IOSR Journal of Mathematics (IOSR-JM)
ISSN: 2278-5728. Volume 3, Issue 3 (Sep-Oct. 2012), PP39-43
www.iosrjournals.org

Common Random Fixed Point theorem for compatible random multivalued operators

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Abstract: The aim of this paper is to prove some common random fixed point theorem for two pairs of compatible random multivalued operators satisfying rational inequality.
Keywords: Random fixed point, Compatible maps, Polish space.
AMS Mathematics Subject Classification (2000): 47H10, 54H25.

I. Introduction

The systematic study of random equations employing the methods of functional analysis was first initiated by Prague School of Probabilistic in 1950's by Spacek [12] and Hans [7,8]. In separable metric space, random fixed point theorems for contraction mappings was proved by Spacek [12] and Hans [7,8]. Bharucha-Reid [6] generalized Mukherjee's [10] result on general probability measure space. For multivalued mappings Itoh [9] obtained random analogues of corresponding deterministic result for different classes of mappings. Papageoriou [11], Beg [2,3], Beg and Shahzad [5] and Beg and Abbas [4] proved some common random fixed point and random coincidence point of a pair of compatible random operators.

Preliminaries: Let (X,d) be a Polish space, that is a separable complete metric space and (Ω, a) be a measurable space. Let 2^X be the family of all subsets of X and CB(X) denote the family of all nonempty bounded closed subsets of X.

A mapping $T: \Omega \to 2^X$, is called measurable, if for any open subset C of X,

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \phi\} \in a.$$

A mapping $\xi : \Omega \to X$ is called measurable selector of a measurable mapping $T: \Omega \to 2^X$, if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$.

A mapping $T: \Omega \times X \to CB(X)$ is called random multivalued operator, if for every $x \in X$, $T(.,x)$ is measurable.

A mapping $f: \Omega \times X \to X$ is called random operator, if for every $x \in X$, $f(.,x)$ is measurable.

A measurable mapping $\xi : \Omega \to X$, is called the random fixed point of a random multivalued operator $T: \Omega \times X \to CB(X)$ ($f: \Omega \times X \to X$), if for every $\omega\in\Omega$, $\xi(\omega) \in T(\omega,\xi(\omega))$ ($f(\omega,\xi(\omega)) = \xi(\omega)$).

A measurable mapping $\xi : \Omega \to X$ is a random coincident point of $T: \Omega \times X \to CB(X)$ and $f: \Omega \times X \to CB(X)$ if for any $\omega \in \Omega$,

$$f(\omega, \xi(\omega)) = T(\omega, \xi(\omega)).$$

Mappings $f, g : X \to X$ are compatible if $\lim_{n\to\infty} d(fg(x_n), gf(x_n)) = 0$, provided that $\lim_{n\to\infty} f(x_n)$ and $\lim_{n\to\infty} g(x_n)$ exists in X and $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n)$.

Random operators $S, T: \Omega \times X \to X$ are compatible if $S(\omega, .)$ and $T(\omega, .)$ are compatible for each $\omega \in \Omega$. (See Beg and Shahzad [5])

Main Result.

Theorem. Let X be a Polish space and let (S, T) and (Q, T) be two pairs of compatible random multivalued operators from $\Omega \times X \to CB(X)$ with $S(\omega, X) \subset T(\omega, X)$ and $Q(\omega, X) \subset T(\omega, X)$ for each $\omega \in \Omega$ and

$$H\big(S(\omega,x),Q(\omega,y)\big) \le \alpha(\omega)\frac{[d(T(\omega,x),S(\omega,x))]^3 + [d(T(\omega,y),Q(\omega,y))]^3}{[d(T(\omega,x),S(\omega,x))]^2 + [d(T(\omega,y),Q(\omega,y))]^2}$$
$$+\beta(\omega)\frac{[d(T(\omega,x),S(\omega,x))]^2 + [d(T(\omega,y),Q(\omega,y))]^2}{[d(T(\omega,x),S(\omega,x))] + [d(T(\omega,y),Q(\omega,y))]}$$
$$+ \gamma(\omega)d(T(\omega, x), T(\omega, y))$$

for each x, y $\in$ X and $\omega \in \Omega$ where $\alpha, \beta, \gamma : \Omega \to (0, 1)$ are measurable mapping such that $\alpha(\omega) + \beta(\omega) + \gamma(\omega) < 1$.

If one of the random multivalued operators S, Q and T is continuous, then S, Q and T have unique common random fixed point. (Here H represents the Hausdorff metric on CB(X) induced by the metric d).

Proof. Let $\xi_0: \Omega \to X$ be an arbitrary measurable mapping and choose a measurable mapping $\xi_1: \Omega \to X$ such that $S(\omega, \xi_0(\omega)) = T(\omega, \xi_1(\omega))$ for each $\omega \in \Omega$.

It further implies that there exists a measurable mapping $\xi_2: \Omega \to X$ such that for any $\omega \in \Omega$

$$Q(\omega, \xi_1(\omega)) = T(\omega, \xi_2(\omega)).$$

In general, we can choose measurable mappings ξ_{2n+1} and ξ_{2n+2} from $\Omega \to X$ such that

$$S(\omega, \xi_{2n}(\omega)) = T(\omega, \xi_{2n+1}(\omega))$$

and

$$Q(\omega, \xi_{2n+1}(\omega)) = T(\omega, \xi_{2n+2}(\omega))$$

for each $\omega \in \Omega$ and $n = 0, 1, 2, \ldots$.

Then for each $\omega \in \Omega$,

$$d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)) = H(S(\omega, \xi_{2n}(\omega)), Q(\omega, \xi_{2n+1}(\omega))$$

$$\leq \alpha(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), S(\omega,\xi_{2n}(\omega)))]^3 + [d(T(\omega,\xi_{2n+1}(\omega)), Q(\omega,\xi_{2n+1}(\omega)))]^3}{[d(T(\omega,\xi_{2n}(\omega)), S(\omega,\xi_{2n}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega)), Q(\omega,\xi_{2n+1}(\omega)))]^2}$$

$$+\beta(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), S(\omega,\xi_{2n}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega)), Q(\omega,\xi_{2n+1}(\omega)))]^2}{[d(T(\omega,\xi_{2n}(\omega)), S(\omega,\xi_{2n}(\omega)))] + [d(T(\omega,\xi_{2n+1}(\omega)), Q(\omega,\xi_{2n+1}(\omega)))]}$$

$$+ \gamma(\omega)d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$\leq \alpha(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega)))]^3 + [d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^3}{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^2}$$

$$+\beta(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^2}{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega)))] + [d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]}$$

$$+ \gamma(\omega)d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$\leq \alpha(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega))) + d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^3}{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega))) + d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^2}$$

$$+\beta(\omega)\frac{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega))) + d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]^2}{[d(T(\omega,\xi_{2n}(\omega)), T(\omega,\xi_{2n+1}(\omega))) + d(T(\omega,\xi_{2n+1}(\omega)), T(\omega,\xi_{2n+2}(\omega)))]}$$

$$+ \gamma(\omega)d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$\leq \alpha(\omega)[d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega))+d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)))]$$

$$+ \beta(\omega)[d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega))) + d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)))]$$

$$+ \gamma(\omega)d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$\leq (\alpha(\omega) + \beta(\omega) + \gamma(\omega))\, d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$+ (\alpha(\omega) + \beta(\omega))\, d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)))$$

i.e. $(1 - \alpha(\omega) - \beta(\omega))\, d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)))$

$$\leq (\alpha(\omega) + \beta(\omega) + \gamma(\omega))d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

$$d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega))) \leq kd(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega)))$$

where

$k = \dfrac{\alpha(\omega)+\beta(\omega)+\gamma(\omega)}{1-\alpha(\omega)-\beta(\omega)} < 1.$

Similarly,

$$d(T(\omega, \xi_{2n+2}(\omega)), T(\omega, \xi_{2n+3}(\omega))) \le kd(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega)) \le k^2 d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega))$$

In general,

$$d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega))) \le k^{2n} d(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))).$$

Furthermore m > n,

$$\begin{aligned} d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2m}(\omega)) &\le d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2n+1}(\omega))) \\ &\quad + d(T(\omega, \xi_{2n+1}(\omega)), T(\omega, \xi_{2n+2}(\omega))) + \ldots \\ &\quad + d(T(\omega, \xi_{2m-1}(\omega)), T(\omega, \xi_{2m}(\omega))) \\ &\le k^{2n} d(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \\ &\quad + k^{2n+1} d(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) + \ldots \\ &\quad + k^{2m-1} d(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \end{aligned}$$

i.e. $d(T(\omega, \xi_{2n}(\omega)), T(\omega, \xi_{2m}(\omega))) \le \dfrac{k^{2n}}{(1-k)} d(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega))) \to 0$ as n, m $\to \infty$.

Thus $\{T(\omega, \xi_{2n}(\omega))\}$ and $\{T(\omega, \xi_{2m}(\omega))\}$ are Cauchy sequence in CB(X), therefore there exists $A(\omega) \in CB(X)$ such that $\{T(\omega, \xi_{2n}(\omega))\} \to A(\omega)$ for some $\omega \in \Omega$.

It further implies that $\{T(\omega, \xi_{2n+1}(\omega))\}$, $\{S(\omega, \xi_{2n}(\omega))\}$ and $\{Q(\omega, \xi_{2n+1}(\omega))\}$ converges to $A(\omega)$ for each $\omega \in \Omega$.

Let $\xi : \Omega \to X$ be a measurable mapping such that for each $\omega \in \Omega$, $\xi(\omega) \in A(\omega)$.

Thus, we have

$T(\omega, \xi_{2n+1}(\omega)) \to A(\omega)$, $S(\omega, \xi_{2n}(\omega)) \to A(\omega)$ and

$Q(\omega, \xi_{2n+1}(\omega)) \to A(\omega)$ as $n \to \infty$.

Now, suppose that T is continuous random multivalued operator, then

$T(\omega, T(\omega, \xi_{2n+1}(\omega))) \to T(\omega, A(\omega))$, $T(\omega, S(\omega, \xi_{2n}(\omega))) \to T(\omega, A(\omega))$ for every $\omega \in \Omega$.

Since pair (S, T) and (Q, T) are compatible random operator,

Since pair (S, T) and (Q, T) are compatible random operator, then for each $\omega \in \Omega$, we have

$T(\omega, T(\omega, \xi_{2n+1}(\omega))) \to T(\omega, A(\omega))$, $S(\omega, T(\omega, \xi_{2n+1}(\omega))) \to T(\omega, A(\omega))$

and $Q(\omega, T(\omega, \xi_{2n+1}(\omega))) \to T(\omega, A(\omega))$.

Consider for each $\omega \in \Omega$

$$\begin{aligned} &H(S(\omega, T(\omega, \xi_{2n}(\omega))), Q(\omega, \xi_{2n+1}(\omega))) \\ &\le \alpha(\omega)\frac{[d(T(\omega,T(\omega,\xi_{2n}(\omega))),S(\omega,T(\omega,\xi_{2n}(\omega)))]^3 + [d(T(\omega,\xi_{2n+1}(\omega))),Q(\omega,\xi_{2n+1}(\omega)))]^3}{[d(T(\omega,T(\omega,\xi_{2n}(\omega))),S(\omega,T(\omega,\xi_{2n}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega))),Q(\omega,\xi_{2n+1}(\omega)))]^2} \\ &+\beta(\omega)\frac{[d(T(\omega,T(\omega,\xi_{2n}(\omega))),S(\omega,T(\omega,\xi_{2n}(\omega)))]^2 + [d(T(\omega,\xi_{2n+1}(\omega))),Q(\omega,\xi_{2n+1}(\omega)))]^2}{[d(T(\omega,T(\omega,\xi_{2n}(\omega))),S(\omega,T(\omega,\xi_{2n}(\omega)))] + [d(T(\omega,\xi_{2n+1}(\omega))),Q(\omega,\xi_{2n+1}(\omega)))]} \\ &+ \gamma(\omega) d(T(\omega, T(\omega,\xi_{2n}(\omega))), T(\omega, \xi_{2n+1}(\omega))). \end{aligned}$$

On taking limit $n \to \infty$ both sides, we get

$d(T(\omega, A(\omega)), A(\omega))$

$$\leq \alpha(\omega)\frac{\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^3+\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^3}{\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^2+\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^2}$$

$$+\beta(\omega)\frac{\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^2+\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]^2}{\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]+\left[d(T(\omega,A(\omega)),T(\omega,A(\omega)))\right]}$$

$+ \gamma(\omega)\ d(T(\omega, A(\omega)), A(\omega))$

$(1-\gamma(\omega))\ d(T(\omega, A(\omega)), A(\omega)) \leq 0$

$d(T(\omega, A(\omega)), A(\omega)) = 0$

i.e., $T(\omega, A(\omega)) = A(\omega)$ for each $\omega \in \Omega$.

But $\xi(\omega) \in A(\omega)$.

Thus $\xi(\omega) \in T(\omega, \xi(\omega))$.

Now, for any $\omega \in \Omega$

$H(S(\omega, A(\omega)), Q(\omega, \xi_{2n+1}(\omega)))$

$$\leq \alpha(\omega)\frac{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^3+\left[d(T(\omega,\xi_{2n+1}(\omega)),Q(\omega,\xi_{2n+1}(\omega)))\right]^3}{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^2+\left[d(T(\omega,\xi_{2n+1}(\omega)),Q(\omega,\xi_{2n+1}(\omega)))\right]^2}$$

$$+\beta(\omega)\frac{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^2+\left[d(T(\omega,\xi_{2n+1}(\omega)),Q(\omega,\xi_{2n+1}(\omega)))\right]^2}{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]+\left[d(T(\omega,\xi_{2n+1}(\omega)),Q(\omega,\xi_{2n+1}(\omega)))\right]}$$

$+ \gamma(\omega)\ d(T(\omega, A(\omega)), T(\omega,\xi_{2n+1}(\omega)))$.

Taking limit $\rightarrow \infty$, we get

$d(S(\omega, A(\omega)), A(\omega))$

$$\leq \alpha(\omega)\frac{\left[d(A(\omega),S(\omega,A(\omega)))\right]^3+\left[d(A(\omega),A(\omega))\right]^3}{\left[d(A(\omega),S(\omega,A(\omega)))\right]^2+\left[d(A(\omega),A(\omega))\right]^2}$$

$$+\beta(\omega)\frac{\left[d(A(\omega),S(\omega,A(\omega)))\right]^2+\left[d(A(\omega),A(\omega))\right]^2}{\left[d(A(\omega),S(\omega,A(\omega)))\right]+\left[d(A(\omega),A(\omega))\right]}$$

$+ \gamma(\omega)\ d(A(\omega), A(\omega))$

$d(S(\omega, A(\omega)), A(\omega)) \leq \alpha(\omega)d(A(\omega), S(\omega, A(\omega))) + \beta(\omega)\ d(A(\omega), S(\omega, A(\omega)))$

$(1 - \alpha(\omega) - \beta(\omega))\ d(S(\omega, A(\omega)), A(\omega)) \leq 0$

i.e. $d(S(\omega, A(\omega)), A(\omega)) \leq 0$

$S(\omega, A(\omega)) = A(\omega)$ for each $\omega \in \Omega$.

But $\xi(\omega) \in A(\omega)$.

Thus, $\xi(\omega) \in S(\omega, A(\omega))$.

Finally,

$H(S(\omega, T(\omega, A(\omega)), Q(\omega, A(\omega)))$

$$\leq \alpha(\omega)\frac{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^3+\left[d(T(\omega,A(\omega)),Q(\omega,A(\omega)))\right]^3}{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^2+\left[d(T(\omega,A(\omega)),Q(\omega,A(\omega)))\right]^2}$$

$$+\beta(\omega)\frac{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]^2+\left[d(T(\omega,A(\omega)),Q(\omega,A(\omega)))\right]^2}{\left[d(T(\omega,A(\omega)),S(\omega,A(\omega)))\right]+\left[d(T(\omega,A(\omega)),Q(\omega,A(\omega)))\right]}$$

$+ \gamma(\omega)\ d(T(\omega, A(\omega)), T(\omega, A(\omega)))$

$d(A(\omega), Q(\omega, A(\omega)))$

$$\leq \alpha(\omega)\frac{\left[d(A(\omega),A(\omega))\right]^3+\left[d(A(\omega),Q(\omega,A(\omega)))\right]^3}{\left[d(A(\omega),A(\omega))\right]^2+\left[d(A(\omega),Q(\omega,A(\omega)))\right]^2}$$

$$+\beta(\omega)\frac{[d(A(\omega),A(\omega))]^2+[d(A(\omega),Q(\omega,A(\omega)))]^2}{[d(A(\omega),A(\omega))]+[d(A(\omega),Q(\omega,A(\omega)))]}$$

$+ \gamma(\omega)\ d(A(\omega), A(\omega))$

$d(A(\omega), Q(\omega, A(\omega))) \le \alpha(\omega)d(A(\omega), Q(\omega, A(\omega))) + \beta(\omega)\ d(A(\omega), Q(\omega, A(\omega)))$

$(1 - \alpha(\omega) - \beta(\omega))\ d(A(\omega), Q(\omega, A(\omega))) \le 0$

i.e. $d(A(\omega), Q(\omega, A(\omega))) \le 0$

$Q(\omega, A(\omega)) = A(\omega)$ for each $\omega \in \Omega$.

But $\xi(\omega) \in A(\omega)$.

Thus, $\xi(\omega) \in Q(\omega, A(\omega))$.

Hence, $\xi(\omega)$ is a random fixed point of random multivalued operator S, Q and T.

Uniqueness :

To prove uniqueness of common random fixed point of random multivalued operator.

Let $\xi_1, \xi_2 : \Omega \to X$ be two common random fixed point of random multivalued operators S, Q and T such that $\xi_1(\omega) = \xi_2(\omega)$ for each $\omega \in \Omega$.

Consider for each $\omega \in \Omega$

$d(\xi_1(\omega), \xi_2(\omega)) \le H(S(\omega, \xi_1(\omega)), Q(\omega, \xi_2(\omega)))$

$$\le \alpha(\omega)\frac{[d(T(\omega,\xi_1(\omega)),S(\omega,\xi_1(\omega)))]^3+[d(T(\omega,\xi_2(\omega)),Q(\omega,\xi_2(\omega)))]^3}{[d(T(\omega,\xi_1(\omega)),S(\omega,\xi_1(\omega)))]^2+[d(T(\omega,\xi_2(\omega)),Q(\omega,\xi_2(\omega)))]^2}$$

$$+\beta(\omega)\frac{[d(T(\omega,\xi_1(\omega)),S(\omega,\xi_1(\omega)))]^2+[d(T(\omega,\xi_2(\omega)),Q(\omega,\xi_2(\omega)))]^2}{[d(T(\omega,\xi_1(\omega)),S(\omega,\xi_1(\omega)))]+[d(T(\omega,\xi_2(\omega)),Q(\omega,\xi_2(\omega)))]}$$

$+ \gamma(\omega)d(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))$

i.e. $d(\xi_1(\omega), \xi_2(\omega)) \le \gamma(\omega)d(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))$

$(1- \gamma(\omega))d(\xi_1(\omega), \xi_2(\omega))\le 0$

$d(\xi_1(\omega), \xi_2(\omega))\le 0.$

Thus, $\xi_1(\omega) = \xi_2(\omega)$ for each $\omega \in \Omega$

which is a contradiction, so the result follows.

Corollary. Let X be a Polish space and let (S, P) and (T, Q) be two pairs of compatible random multivalued operators from $\Omega \times X \to CB(X)$ with $S(\omega, X) \subset Q(\omega, X)$ and $T(\omega, X) \subset P(\omega, X)$ for each $\omega \in \Omega$ and

$\omega \in \Omega$ and

$H(S(\omega, x), T(\omega, y))$

$$\le \alpha(\omega)\frac{[d(P(\omega,x),S(\omega,x))]^3+[d(Q(\omega,y),T(\omega,y))]^3}{[d(P(\omega,x),S(\omega,x))]^2+[d(Q(\omega,y),T(\omega,y))]^2}$$

$$+\beta(\omega)\frac{[d(P(\omega,x),S(\omega,x))]^2+[d(Q(\omega,y),T(\omega,y))]^2}{[d(P(\omega,x),S(\omega,x))]+[d(Q(\omega,y),T(\omega,y))]}$$

$+ \gamma(\omega)d(P(\omega, x), Q(\omega, y))$

for each $x, y \in X$ and $\omega \in \Omega$, where $\alpha, \beta, \gamma : \Omega \to (0, 1)$ are measurable mappings such that $\alpha(\omega) + \beta(\omega) + \gamma(\omega) < 1$. If one of the random multivalued operators P, Q, T or S is continuous then P, Q, S and T have unique common random fixed point (where H represents Hausdorff metric on CB(X) induced by metric d).

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