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 & 10x_n^9 & 11x_n^{10} & 12x_n^{11} & 13x_n^{12} \\ 0 & 1 & 2x_n^3 & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 & 10x_n^9 & 11x_n^{10} & 12x_n^{11} & 13x_n^{12} \end{bmatrix} \]

Solving (5) for the \( a_i \) and substituting back into (2) gives a continuous multistep method in the form

\[ y(x) = a_0y_n + h \sum_{j=0}^{13} \beta_j f(x_{n+j}) \quad (6) \]

Where \( a_0 = 1 \) and the coefficients of \( f(x_{n+j}) \) gives

\[ \beta_0 = \frac{b}{63063000} \]

\[ \beta_1 = \left( \frac{1}{1576575} \right) \]

\[ \beta_2 = \left( \frac{1}{875875} \right) \]

\[ \beta_3 = \left( \frac{1}{49628858880} \right) \]

\[ \beta_4 = \left( \frac{1}{1401400} \right) \]

\[ \beta_5 = \left( \frac{1}{875875} \right) \]

\[ \beta_6 = \left( \frac{1}{49628858880} \right) \]

\[ \beta_7 = \left( \frac{1}{1401400} \right) \]

\[ \beta_8 = \left( \frac{1}{875875} \right) \]

\[ \beta_9 = \left( \frac{1}{49628858880} \right) \]

\[ \beta_{10} = \left( \frac{1}{1576575} \right) \]

\[ \beta_{11} = \left( \frac{1}{49628858880} \right) \]

\[ \beta_{12} = \left( \frac{1}{1401400} \right) \]

\[ \beta_{13} = \left( \frac{1}{3753755} \right) \]

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\[ \beta_{i+} = \left( \frac{1}{1401400} \right) \left( 2093.257717760 \alpha^{12} - 636.506056920 \alpha^{11} + 1809.85446300 \alpha^{10} - 3006.156514080 \alpha^9 + 32370.33739560 \alpha^8 - 2368.6439016240 \alpha^7 + 12002.205758400 \alpha^6 - 4208.405401200 \alpha^5 + 1001.0304761760 \alpha^4 + 154.528133760 \alpha^3 + 14.081667600 \alpha^2 - 5945.94000 \alpha \right) \]

\[ \beta_{i} = \left( \frac{1}{1575657} \right) \left( 496.628858800 \alpha^{12} - 336.259123200 \alpha^{11} + 1015.763454720 \alpha^{10} - 619.46452800 \alpha^9 - 601.53029800 \alpha^8 + 471.062312960 \alpha^7 - 408.497537120 \alpha^6 + 2526.651372960 \alpha^5 - 1125.143019000 \alpha^4 + 358.83924320 \alpha^3 - 80.420967550 \alpha^2 + 12.206383280 \alpha - 1174.1866500 \alpha + 63063000 \right) \]

\[ \beta_{i} = \left( \frac{1}{1575657} \right) \left( 496.628858800 \alpha^{12} - 336.259123200 \alpha^{11} + 1015.763454720 \alpha^{10} - 619.46452800 \alpha^9 - 601.53029800 \alpha^8 + 471.062312960 \alpha^7 - 408.497537120 \alpha^6 + 2526.651372960 \alpha^5 - 1125.143019000 \alpha^4 + 358.83924320 \alpha^3 - 80.420967550 \alpha^2 + 12.206383280 \alpha - 1174.1866500 \alpha + 63063000 \right) \]

\[ \beta_{i} = \left( \frac{1}{1575657} \right) \left( 496.628858800 \alpha^{12} - 336.259123200 \alpha^{11} + 1015.763454720 \alpha^{10} - 619.46452800 \alpha^9 - 601.53029800 \alpha^8 + 471.062312960 \alpha^7 - 408.497537120 \alpha^6 + 2526.651372960 \alpha^5 - 1125.143019000 \alpha^4 + 358.83924320 \alpha^3 - 80.420967550 \alpha^2 + 12.206383280 \alpha - 1174.1866500 \alpha + 63063000 \right) \]

Where \( t = \frac{x-s}{\alpha} \). Solving (6) for the independent solution gives a continuous block method in the form

\[ y_{n+k} = \sum_{j=0}^{\mu} \left( \frac{(jh)^m}{m!} \right) y_{n+k}^m + \sum_{j=0}^{h^2} \sigma_j(x)f_{n+j} \]  

(7)

Where \( \mu \) is the order of the differential equation, \( s \) is the collocation points. Hence the coefficient of \( f_{n+j} \) in (7)

\[ \sigma_0 = \left( \frac{b}{a} \right) \left( 90.296151660 \alpha^{12} - 635.835432960 \alpha^{11} + 2013.74887104 \alpha^{10} - 3788.519454720 \alpha^9 + 471.062312960 \alpha^8 - 408.497537120 \alpha^7 + 2526.651372960 \alpha^6 - 1125.143019000 \alpha^5 + 358.83924320 \alpha^4 - 80.420967550 \alpha^3 + 12.206383280 \alpha^2 - 1174.1866500 \alpha + 63063000 \right) \]

\[ \sigma_1 = \left( \frac{1}{1401400} \right) \left( 15.049359360 \alpha^{12} - 91.02772240 \alpha^{11} + 244.55208960 \alpha^{10} - 384.829885440 \alpha^9 + 393.720687360 \alpha^8 - 274.751530680 \alpha^7 + 133.30128400 \alpha^6 - 44.965520600 \alpha^5 + 10.3437749160 \alpha^4 - 155.14699200 \alpha^3 + 138.1562000 \alpha^2 - 573.30000 \alpha \right) \]

\[ \sigma_2 = \left( \frac{1}{63063000} \right) \left( 90.296151660 \alpha^{12} - 635.835432960 \alpha^{11} + 2013.74887104 \alpha^{10} - 3788.519454720 \alpha^9 + 471.062312960 \alpha^8 - 408.497537120 \alpha^7 + 2526.651372960 \alpha^6 - 1125.143019000 \alpha^5 + 358.83924320 \alpha^4 - 80.420967550 \alpha^3 + 12.206383280 \alpha^2 - 1174.1866500 \alpha + 63063000 \right) \]

\[ \sigma_3 = \left( \frac{1}{63063000} \right) \left( 90.296151660 \alpha^{12} - 635.835432960 \alpha^{11} + 2013.74887104 \alpha^{10} - 3788.519454720 \alpha^9 + 471.062312960 \alpha^8 - 408.497537120 \alpha^7 + 2526.651372960 \alpha^6 - 1125.143019000 \alpha^5 + 358.83924320 \alpha^4 - 80.420967550 \alpha^3 + 12.206383280 \alpha^2 - 1174.1866500 \alpha + 63063000 \right) \]

\[ \sigma_4 = \left( \frac{1}{63063000} \right) \left( 90.296151660 \alpha^{12} - 635.835432960 \alpha^{11} + 2013.74887104 \alpha^{10} - 3788.519454720 \alpha^9 + 471.062312960 \alpha^8 - 408.497537120 \alpha^7 + 2526.651372960 \alpha^6 - 1125.143019000 \alpha^5 + 358.83924320 \alpha^4 - 80.420967550 \alpha^3 + 12.206383280 \alpha^2 - 1174.1866500 \alpha + 63063000 \right) \]

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\[ \sigma_{ij} = \left( \frac{1}{875875} \right) \left( \begin{array}{c} 993 \times 257717760t^{ij} - 6366 \times 506065920t^{ij} + 18096 \times 854630400t^{ij} - 30061 \times 65614080t^0 + 32370 \times 337359360t^0 - 23686 \times 439016240t^0 + 12002 \times 205758400t^0 - 4208 \times 405401200t^0 + 1001 \times 300476176t^0 - 154 \times 528133760t^0 + 14 \times 081667600t^0 - 594594000t^0 \end{array} \right) \]

\[ \sigma_{ij} = \left( \frac{1}{1401400} \right) \left( \begin{array}{c} 993 \times 257717760t^{ij} - 6276 \times 836966400t^{ij} + 17583 \times 295242240t^{ij} - 28779 \times 297490944t^0 + 30536 \times 687301120t^0 - 22026 \times 957432480t^0 + 11910 \times 721344480t^0 - 3812 \times 678829960t^0 + 897 \times 0411930486t^0 - 137 \times 108736765t^0 + 12 \times 395783400t^0 - 520269755t^0 \end{array} \right) \]

\[ \sigma_{ij} = \left( \frac{1}{1576575} \right) \left( \begin{array}{c} 496 \times 628858880t^{ij} - 3093 \times 583933440t^{ij} + 8543 \times 019663360t^{ij} - 13788 \times 865778688t^0 + 14436 \times 621239040t^0 - 10283 \times 837283720t^0 + 5081 \times 864879520t^0 - 1741 \times 654834920t^0 + 406 \times 102777000t^0 - 61 \times 600178640t^0 + 5534929400t^0 - 231231000t^0 \end{array} \right) \]

\[ \sigma_{ij} = \left( \frac{1}{875875} \right) \left( \begin{array}{c} 82 \times 771476480t^{ij} - 508 \times 124897280t^{ij} + 1383 \times 757240320t^{ij} - 2204 \times 365363200t^0 + 2280 \times 144746880t^0 - 1606 \times 487042160t^0 + 796 \times 109432660t^0 - 267 \times 108041200t^0 + 61 \times 823980218t^0 - 93202759665t^0 + 833322490t^0 - 34684650t^0 \end{array} \right) \]

\[ \sigma_{ij} = \left( \frac{1}{63063000} \right) \left( \begin{array}{c} 15 \times 0499359360t^{ij} - 91 \times 0277722240t^{ij} + 244 \times 552089600t^{ij} - 384 \times 829885440t^0 + 393 \times 720687360t^0 - 274 \times 725130680t^0 + 133 \times 301282400t^0 - 44 \times 965520600t^0 + 10 \times 343749416t^0 - 1551469920t^0 + 138156200t^0 - 5733000t^0 \end{array} \right) \]

\[ \sigma_{ij} = \left( \frac{1}{63063000} \right) \left( \begin{array}{c} 90 \times 296156160t^{ij} - 538 \times 014597120t^{ij} + 1426 \times 553856000t^{ij} - 2219 \times 310213120t^0 + 2248 \times 162076160t^0 - 1555 \times 315201440t^0 + 749 \times 148285600t^0 - 251 \times 1366858000t^0 + 57 \times 477090496t^0 - 8583459885t^0 + 761770100t^0 - 31531500t^0 \end{array} \right) \]

where \( t = \frac{x-t_a}{h} \). Evaluating (7) at \( t = \frac{1}{12} \left( \frac{1}{12} \right) \) gives a discrete block formula of the form

\[ Y_m = e y_n + hd(y_n) + hd(y_m) \quad (B) \]

where \( e, d \), are \( r \times r \) matrix

\[ d = \begin{bmatrix} 703.604254357 & 538990963 & 2846527447 & 337524401 & 337524401 & 22226233 \\ 31384184832000 & 245188944000 & 2846527447 & 337524401 & 251073478656 & 1009080000 \\ 14.110554661 & 42194069 & 316182879 & 431889735 & 62.984859487 & 1364651 \end{bmatrix} \]

\[ \begin{bmatrix} 640.493568000 & 19.15538625 & 14.350336000 & 196.1511572853 & 10771200063063000 \end{bmatrix} \]

Where

\[ Y_m = \begin{bmatrix} y_{n+1/12} & y_{n+1/6} & y_{n+1/4} & y_{n+1/3} & y_{n+1/2} & y_n & y_{n+1/2} & y_{n+1/4} & y_{n+1/6} & y_{n+1/12} \end{bmatrix}^T \]

\[ e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
### III. ANALYSIS OF THE BASIC PROPERTIES OF THE NEW BLOCK METHOD

**ORDER OF THE METHOD**

Let the linear operator \( L(y; \gamma) \) associated with the block formula be defined as

\[
L(y; \gamma) = A^{(0)} y - c_n h^n f(y_n) - h^\rho b F(y_m) \quad (9)
\]

expanding in Taylor series expansion and comparing the coefficient of \( h \) gives

\[
L(y; \gamma) = c_0 y + c_1 h y + c_2 h^2 y + \ldots + c_\rho h^\rho y + c_{\rho+1} h^{\rho+1} y + \ldots + c_{\rho+2} h^{\rho+2} y^2 + \ldots \quad (10)
\]

**Definition:**

The linear operator \( L \) and the associated continuous linear multistep method (9) are said to be of order \( p \) if

\[
c_0 = c_1 = c_2 = \ldots = c_p = 0 \quad \text{and} \quad c_{p+1} \neq 0\]

is called the error constant and implies that the local truncation error is given by \( t_{n+1} = C_{p+1} h^{p+1} \rho_{p+1}(X_1) + O(h^{p+2}) \). For our method, expanding in Taylor series expansion gives

\[
L(y; \gamma) \quad \text{zero expansion to zero yield a constant order } 13 \text{ with the following error constants}
\]

\[
c_0 = c_1 = \ldots = c_{12} = 0, c_{14} = \left[ -2.659(-11) - 2.508(-11) - 2.492(-11) - 2.495(-11) - 2.494(-11) - 2.494(-11) - 2.498(-11) - 2.487(-11) - 2.458(-11) - 2.336(-11) \right]^T
\]

**Zero Stability**

**Definition:** The block (8) is said to be zero stable, if the roots \( Z_s, s = 1, 2, \ldots, N \) of the characteristic polynomial \( \rho(z) \) determined by \( \rho(z) = det(A^{(0)} - E) \) satisfies \( |z| \leq 1 \) and every root satisfying \( |z| \leq 1 \) have multiplicity not exceeding the order of the differential equation. Moreover \( \rho = 0 \), \( \rho(z) = z^r \rho(z - 1)^w \) where \( w \) is the order of the differential equation, \( r \) is the order of the matrix \( A^{(0)} \) and \( E \) (see Awoyemi et al.[6] for details).

For our method

\[
\rho(z) = z^{12}(z - 1) \]

Since \( \rho(z) = z^{12}(z - 1) \) gives roots that lie within 0 and 1, hence our method is zero stable.
Notation used in the table
ERB→Error in Badmus and Mishelia (2012)
ERA→Error in Areo et al. (2012)

**Problem 1**
We consider a linear first order ordinary differential equation
\[ y' = x - y, y(0) = 0, 0 \leq x \leq 1, h = 0.1 \]
Exact solution \( y(x) = x + e^{-x} - 1 \)
This problem was solved by Areo et al. (2011) using block method of order seven. They adopted classical RungeKutta method to provide the starting values. The result is shown in Table 1

<table>
<thead>
<tr>
<th>x</th>
<th>Exact Result</th>
<th>Computed Result</th>
<th>Error in our method</th>
<th>ERA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.004837418035959</td>
<td>0.004837418055555</td>
<td>1.9595x10^-11 (-1)</td>
<td>0.000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.01873075307798</td>
<td>0.01873075311344</td>
<td>3.54623x10^-11 (-1)</td>
<td>0.000</td>
</tr>
<tr>
<td>0.3</td>
<td>0.04081822068171</td>
<td>0.04081822072989</td>
<td>4.8131x10^-11 (-1)</td>
<td>6.0x10^-10</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0703204603563</td>
<td>0.07032046049377</td>
<td>5.8068x10^-11 (-1)</td>
<td>2.0x10^-10</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10653065971263</td>
<td>0.10653065977831</td>
<td>6.5677x10^-11 (-1)</td>
<td>7.0x10^-10</td>
</tr>
<tr>
<td>0.6</td>
<td>0.14881163615352</td>
<td>0.14881163618553</td>
<td>7.1313x10^-11 (-1)</td>
<td>1.0x10^-10</td>
</tr>
<tr>
<td>0.7</td>
<td>0.19655303866690</td>
<td>0.19655303866690</td>
<td>7.5281x10^-11 (-1)</td>
<td>8.0x10^-10</td>
</tr>
<tr>
<td>0.8</td>
<td>0.24932896411722</td>
<td>0.24932896419507</td>
<td>7.7848x10^-11 (-1)</td>
<td>2.0x10^-10</td>
</tr>
<tr>
<td>0.9</td>
<td>0.30656969540597</td>
<td>0.30656969581984</td>
<td>7.9240x10^-11 (-1)</td>
<td>9.0x10^-10</td>
</tr>
<tr>
<td>1.0</td>
<td>0.36787944117144</td>
<td>0.367879441251113</td>
<td>7.9671x10^-11 (-1)</td>
<td>4.0x10^-10</td>
</tr>
</tbody>
</table>

**Problem 2**
\( y' = xy, y(0) = 1, h = 0.1 \)
Exact solution: \( y(x) = e^{x^2} \)
This problem was solved by Badmus and Mishelia (2011) using self-starting block method of order six, the result is shown in Table 2

<table>
<thead>
<tr>
<th>x</th>
<th>Exact Result</th>
<th>Computed Result</th>
<th>Error in our method</th>
<th>ERB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.00501252085940</td>
<td>1.00012520833553</td>
<td>2.6067x10^-11 (-1)</td>
<td>5.29x10^-7</td>
</tr>
<tr>
<td>0.2</td>
<td>1.02020134002675</td>
<td>1.02020133994199</td>
<td>8.4790x10^-11 (-1)</td>
<td>1.77x10^-7</td>
</tr>
<tr>
<td>0.3</td>
<td>1.04602785909871</td>
<td>1.04602785972213</td>
<td>1.8684x10^-11 (-10)</td>
<td>8.99x10^-7</td>
</tr>
<tr>
<td>0.4</td>
<td>1.08327067649581</td>
<td>1.08327067632539</td>
<td>3.5701x10^-11 (-10)</td>
<td>3.09x10^-6</td>
</tr>
<tr>
<td>0.5</td>
<td>1.13314845306682</td>
<td>1.13314832456272</td>
<td>6.1054x10^-10 (-9)</td>
<td>1.91x10^-6</td>
</tr>
<tr>
<td>0.6</td>
<td>1.19721736312118</td>
<td>1.19721736210600</td>
<td>1.0157x10^-9 (-9)</td>
<td>4.48x10^-6</td>
</tr>
<tr>
<td>0.7</td>
<td>1.27762131320488</td>
<td>1.27762131156033</td>
<td>1.6445x10^-9 (-9)</td>
<td>1.02x10^-5</td>
</tr>
<tr>
<td>0.8</td>
<td>1.37712776433595</td>
<td>1.37712776417200</td>
<td>2.6158x10^-9 (-9)</td>
<td>7.74x10^-6</td>
</tr>
<tr>
<td>0.9</td>
<td>1.49930250056767</td>
<td>1.49930249594572</td>
<td>4.1110x10^-9 (-9)</td>
<td>1.44x10^-5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.64872126429389</td>
<td>1.64872126429389</td>
<td>6.4070x10^-9 (-9)</td>
<td>2.93x10^-5</td>
</tr>
</tbody>
</table>

**V. DISCUSSION OF THE RESULT**
We have considered two numerical examples to test the efficiency of our method. Problem 1 was solved by Areo et al. (2012). They proposed a hybrid method of order seven and adopted classical RungeKutta method to provide the starting values. The new method gave better approximation because the proposed method is self-starting and does not require starting values. Problem 2 was solved by Badmus and Mishelia (2012). They adopted self-starting block methods of order six. Our method gave better approximation because the iteration per step in the new method was lower than the method proposed by Badmus and Mishelia (2012)

**VI. CONCLUSION**
We have proposed an order seven continuous hybrid method for the solution of first order ordinary differential equations. Our method was found to be zero stable, consistent and converges. The numerical examples show that our method gave better accuracy than the existing methods.

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