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Oscillatory Unsteady Hydrodynamic Viscoelastic Flow in a Porous Channel with Radiative Heat Transfer

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Abstract: *This analysis examines the problem of oscillatory flow of a viscoelastic fluid and heat transfer along a porous oscillating channel with radiative heat transfer. Here we consider the flow through a channel in which the fluid is injected on one boundary of the channel with a constant velocity, while it is sucked off at the other boundary with the same velocity. The two boundaries are considered to be in close contact with two plates parallel to each other. The plates are supposed to be oscillating with a given velocity in their own planes. The analytical expressions for the velocity, the temperature and the wall shear stress have been obtained. The effects of viscoelastic parameter on the velocity profile, shear stress are presented graphically with the combinations of the other flow parameters. It is also observed that the temperature field is not significantly affected by the viscoelastic parameter.*
Keywords: *Viscoelastic fluid, radiative heat transfer, porous wall, oscillating channel.*

I. INTRODUCTION

The problem of hydrodynamic flow in a porous channel with radiative heat transfer received much attention because of its various applications in physiology and in engineering devices such as blood flow in arteries, transpiration cooling of re-entry vehicles and rocket busters, cross-hatching on ablative surfaces. Pulsatile flow of a fluid in a porous channel has been investigated by Wang [1], as well as Bhuyan and Hazarika [2] by considering the periodic pressure gradient. Raptis [3] studied the unsteady free convective flow through a porous medium bounded by an infinite vertical limiting surface with constant suction and time dependent temperature. The effect of Hall current and wall temperature oscillation on convective flow in a rotating fluid through porous medium was studied by Ram [4]. On the other hand, several other researchers (e.g. Makinde and Mhone [5], Prakash and Ogulu [6] as well as Mehmood and Ali [7]) investigated the effects of heat transfer in the flow of fluids. Adhikary and Misra [8] investigated the effects of porosity of the channel wall, magnetic field and radiative heat transfer on unsteady flow of an electrically conducting fluid through a channel. Ghosh [9] investigated the hydrodynamic fluctuating flow of a viscoelastic fluid in a porous channel, where the channels oscillate with a given velocity in their own planes.

The aim of the present work is to investigate the effects of non-Newtonian parameter on the unsteady two dimensional hydrodynamic flow and heat transfer of a viscoelastic fluid. One of the most popular models for non-Newtonian fluids is the model that is called the second-order fluid or fluid of second grade. It is reasonable to use the second-order fluid model to do numerical calculations. The effects of visco-elastic parameter with the combinations of the other flow parameters have been studied thoroughly and presented graphically. The constitutive equation for the incompressible second-order fluid is

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1)$$

where S is the stress tensor, p is the hydrostatic pressure, $A_n, n = 1,2$ are the kinematic Rivlin-Ericksen tensors, μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively, where μ_1 and μ_3 are positive and μ_2 is negative (Coleman and Markovitz [10]). The equation (1) was derived by Coleman and Noll [11] from that of the simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in the distant past.

II. MATHEMATICAL FORMULATIONS

Consider the channel between two oscillating porous plate $y = 0$ and $y = h,$ the fluid is being injected by one plate with constant velocity V and sucked off by the other plate with the same velocity. Then the continuity equation reduces to $\frac{\partial u^*}{\partial x^*} = 0$ so that u^* is the function of y^* and t^* only.

The momentum equations are given by

$$\rho\left(\frac{\partial u^*}{\partial t^*}+V\frac{\partial u^*}{\partial y^*}\right)=-\frac{\partial p^*}{\partial x^*}+\mu_1\frac{\partial^2 u^*}{\partial y^{*2}}+\mu_2\left(\frac{\partial^3 u^*}{\partial y^{*2}\partial t^*}+V\frac{\partial^3 u^*}{\partial y^{*3}}\right)-\frac{\mu_1 u^*}{k^*}+g\rho\beta(T-T_0) \qquad (2)$$

$$0=-\frac{\partial p^*}{\partial y^*}+(2\mu_2+\mu_3)\frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)^2$$ so that

$0=-\frac{\partial p^*}{\partial y^*}$, assuming $2\mu_2+\mu_3=0$ as $\mu_2<0$ and $\mu_3>0.$ (3)

The heat transfer equation may be put in the form

$$\frac{\partial T}{\partial t^*}+V\frac{\partial T}{\partial y^*}=\frac{k'}{\rho C_p}\frac{\partial^2 T}{\partial y^{*2}}-\frac{1}{\rho C_p}\frac{\partial q}{\partial y^*} \qquad (4)$$

where p^* is the pressure, ρ the density of the fluid, k permeability factor, q the radiative heat flux, β the coefficient of volume expansion due to temperature, g the gravitational acceleration, k' the coefficient of thermal conductivity, C_p the specific heat at constant pressure. The last term on the right hand side of equation (4) arises owing to the radiation effect of the heat transfer.
The corresponding boundary conditions of the oscillatory motion are:

$$u^*=U_0e^{i\omega^* t^*}, T=T_w+e^{i\omega^* t^*}(T_w-T_0) \quad \text{at} \quad y^*=h$$
$$u^*=U_0e^{i\omega^* t^*}, T=T_0 \quad \text{at} \quad y^*=0 \qquad (5)$$

In these equations, we have taken into account the temperature oscillation on the upper plate $y^*=h$, while the lower plate $y^*=0$ is maintained at the fixed temperature T_0.
The heat flux may be expressed (Cogley et al. [12]) as

$$\frac{\partial q}{\partial y^*}=4\alpha_1^2(T-T_0) \qquad (6)$$

where α_1 is the mean radiation and absorption coefficient.
Introduce the following non-dimensional quantities:

$$y=\frac{y^*}{h},\ x=\frac{x^*}{h},\ u=\frac{u^*}{h},\ \mathrm{Re}=\frac{Vh}{k_1},\ k=\frac{k^*}{h^2\rho},\ p=\frac{hp^*}{\rho\upsilon_1 V},\ \theta=\frac{T-T_0}{T_w-T_0},$$
$$t=\frac{t^*V}{h},\ Gr=\frac{g\beta(T_w-T_0)h^2}{\upsilon_1 V},\ \mathrm{Pr}=\frac{Vh\rho C_p}{k'},\ N^2=\frac{4\alpha^2h^2}{k'},\ \omega=\frac{\omega^* h}{V}. \qquad (7)$$

where Re the Reynolds number, Gr the Grashof number, Pr the Prandtl number, N the radiation Parameter, ω the angular frequency.
The governing equations together with the heat equation can be re-written in terms of dimensionless quantities given in (7) as

$$\mathrm{Re}\left(\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-\frac{u}{k}+Gr\theta+\alpha\left(\frac{\partial^3 u}{\partial y^2\partial t}+\frac{\partial^3 u}{\partial y^3}\right) \qquad (8)$$

$$0=-\frac{\partial p}{\partial y} \qquad (9)$$

and

$$\mathrm{Pr}\left(\frac{\partial\theta}{\partial t}+\frac{\partial\theta}{\partial y}\right)=\frac{\partial^2\theta}{\partial y^2}+N^2\theta \qquad (10)$$

while the boundary conditions will assume the form

$$u=U_0e^{i\omega t}, \theta=1+e^{i\omega t} \quad \text{at} \quad y=1$$
$$u=U_0e^{i\omega t}, \theta=0 \quad \text{at} \quad y=0 \qquad (11)$$

where $\alpha = \dfrac{\mu_2 V}{\mu_1 h}$ is the viscoelastic parameter.

III. METHOD OF SOLUTION

From (8) and (9), it follows that $\dfrac{\partial p}{\partial x}$ is a function of t alone. For the present study, we consider

$$\frac{\partial p}{\partial x} = A + Be^{i\omega t},$$

A and B being undetermined constants. To solve equations (8) and (10) subject to boundary conditions (11), we write the velocity and temperature in temperature in the form:

$$\begin{aligned} u(y,t) &= u_s(y) + u_p(y,t) \\ &= u_s(y) + u_f(y)e^{i\omega t} \end{aligned} \tag{12}$$

and

$$\begin{aligned} \theta(y,t) &= \theta_s(y) + \theta_p(y,t) \\ &= \theta_s(y) + \theta_f(y)e^{i\omega t} \end{aligned} \tag{13}$$

where $u_s(y), u_p(y,t), \theta_s(y),\ \theta_p(y,t)$ respectively represent the steady and unsteady parts of the velocity and temperature.

Substituting the above expressions in (8) and (10) and comparing the like terms, we have derived the equations that govern the corresponding steady and unsteady flow and heat transfer of the problem under consideration. They are given below:

Steady Case:

$$\alpha \frac{d^3 u_s}{dy^3} + \frac{d^2 u_s}{dy^2} - \mathrm{Re}\frac{du_s}{dy} - \frac{u_s}{k} = A - Gr\theta_s \quad , \tag{14}$$

$$\frac{d^2\theta_s}{dy^2} - \mathrm{Pr}\frac{d\theta_s}{dy} + N^2\theta_s = 0 . \tag{15}$$

with the boundary conditions:

$$\begin{aligned} &u_s = 0,\ \theta_s = 1, && \text{at} \quad y = 1 \\ &u_s = 0,\ \theta_s = 0. && \text{at} \quad y = 0 \end{aligned} \tag{16}$$

Unsteady Case:

$$\alpha \frac{d^3 u_f}{dy^3} + (1 + i\alpha\omega)\frac{d^2 u_f}{dy^2} - \mathrm{Re}\frac{du_f}{dy} - (i\omega \mathrm{Re} + \frac{1}{k})u_f = B - Gr\theta_f \quad , \tag{17}$$

$$\frac{d^2\theta_f}{dy^2} - \mathrm{Pr}\frac{d\theta_f}{dy} + \left(N^2 - i\omega \mathrm{Pr}\,\theta_s\right)\theta_f = 0 . \tag{18}$$

with the boundary conditions:

$$\begin{aligned} &u_f = U_0\ ,\ \ \theta_f = 1, && \text{at} \quad y = 1 \\ &u_f = U_0\ ,\ \ \theta_f = 0. && \text{at} \quad y = 0 \end{aligned} \tag{19}$$

On solving the equations (15) and (18) along with the boundary conditions (16) and (19) respectively, are found as

$$\theta_s = \frac{1}{e^{m_1} - e^{m_2}}\left(e^{m_1 y} - e^{m_2 y}\right), \tag{20}$$

$$\theta_f = \frac{e^{i\omega t}}{e^{m_3} - e^{m_4}}\left(e^{m_3 y} - e^{m_4 y}\right) \tag{21}$$

where

$$m_1 = \frac{\Pr+\sqrt{\Pr^2-4N^2}}{2},\ m_2 = \frac{\Pr-\sqrt{\Pr^2-4N^2}}{2},$$

$$m_3 = \frac{\Pr+\sqrt{\Pr^2-4(N^2-i\omega\Pr)}}{2},\ m_4 = \frac{\Pr-\sqrt{\Pr^2-4(N^2-i\omega\Pr)}}{2}.$$

We note that $|\alpha| < 1$ for small shear and so we can assume that

$$u_s(y) = u_{s0}(y) + \alpha u_{s1}(y) + O(\alpha^2),$$

$$u_f(y) = u_{f0}(y) + \alpha u_{f1}(y) + O(\alpha^2) \qquad (22)$$

Substituting (22) in (14) and (17) together with boundary conditions (16) and (19) up to first order of α and equating the co-efficient of like powers of α, we obtain the following sets of ordinary differential equations and corresponding boundary conditions:

$$\frac{d^2u_{s0}}{dy^2} - \mathrm{Re}\frac{du_{s0}}{dy} - \frac{u_{s0}}{k} = A - Gr\theta_s, \qquad (23)$$

$$\frac{d^3u_{s0}}{dy^3} + \frac{d^2u_{s1}}{dy^2} - \mathrm{Re}\frac{du_{s1}}{dy} - \frac{u_s}{k} = 0. \qquad (24)$$

with

$$u_{s0} = u_{s1} = 0, \quad \text{at} \quad y = 1$$

$$u_{s0} = u_{s1} = 0, \quad \text{at} \quad y = 0 \qquad (25)$$

and

$$\frac{d^2u_{f0}}{dy^2} - \mathrm{Re}\frac{du_{f0}}{dy} - (i\omega\mathrm{Re} + \frac{1}{k})u_{f0} = B - Gr\theta_f, \qquad (26)$$

$$\alpha\frac{d^3u_{f0}}{dy^3} + \frac{d^2u_{f1}}{dy^2} - \mathrm{Re}\frac{du_{f1}}{dy} - (i\omega\mathrm{Re} + \frac{1}{k})u_{f1} = -i\,\omega\frac{d^2u_{f0}}{dy^2}. \qquad (27)$$

with

$$u_{f0} = U_0,\ u_{f1} = 0, \quad \text{at} \quad y = 1$$

$$u_{f0} = U_0,\ u_{f1} = 0, \quad \text{at} \quad y = 0 \qquad (28)$$

The equations (23), (24) and (26), (27) are solved under the boundary conditions (25) and (28) respectively. Substituting these solutions in (22), we get the expressions for u_s and u_f, and thus the expression for u but due brevity the solutions are not presented here.

The non-dimensional wall shear stress at the upper plate is given by

$$\tau_w = \mu_1\left(\frac{\partial u}{\partial y}\right) + \mu_2\left(\frac{\partial^2 u}{\partial y \partial t} + v\frac{\partial^2 u}{\partial y^2}\right) \qquad (29)$$

IV. RESULTS AND CONCLUSIONS

The purpose of this study is to bring out the effects of the visco-elastic parameter α on the governing flow and heat transfer characteristics. We have considered the real parts of the results throughout for numerical validation. The effects of viscoelastic parameter on velocity, shear stress and flow rate of viscoelastic fluid are evaluated numerically and the results are presented in figures 1-4, and 5. The predicted variation of velocity with different values of α and for $\mathrm{N}=2, \Pr=2, \mathrm{k}=1$; $\mathrm{N}=3, \Pr=2, \mathrm{k}=1$; $\mathrm{N}=2, \Pr=5, \mathrm{k}=1$; $\mathrm{N}=2, \Pr=2, \mathrm{k}=2$ are shown in figures 1-4 respectively.

It is evident from the figures 1-4 that the velocity profile is parabolic in nature and the values of the velocity u decrease with the increasing values of the viscoelastic parameter $|\alpha|(\alpha = 0, -0.1, -0.2)$ in comparison with Newtonian fluid. It is also noted from the figures that the behaviours of the velocity profiles remain the same with the increasing values of the viscoelastic parameter $|\alpha|$ when (i) The values of N increase

(Fig.1 and Fig.2) and (ii) $\Pr$ increase (Fig.1 and Fig.3) (iii) k increase (Fig.1 and Fig.4). It is also seen that the velocity u increase with the increasing values of the radiative parameter N and the permeability parameter k for both Newtonian and non-Newtonian cases.

The wall shear stresses are calculated from the equation (29). Figure 5 show that the wall shear stress τ_w increases as the values of the viscoelastic parameter $|\alpha|(\alpha = 0, -0.1, -0.2)$ increase in comparison to Newtonian fluid.

It is also observed that the temperature field is not significantly affected by the visco-elastic parameter.

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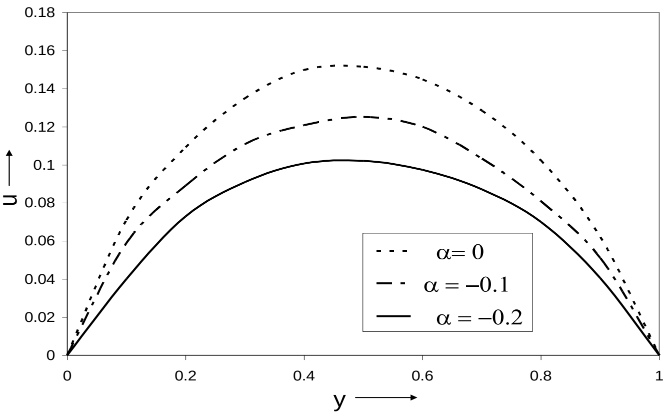

Fig. 1: Variation of u against y when $\mathrm{N} = 2, \Pr = 2, \mathrm{k} = 1, \mathrm{Re} = 1, \mathrm{t} = \frac{\pi}{2}$.

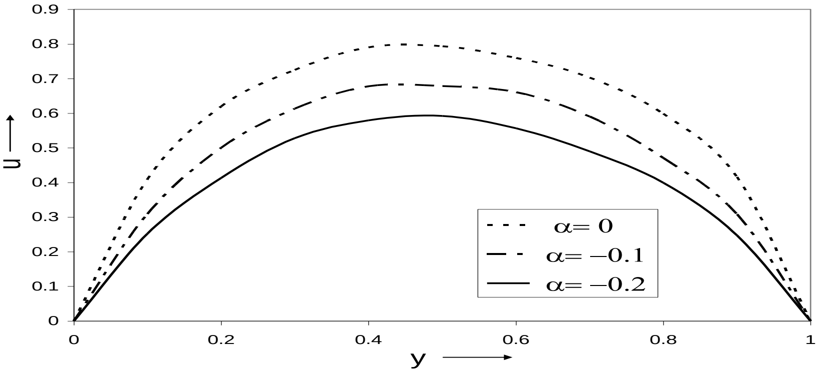

Fig. 2: Variation of u against y when $\mathrm{N} = 3, \Pr = 2, \mathrm{k} = 1, \mathrm{Re} = 1, \mathrm{t} = \frac{\pi}{2}$.

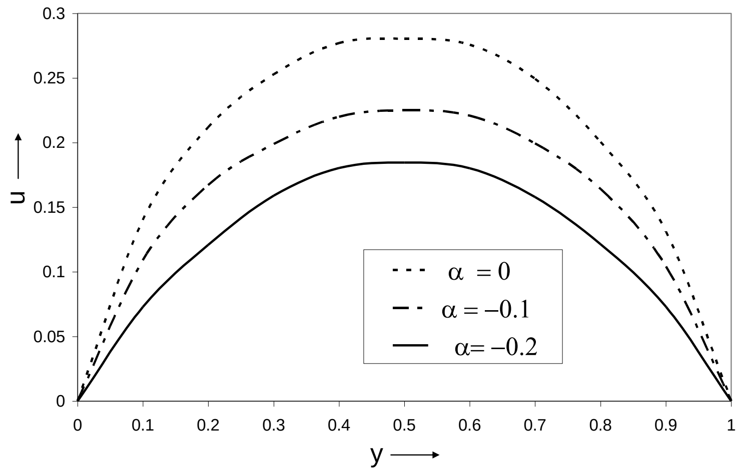

Fig. 3: Variation of u against y when $\mathrm{N} = 2, \mathrm{Pr} = 5, \mathrm{k} = 1, \mathrm{Re} = 1, \mathrm{t} = \frac{\pi}{2}$.

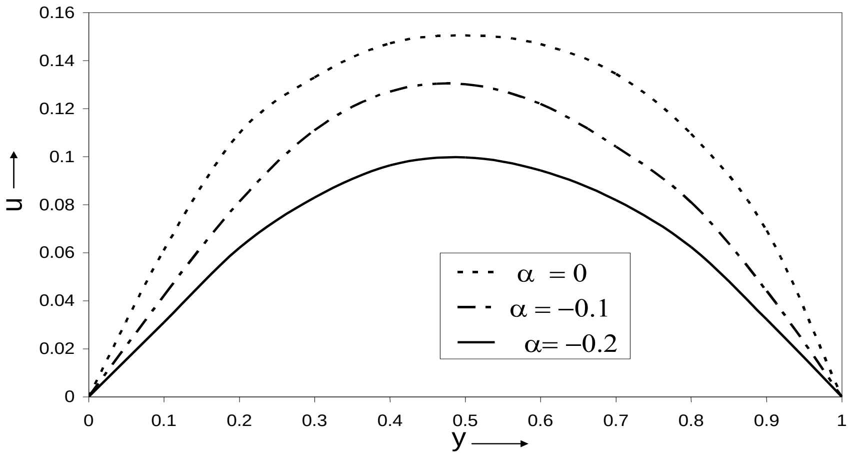

Fig. 4: Variation of u against y when $\mathrm{N} = 2, \mathrm{Pr} = 2, \mathrm{k} = 2, \mathrm{Re} = 1, \mathrm{t} = \frac{\pi}{2}$.

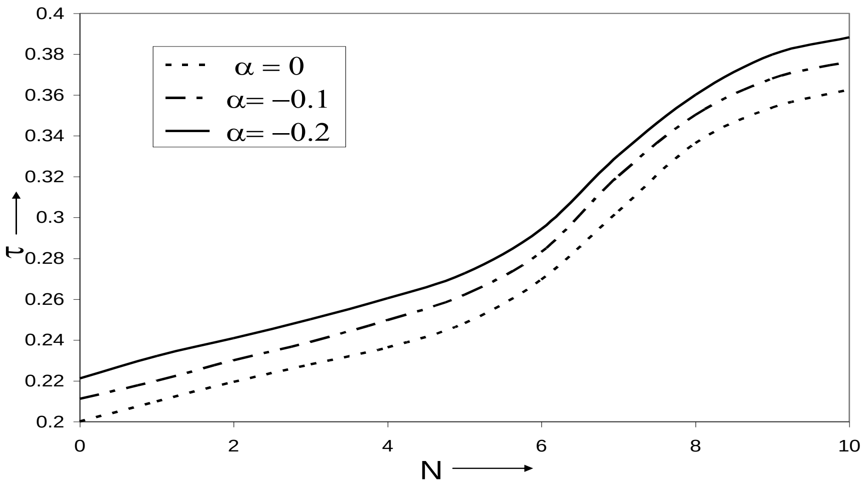

Fig. 5: Wall shear stress versus N when t= π..